

Homogeneous Codes for Energy-Efficient Illumination and Imaging: Supplemental Document

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1 Derivation of Homogeneous Low-Rank Factorization Formulation

The homogeneous factorization equation maximizes energy efficiency subject to all imaging constraints, by maximizing the scalar γ or, equivalently, minimizing its reciprocal γ^{-1} :

$$\begin{aligned} \min_{\gamma, t, \mathbf{M}, \mathbf{L}} \quad & \gamma^{-1} \\ \text{subject to} \quad & 0 \leq \mathbf{m}_k, \quad \|\mathbf{m}_k\|_\infty \leq 1 \\ & 0 \leq \mathbf{l}_k, \quad \|\mathbf{l}_k\|_{\dagger\sigma} \leq t_k \\ & 0 \leq t_k, \quad \sum_{k=1}^K t_k \leq T \\ & \gamma \mathbf{\Pi} = \mathbf{M} \mathbf{L}^T \end{aligned} \quad (1)$$

This is a difficult problem to solve in general, given the nonlinear nature of the equality constraint. Instead, we reformulate and then relax this problem into a simpler form.

Reformulation There is a bilinear relation between the masks \mathbf{m}_k and illumination patterns \mathbf{l}_k . Substituting the mask and illumination variables with $a_k \mathbf{l}_k$ and $a_k^{-1} \mathbf{m}_k$ satisfies the equality constraint for all scalars $a_k > 0$. Eq. (1) can be rewritten as follows:

$$\begin{aligned} \min_{\gamma, t, \mathbf{M}, \mathbf{L}} \quad & \gamma^{-1} \\ \text{subject to} \quad & 0 \leq \mathbf{m}_k, \quad a_k^{-1} \|\mathbf{m}_k\|_\infty \leq 1 \\ & 0 \leq \mathbf{l}_k, \quad a_k \|\mathbf{l}_k\|_{\dagger\sigma} \leq t_k \\ & 0 \leq t_k, \quad \sum_{k=1}^K t_k \leq T \\ & \gamma \mathbf{\Pi} = \mathbf{M} \mathbf{L}^T \end{aligned} \quad (2)$$

The formulation can be relaxed by combining the inequality constraints on the mask and illumination patterns:

$$\begin{aligned} \min_{\gamma, t, \mathbf{M}, \mathbf{L}} \quad & \gamma^{-1} \\ \text{subject to} \quad & 0 \leq \mathbf{m}_k, 0 \leq \mathbf{l}_k \\ & \|\mathbf{m}_k\|_\infty \|\mathbf{l}_k\|_{\dagger\sigma} \leq t_k \\ & 0 \leq t_k, \quad \sum_{k=1}^K t_k \leq T \\ & \gamma \mathbf{\Pi} = \mathbf{M} \mathbf{L}^T \end{aligned} \quad (3)$$

Similarly, we can eliminate the time variable t_k from the formulation by relaxing the inequality constraints, as follows:

$$\begin{aligned} \min_{\gamma, \mathbf{M}, \mathbf{L}} \quad & \gamma^{-1} \\ \text{subject to} \quad & 0 \leq \mathbf{m}_k, 0 \leq \mathbf{l}_k \\ & \frac{1}{T} \sum_{k=1}^K \|\mathbf{m}_k\|_\infty \|\mathbf{l}_k\|_{\dagger\sigma} \leq 1 \\ & \gamma \mathbf{\Pi} = \mathbf{M} \mathbf{L}^T \end{aligned} \quad (4)$$

By replacing the mask with $\gamma^{-1} \mathbf{m}_k$, the formulation reduces to the following form:

$$\begin{aligned} \min_{\gamma, \mathbf{M}, \mathbf{L}} \quad & \gamma^{-1} \\ \text{subject to} \quad & 0 \leq \mathbf{m}_k, 0 \leq \mathbf{l}_k \\ & \frac{1}{T} \sum_{k=1}^K \|\mathbf{m}_k\|_\infty \|\mathbf{l}_k\|_{\dagger\sigma} \leq \gamma^{-1} \\ & \mathbf{\Pi} = \mathbf{M} \mathbf{L}^T \end{aligned} \quad (5)$$

