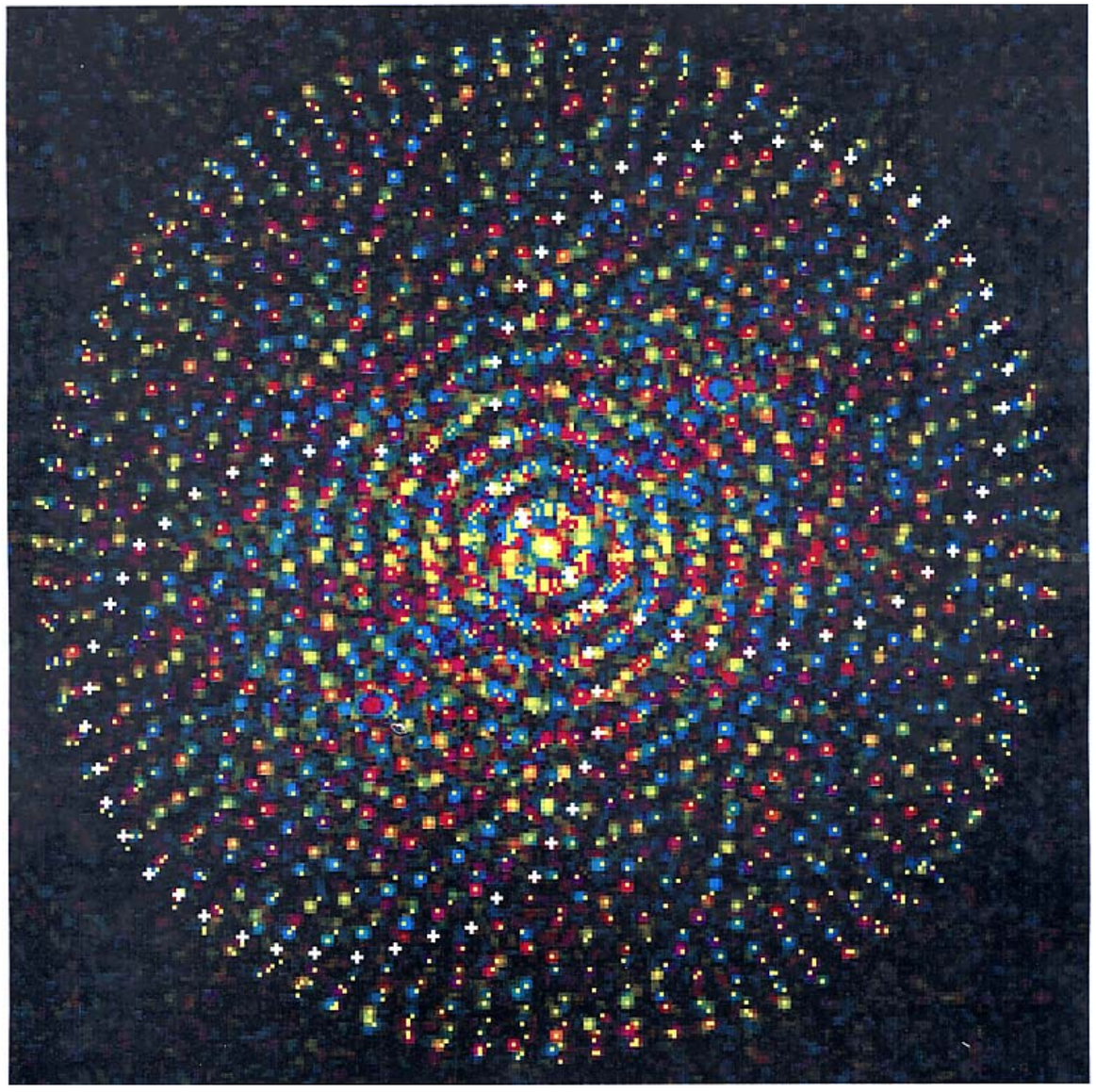
COMPUTATIONAL IMAGING



Berthold K.P. Horn

What is Computational Imaging?

- Computation inherent in image formation

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(1) Computing is getting faster and cheaper

—precision physical apparatus is not

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What is Computational Imaging?

- Computation inherent in image formation

(1) Computing is getting faster and cheaper

—precision physical apparatus is not

(2) Can’t refract or reflect some radiation

(3) Detection is at times inherently coded

Computational Imaging System

Examples of Computational Imaging:

(1) Synthetic Aperture Imaging

(2) Coded Aperture Imaging

(3) Exact Cone Beam Reconstruction

(4) Diaphanography—DiffuseTomography

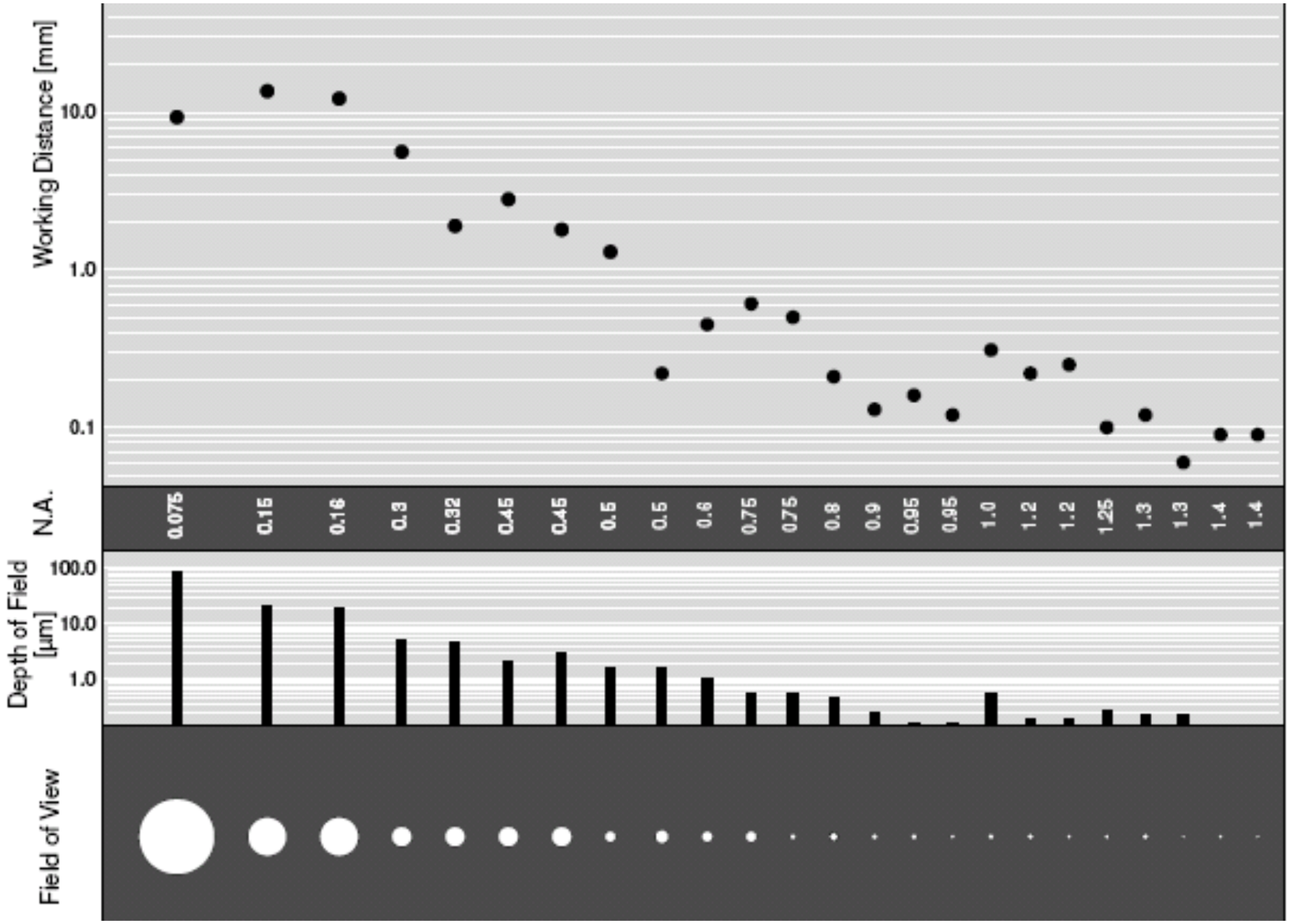
(1) SYNTHETIC APERTURE IMAGING

Traditional approach:

