A Practical Analytic Model for the Radiosity of Translucent Scenes

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Rendering Translucent Objects

Monte-Carlo
[Jensen et al. 1999; Pharr and Hanrahan 2000]

Analytic model
[Jensen et al. 2001; Donner and Jensen 2007]

Radiosity based method
[Lensch et al. 2002; Carr et al. 2003]

Precomputed Radiance Transfer
[Wang et al. 2005; Wang et al. 2008]
Optical Interactions between Translucent Objects
Light Transport Pathways

Subsurface scattering: $S$

Inter-reflection: $F$
Analytic Model

First bounce:

\[
\begin{bmatrix}
S_{11} & \cdots & S_{1n} \\
\vdots & \ddots & \vdots \\
S_{n1} & \cdots & S_{nn}
\end{bmatrix}
\begin{bmatrix}
L_1 \\
\vdots \\
L_n
\end{bmatrix}
\]

Second bounce:

\[
\begin{bmatrix}
S_{11} & \cdots & S_{1n} \\
\vdots & \ddots & \vdots \\
S_{n1} & \cdots & S_{nn}
\end{bmatrix}
\begin{bmatrix}
F_{11} & \cdots & F_{1n} \\
\vdots & \ddots & \vdots \\
F_{n1} & \cdots & F_{nn}
\end{bmatrix}
\begin{bmatrix}
S_{11} & \cdots & S_{1n} \\
\vdots & \ddots & \vdots \\
S_{n1} & \cdots & S_{nn}
\end{bmatrix}
\begin{bmatrix}
L_1 \\
\vdots \\
L_n
\end{bmatrix}
\]

N-th bounce: \((SF)^{N-1}SL\)

All bounces: \(B = \sum_{N=1}^{\infty} (SF)^{N-1}SL = (I - SF)^{-1}SL\)
Comparing with Traditional Radiosity

Radiosity: \[ B = (I - F)^{-1} L \]
\[ S = I \]

Analytic model: \[ B = (I - SF)^{-1} SL \]

Pros:
- Realtime changing viewpoints
- Fast relighting and changing materials

Cons:
- Precomputating \( S \) and \( F \): 10+ minutes
- Memory consumption: 1.5 GB

[Willmott et al. 1999]
Subsurface Scattering Matrix

Subsurface scattering matrix:

\[
S = \begin{bmatrix}
S_{11} & \cdots & S_{1n} \\
\vdots & \ddots & \vdots \\
S_{n1} & \cdots & S_{nn}
\end{bmatrix}
\]

Subsurface scattering factor: [Carr et al. 2003]

\[
S_{ji} = \frac{1}{A_i} \int_{A_i} \int_{A_j} s(i, j) \, dA_j \, dA_i
\]

BSSRDF \( s(i, \omega_i; j, \omega_j) \)
Homogeneous Materials

Subsurface scattering factor:

\[ S_{ji} = \frac{1}{A_i} \int_{A_i} \int_{A_j} s(r) \, dA_j \, dA_i \]

Computing distance \( r \):
Changing Translucencies

Subsurface scattering factor:

\[
S_{ji} = \frac{1}{A_i} \int_{A_i} \int_{A_j} s(r) \, dA_j \, dA_i \approx \frac{1}{A_i} \sum_{\Delta A_i} \sum_{\Delta A_j} s(r) \Delta A_j \Delta A_i
\]

\[
\begin{bmatrix}
S_{11} & \cdots & S_{1n} \\
\vdots & \ddots & \vdots \\
S_{n1} & \cdots & S_{nn}
\end{bmatrix}
\]

Profile
Changing Translucencies

Performance: 2fps
Heterogeneous materials

Subsurface scattering factor:

\[ S_{ji} = \frac{1}{A_i} \int_{A_i} \int_{A_j} s(i, j) dA_j dA_i \]

4D BSSRDF Function:

- **Measurement:** DISCO [Goesele et al. 2004]
- **Simulation:** [wang et al. 2008], [Arbree et al. 2011]
Relighting

Analytic model:

\[ B = (I - SF)^{-1}SL = TL \]

Iterative solver: 5fps
Relighting only (>20 fps)

Translucent wax bowl
Diffuse bowl
Validation using Material Capture

Reflection profile

Transmission profile

Reflection distribution

Transmission distribution
Validation Results

Photo

Rendering
Rose

<table>
<thead>
<tr>
<th># of polys</th>
<th>Pre-computation (min)</th>
<th>Relighting</th>
<th>Change Trans (fps)</th>
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<tbody>
<tr>
<td>72k</td>
<td>15 + 20</td>
<td>3</td>
<td>0.5</td>
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</table>

First bounce removed

\[ B - SL \]

Diffuse Inter-reflection

\[ \sum_{i=1}^{\infty} F^i L \]
Bunny

Environment light

Spot lights

Point lights
## Bunny

<table>
<thead>
<tr>
<th>Camera 1</th>
<th>Camera 2</th>
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</table>

<table>
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<tr>
<th># of polys</th>
<th>Pre-computation (min)</th>
<th>Relighting (fps)</th>
<th>Change Trans (fps)</th>
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<tr>
<td>73k</td>
<td>10 + 12</td>
<td>3</td>
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Horses

Our result

First bounce removed

$B \rightarrow SL$
Our result

First bounce removed $B - SL$

<table>
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<th>Pre-computation (min)</th>
<th>fps</th>
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<tr>
<td>63k</td>
<td>$(9 + 10) \times 48$</td>
<td>0.05</td>
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</table>
Future Work

• Reduce memory consumption
  – Hierarchical method ([Willmott et al. 1999; Mertens et al. 2003])
• Multiple bounces of specular reflection
• Interactive rendering of dynamic scenes with translucent objects
Radiosity in Computer Games

Coarse radiosity solution
Fine interpolated result

DICE & Geometrics
(Battle field 3)
Summary

• Analytic model combining subsurface scattering and inter-reflection
• Supports both homogeneous and heterogeneous materials
• Interactive change in viewpoint, lighting and translucency