Note that the solution of Eq. (5) is closely related to that of Eq. (1). Suppose that γ' , \mathbf{m}'_k , and \mathbf{l}'_k minimize Eq. (5). The solution to Eq. (1) is then given by the following:

$$\gamma = \gamma' \quad (6)$$

$$t_k = \|\mathbf{m}'_k\|_\infty \|\mathbf{l}'_k\|_{\dagger\sigma} \quad (7)$$

$$\mathbf{m}_k = \frac{\mathbf{m}'_k}{\|\mathbf{m}'_k\|_\infty} \quad (8)$$

$$\mathbf{l}_k = t_k \frac{\mathbf{l}'_k}{\|\mathbf{l}'_k\|_{\dagger\sigma}} \quad (9)$$

Relaxation The final step is to relax the equality constraint through Lagrangian relaxation. First, we rewrite Eq. (5) equivalently in the following form:

$$\begin{aligned} \min_{\mathbf{M}, \mathbf{L}} \quad & \frac{1}{T} \sum_{k=1}^K \|\mathbf{m}_k\|_\infty \|\mathbf{l}_k\|_{\dagger\sigma} \\ \text{subject to} \quad & 0 \leq \mathbf{m}_k, 0 \leq \mathbf{l}_k \\ & \|\mathbf{\Pi} - \mathbf{M} \mathbf{L}^T\|_F^2 \leq 0 \end{aligned} \quad (10)$$

The inequality with respect to the homogeneous factorization can be brought into the objective using a Lagrange multiplier, as follows:

$$\begin{aligned} \min_{\mathbf{M}, \mathbf{L}} \quad & \|\mathbf{\Pi} - \mathbf{M} \mathbf{L}^T\|_F^2 + \lambda \sum_{k=1}^K \|\mathbf{m}_k\|_\infty \|\mathbf{l}_k\|_{\dagger\sigma} \\ \text{subject to} \quad & 0 \leq \mathbf{m}_k, 0 \leq \mathbf{l}_k \end{aligned} \quad (11)$$

for some scalar $\lambda > 0$. When λ is sufficiently small, the solution to Eq. (11) is a close approximation to the solution of the original problem.

2 Analysis Of Projector Power, Working Range and Ambient Light Blocking

Our analysis of the working range of a projector-camera system is based on simulations of expected camera intensity values and noise levels (read noise and shot noise) using the parameter values from Table 1. Given the short exposures, we did not model the effects of dark current noise. We also ignored discretization noise in our analysis. We assumed the projector source was monochromatic and that a bandpass filter placed in front of the camera would block 95% of the ambient light.

	Parameter	Value
Scene	Albedo	0.25
	Pixel Pitch	5 μ m
	Quantum Efficiency	0.7
	Filter Transmission	0.8
Camera	Focal Length	5mm
	Aperture (f-number)	1.6
	Exposure time	16.67ms
	RMS Read noise	10e ⁻
	Output Power	20 Lumens
Projector	Field of View	45°H × 30°V
	Wavelength	532nm

Table 1: Range Analysis Parameters: parameters for the camera, projector and scene used to generate range analysis plots.

We say a projector-camera system can work at a range z if binary patterns projected onto a fronto-parallel surface at depth z can be detected by the camera. If a camera pixel receives light from its corresponding projector pixel, the intensity value the camera sees will be

$$I_1(z) = \bar{I}_1(z) + \eta(\bar{I}_1(z)) \quad (12)$$

where $\bar{I}_1(z)$ is the expected camera intensity value due to ambient light and the projector; η is an intensity-dependent, zero-mean Gaussian additive noise term that contains read noise and shot noise; and $I_1(z)$ is the observed intensity at the camera. Let $s_1(z)$ be the standard deviation of the Gaussian $\eta(\bar{I}_1(z))$.

Similarly, if the pixel is off

$$I_0(z) = \bar{I}_0(z) + \eta(\bar{I}_0(z)) \quad (13)$$

where $\bar{I}_0(z)$ is the expected camera intensity value due to just the ambient light that reaches the camera sensor. Let $s_0(z)$ be the standard deviation of the Gaussian $\eta(\bar{I}_0(z))$.