- Coupling of resolution, DOF, FOV to NA
- Precision imaging — “flat” illumination

with: Michael Mermelstein, Jekwan Ryu,
Stanley Hong, and Dennis Freeman

Objective Lens Parameter Coupling



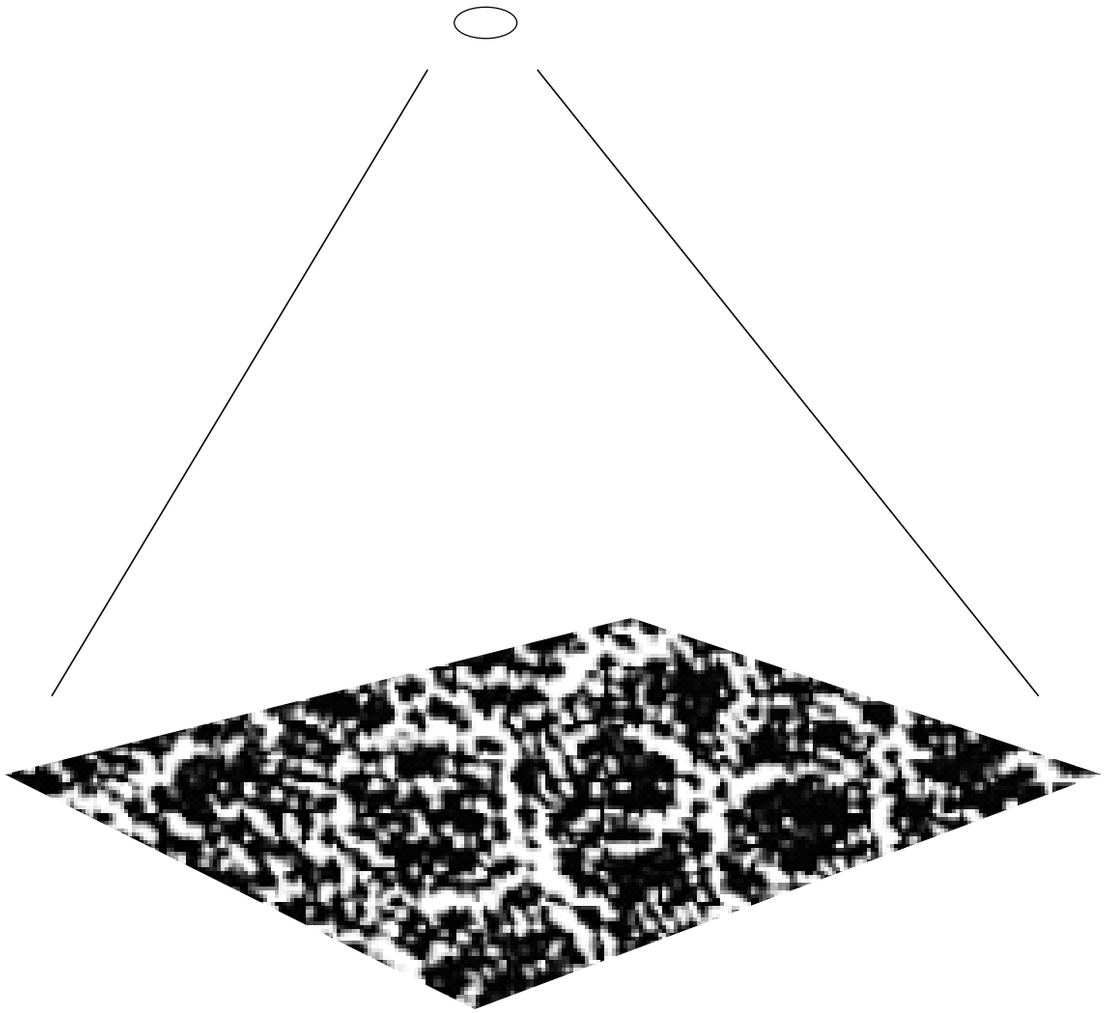
Synthetic Aperture Imaging

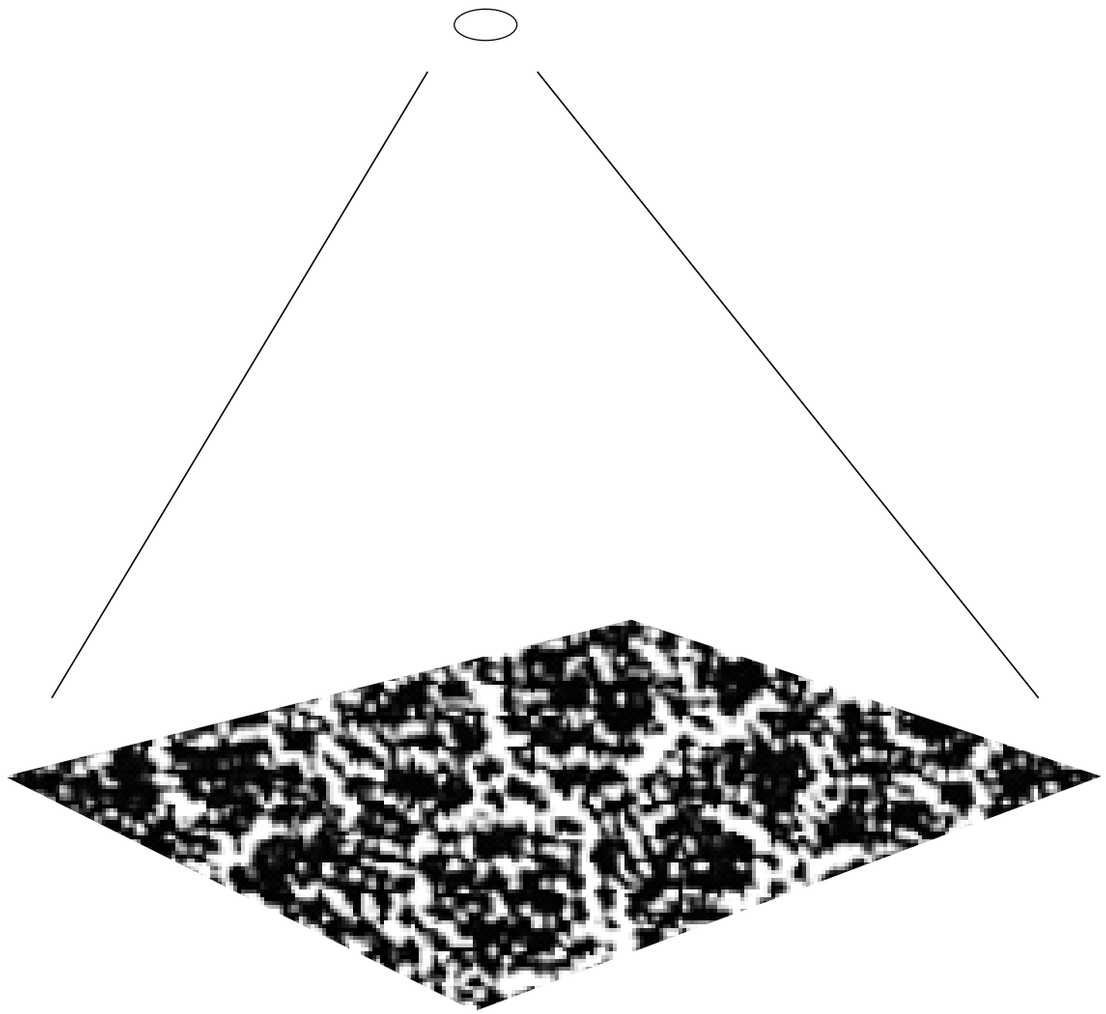
Traditional approach:

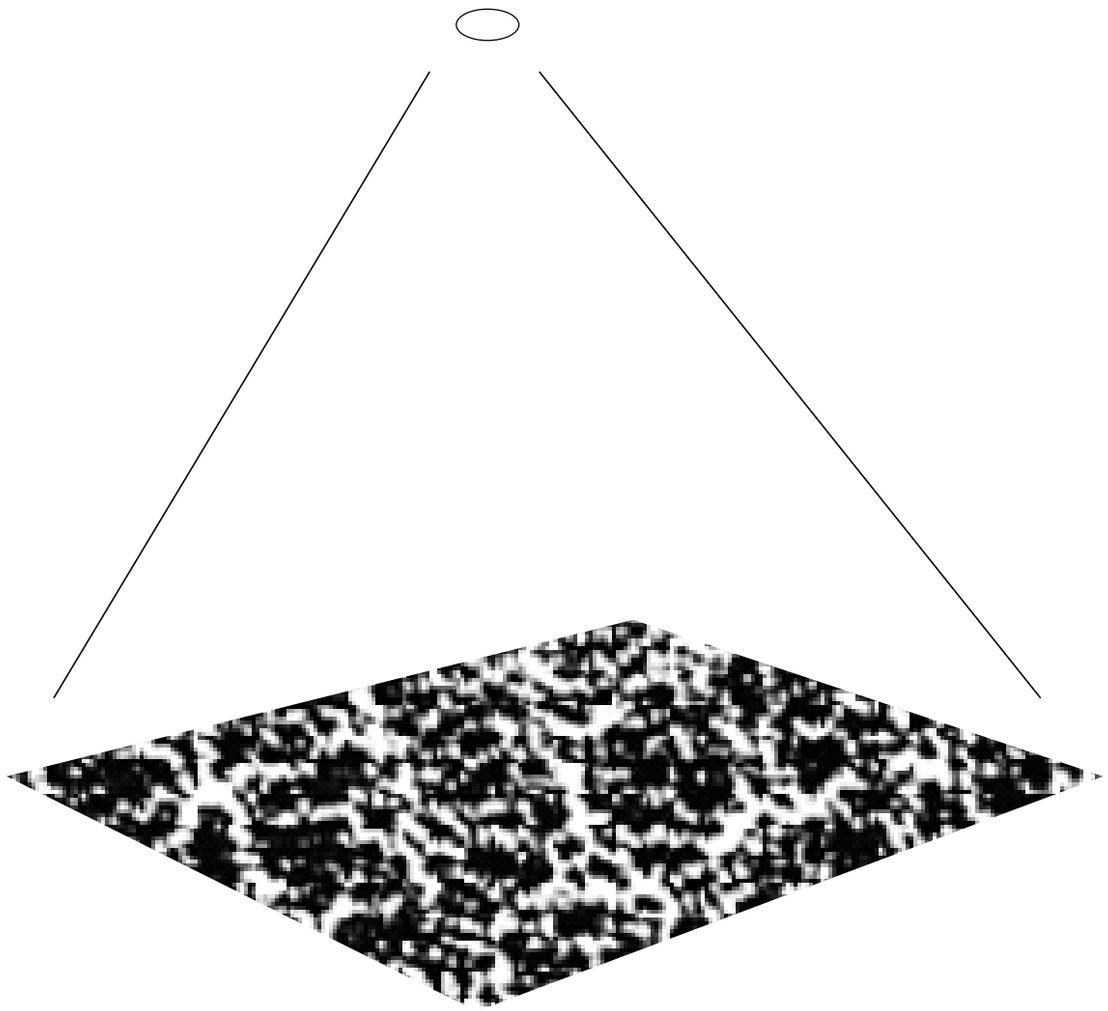
- Coupling of resolution, DOF, FOV to NA
- Precision imaging — “flat” illumination

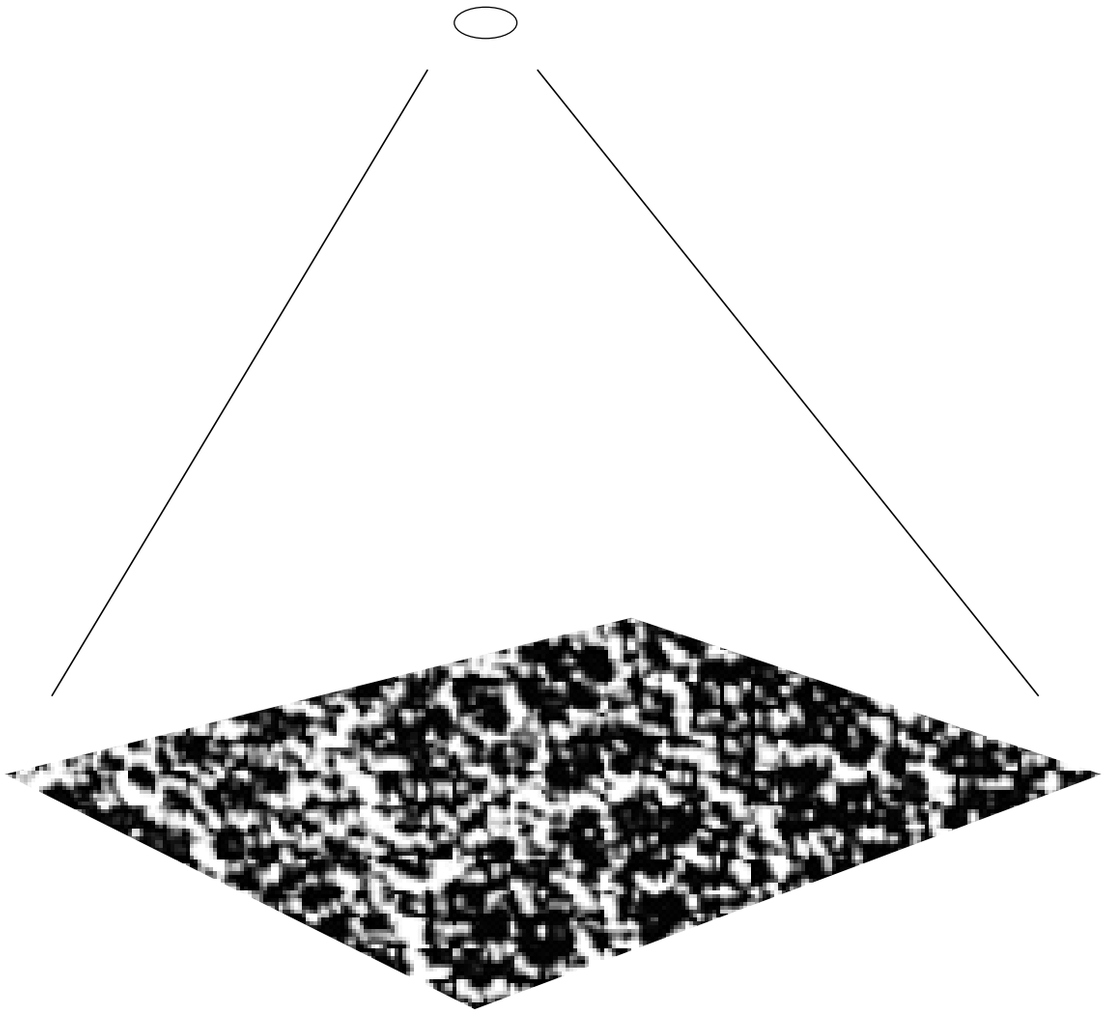
New approach:

- Precision illumination — Simple imaging
- Multiple images — Textured illumination







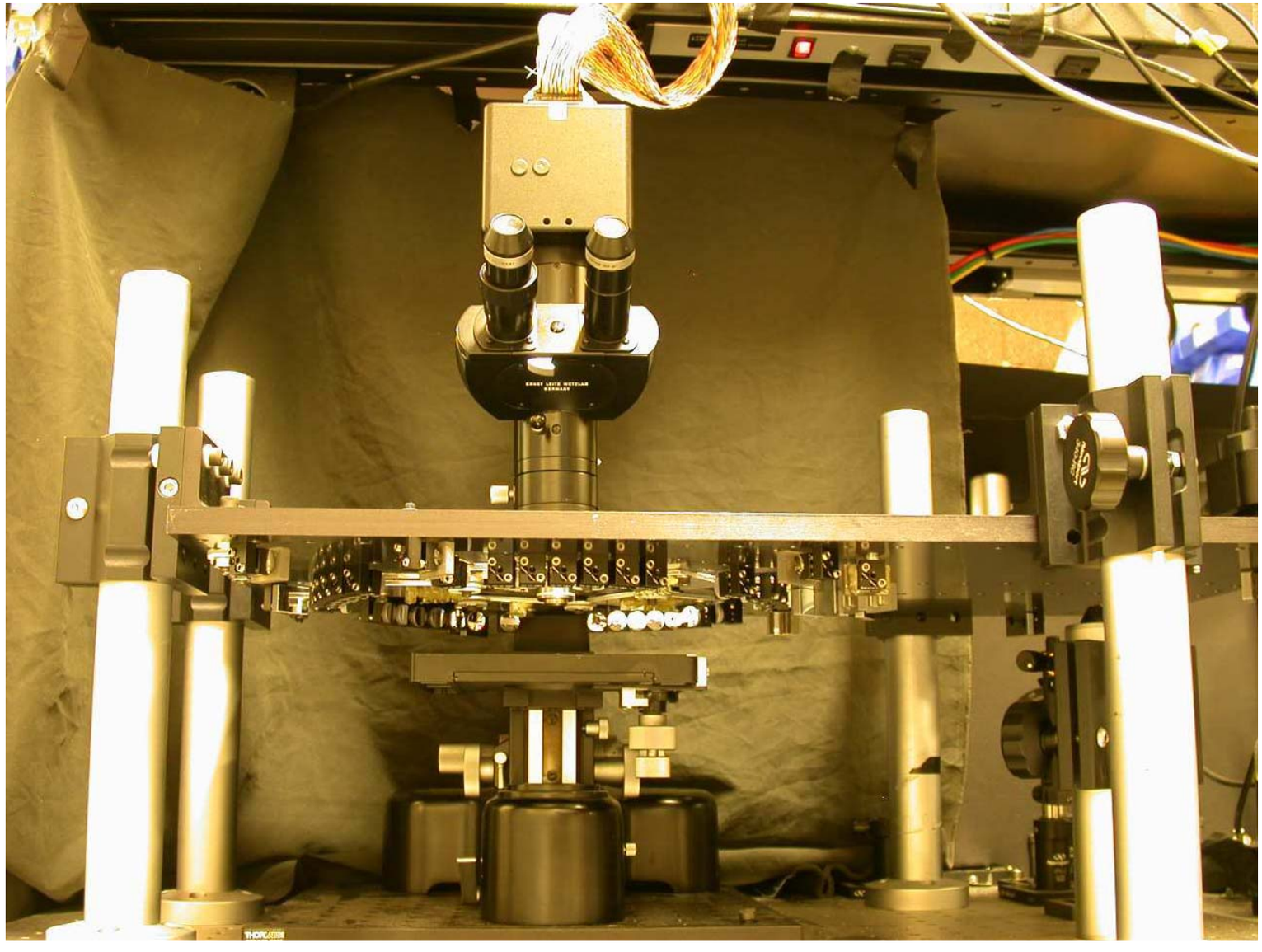


Synthetic Aperture Imaging

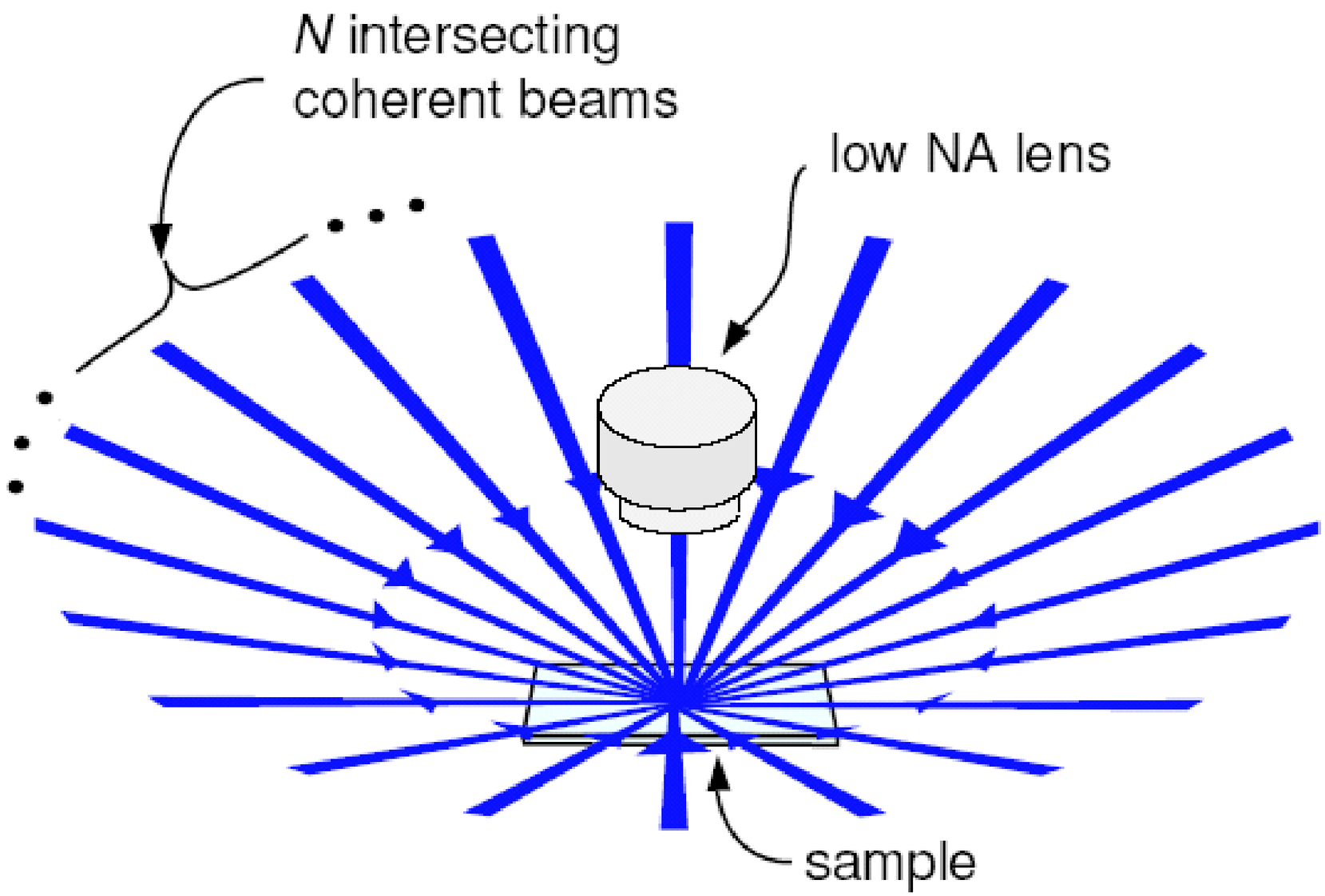
- Precision illumination — Simple imaging
- Multiple images — Textured illumination

- Image detail in response to textures
- Non-uniform samples in FT space

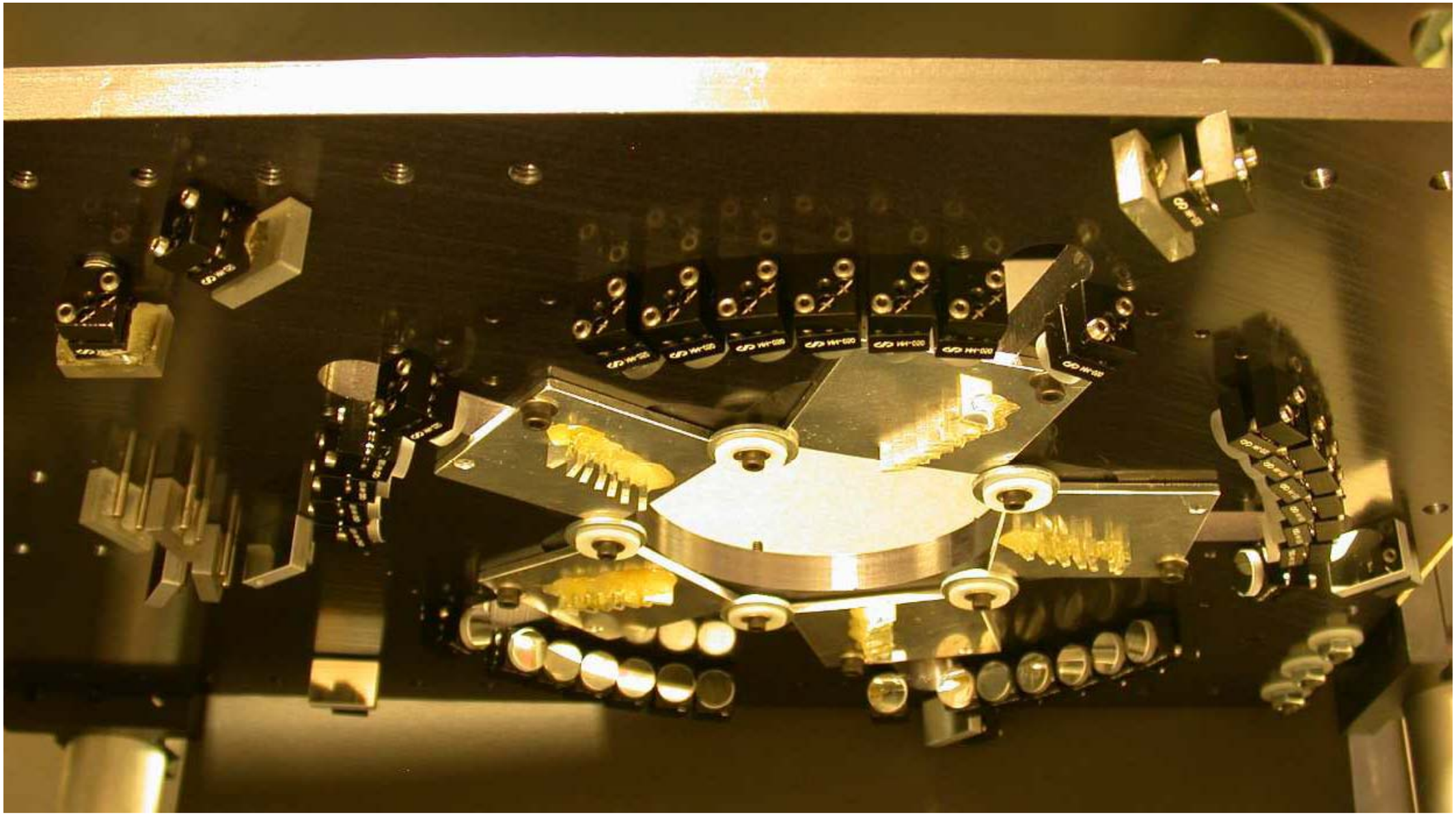
SAM M6



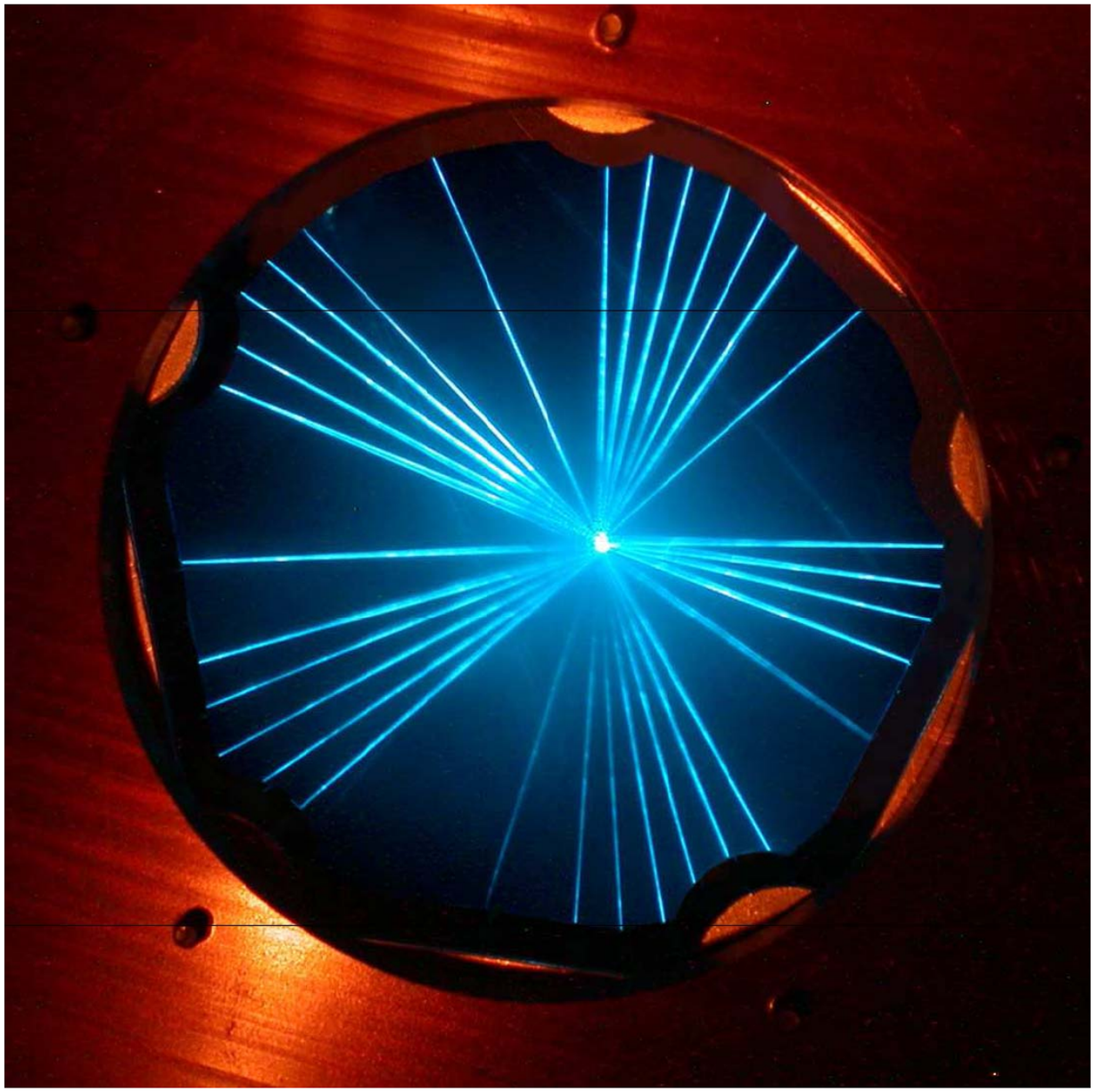
Creating Interference Pattern



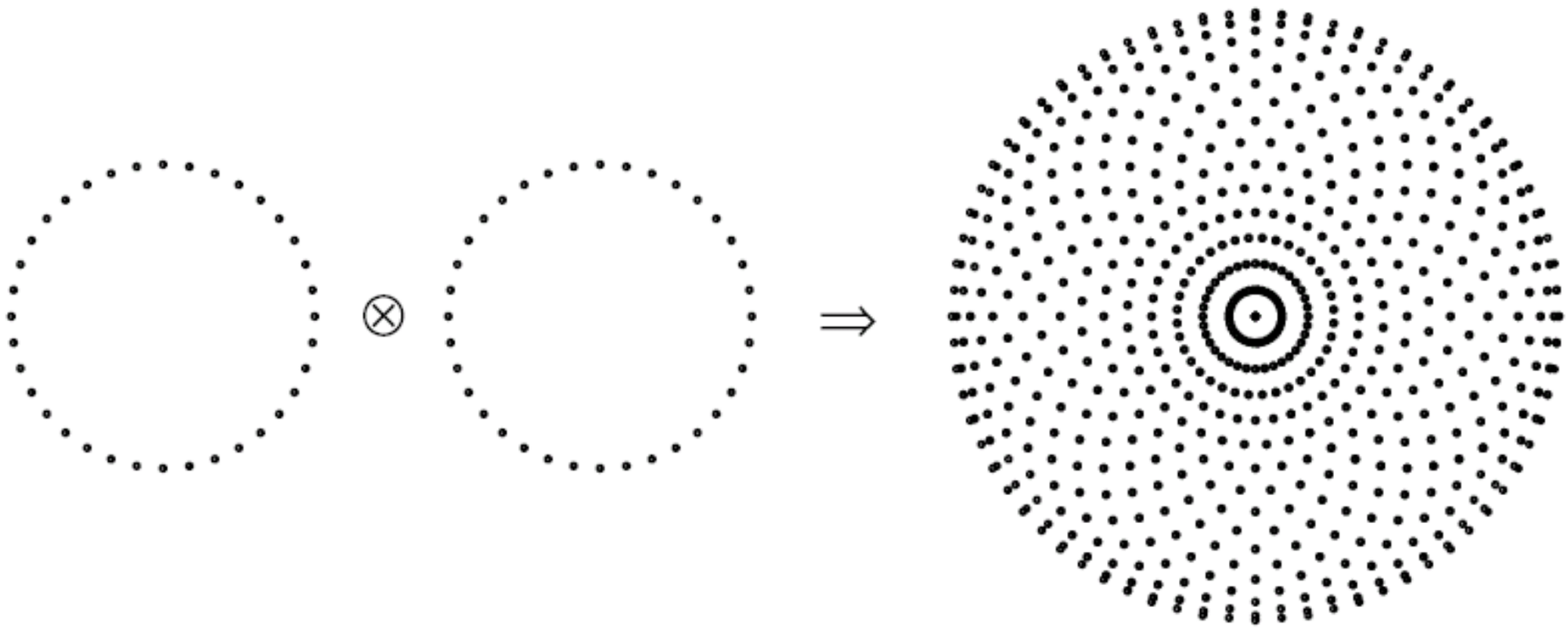