Our condition for reliable decoding is that the expected difference between $I_1(z)$ and $I_0(z)$ must be at least three times the standard deviation of the difference between $I_1(z)$ and $I_0(z)$. At this threshold, a pixel can be reliably decoded as being on or off 99.8% of the time. At the maximum working range z^* :

$$\bar{I}_1(z^*) - \bar{I}_0(z^*) = 3(s_1(z^*) + s_0(z^*)) \quad (14)$$

3 Parts List

The DMD-based experiments use the setup described in [O’Toole et al. 2014]. The list of parts used to build our laser-based prototype is in Table 2. To synchronize the projector and camera we needed a VSYNC signal from the laser projector. The projector we used does not have a VSYNC output so we modified it to gain access to a VSYNC signal. The green laser output (any of pins 26, 28 and 30 on the MAX3600 laser driver IC) was tapped (the yellow wire in Figure 1). A Schmitt-triggered inverter and PLD are used to condition the laser driver output to generate a sync signal from it (see Figure 2).

References

O’TOOLE, M., MATHER, J., AND KUTULAKOS, K. N. 2014. 3D Shape and Indirect Appearance by Structured Light Transport. In *Proc. CVPR*, 3246–3253.

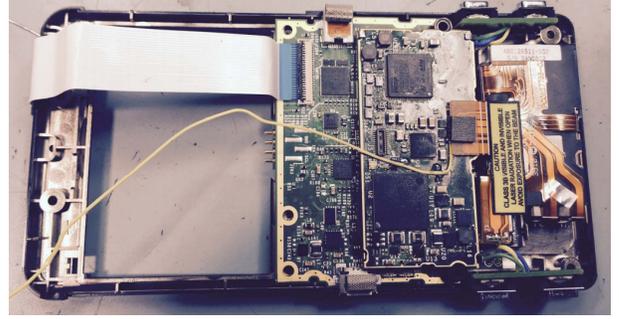


Figure 1: Tapping The Laser Signal: The VSYNC signal is generated from the green laser output of the MAX3600 laser driver IC. Any one of pins 26, 28 or 30 on the IC provides access to the laser output. A via located under the flex cable provides convenient access to the output. The yellow wire is soldered to this via.

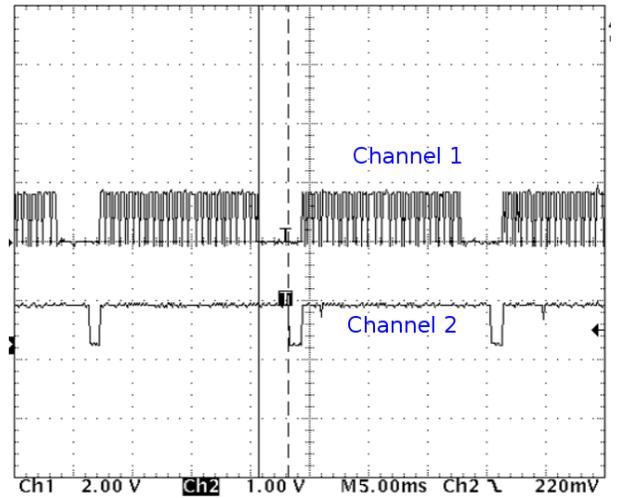


Figure 2: Generating a VSYNC from the Laser Output: The laser output (channel 1) stops at the end of a frame while the MEMS mirror returns to the home position. This indicates the start of a new frame. We use a Schmitt-triggered inverter and a PLD to condition the laser output and extract a VSYNC signal (channel 2).

Component	#	Part Number	Manufacturer
Rolling shutter camera	1	UI-3250-M-GL	IDS Imaging
Tripod adapter	1	IDS-ST-CP-USB3	IDS Imaging
Camera I/O cable	1	CV-HIR8-IO-02M	IDS Imaging
Microvision Showwx+ projector	1		Microvision
USB 3.0 A to Micro-B cable	1		
2 axis translation stage	1	DT12XZ	Thorlabs
12 inch optical rail	1	RLA1200	Thorlabs
Rail carrier	1	RC2	Thorlabs
Perpendicular rail carrier	1	RC3	Thorlabs
Right angled bracket	1	AB90A	Thorlabs
Counterbore adapter	1	SD1	Thorlabs
532nm laser line filter	1	FL53210	Thorlabs
Low distortion varifocal lens	1	CMV0411ND	Lensation
Arduino Pro Mini 328	1	DEV-11113	Sparkfun
FTDI Basic Breakout 5V	1	DEV-09716	Sparkfun
Break away headers	2	PRT-00116	Sparkfun

Table 2: Parts list for our laser-based prototype.