Reflective Optics M6



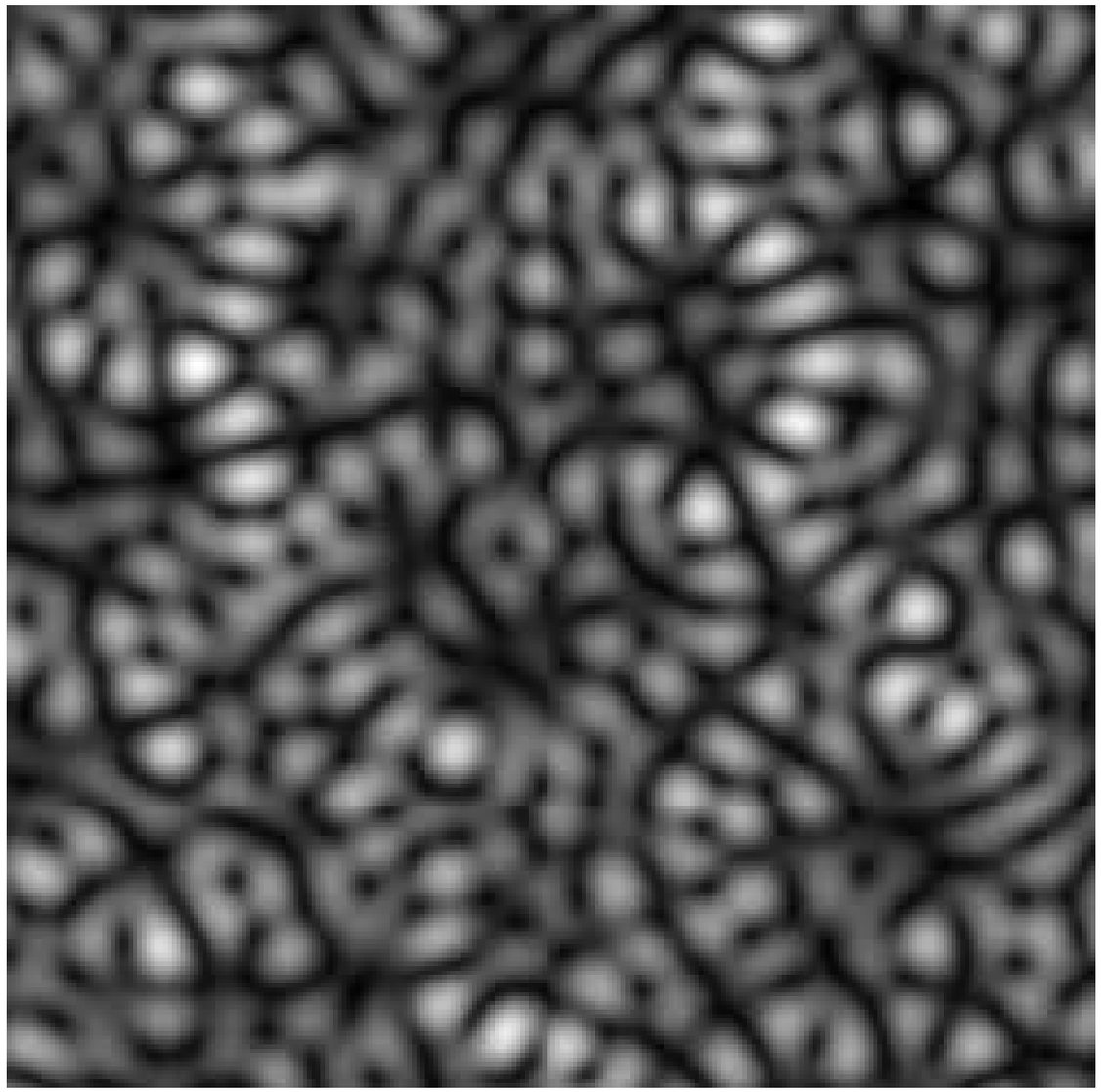
Creating Interference Pattern



Fourier Transform of Texture Pattern



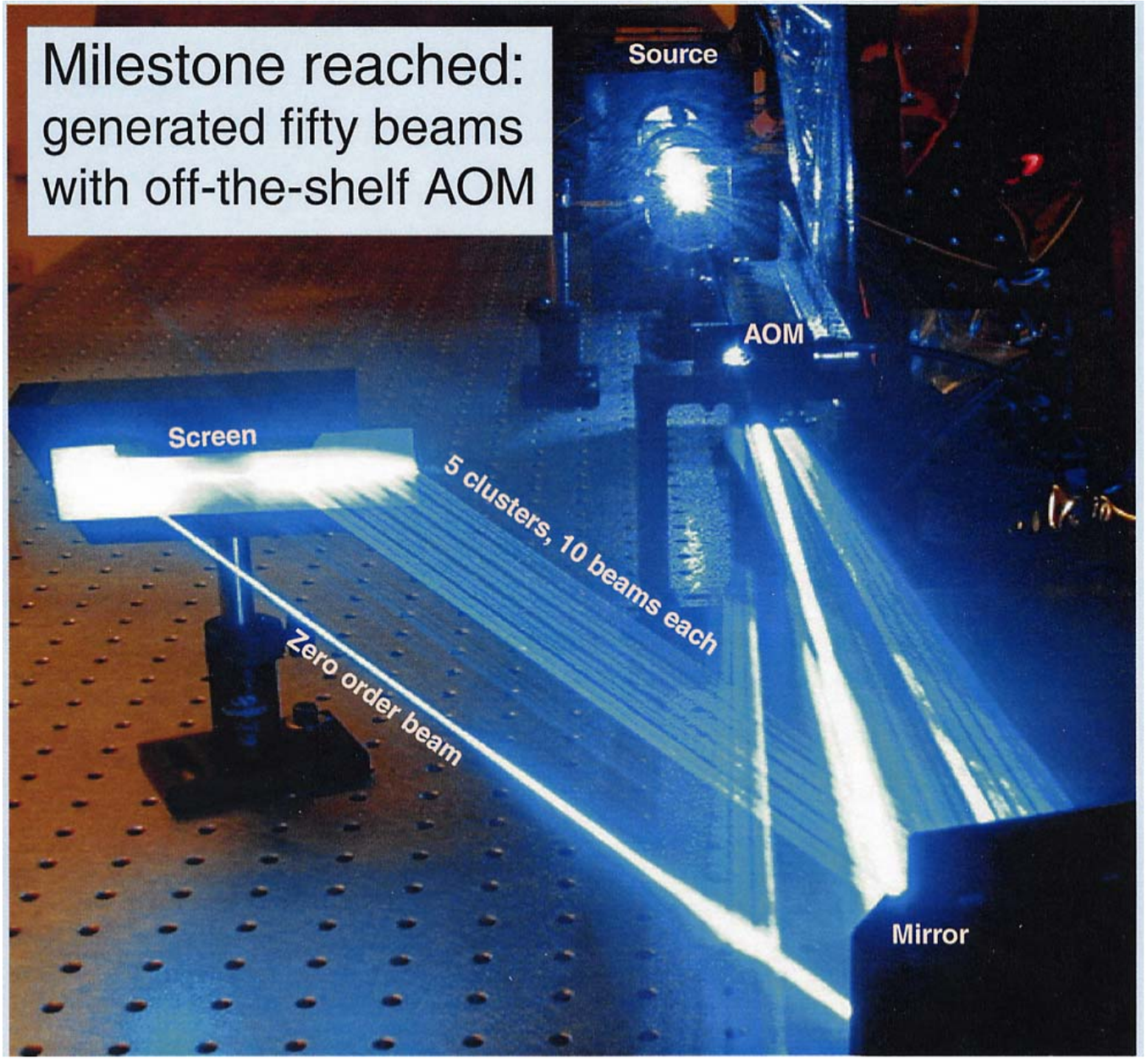
Interference Pattern Texture



Synthetic Aperture Microscopy

- Interference of many Coherent Beams
- Amplitude and Phase Control of Beams

Amplitude and Phase Control

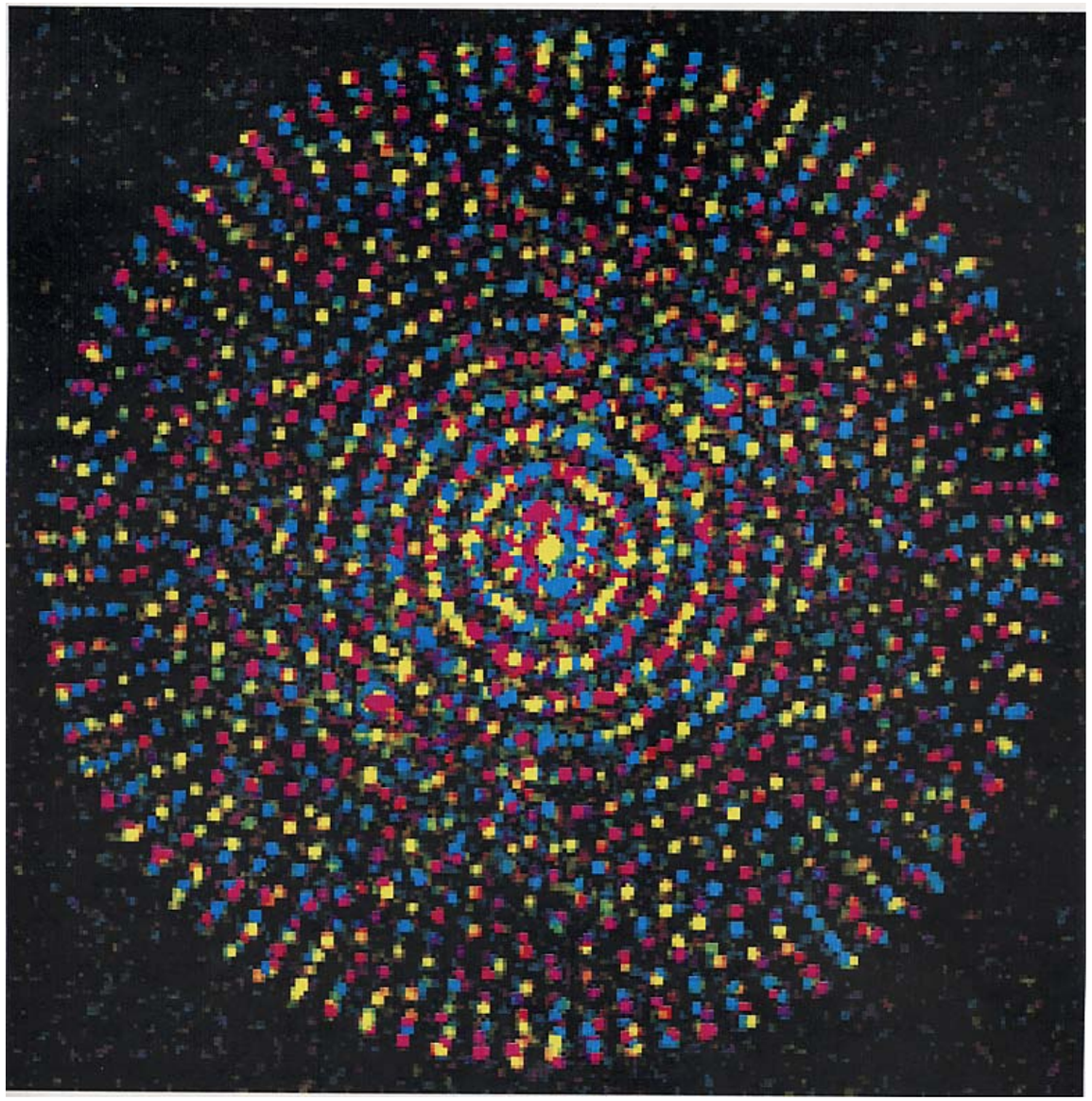


Synthetic Aperture Microscopy

- Interference of many Coherent Beams
- Amplitude and Phase Control of Beams

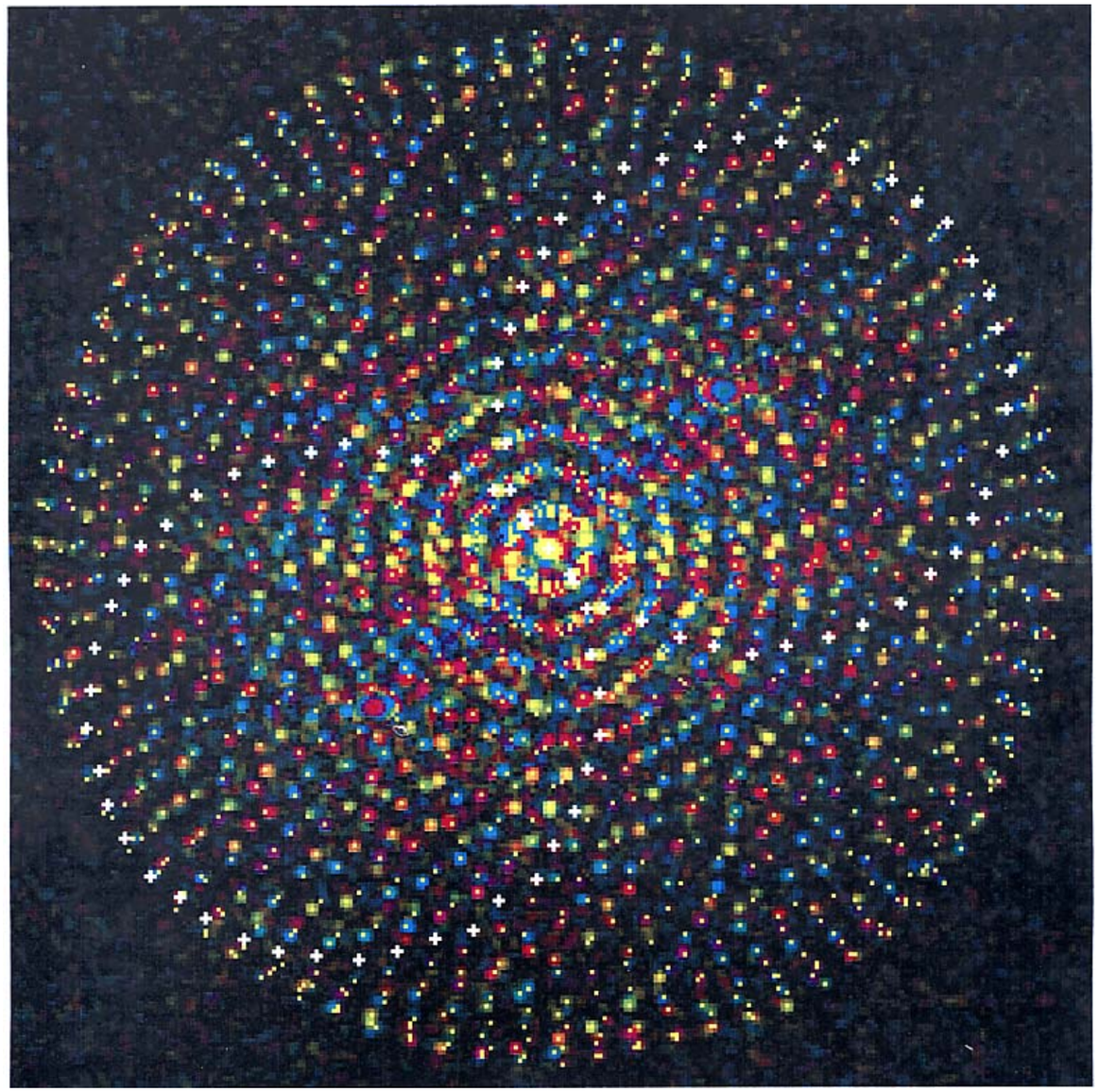
- On the fly calibration
- Non-uniform inverse FT Least Squares

Wavenumber Calibration using FT

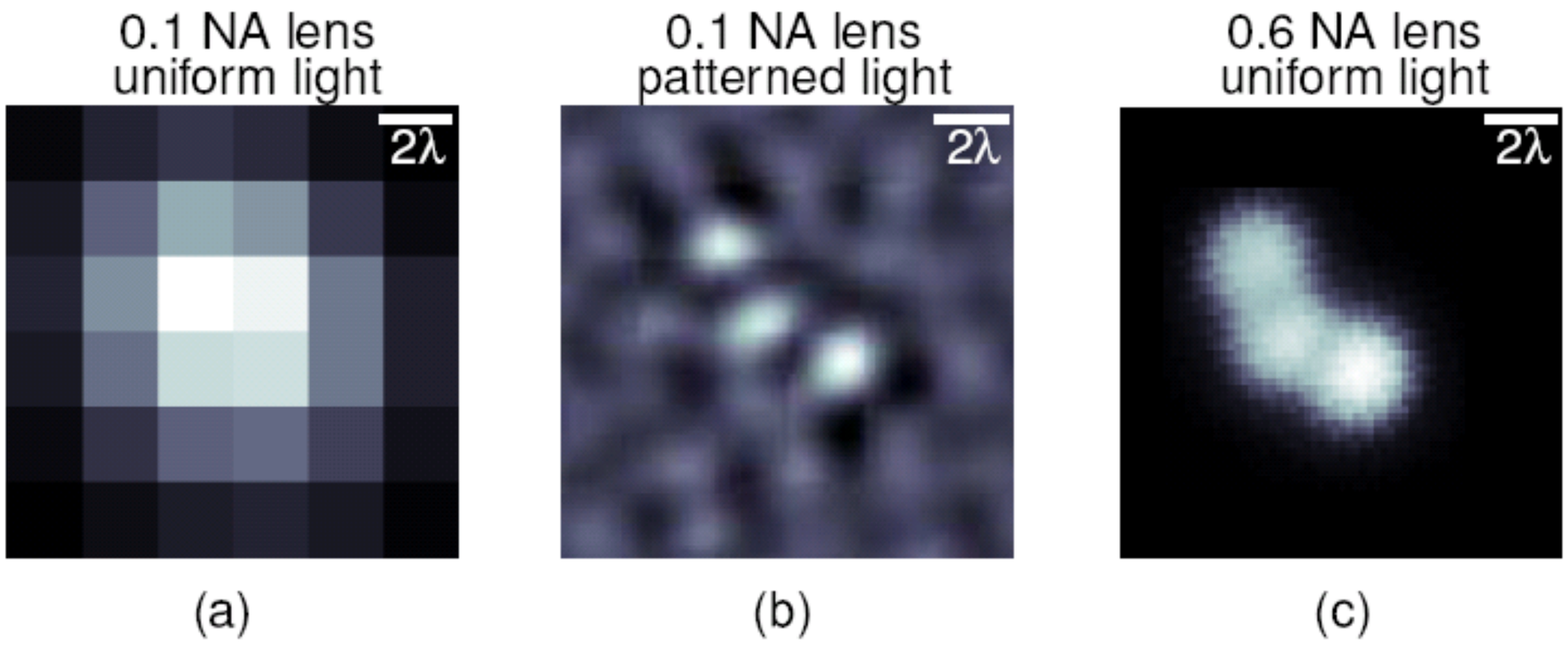


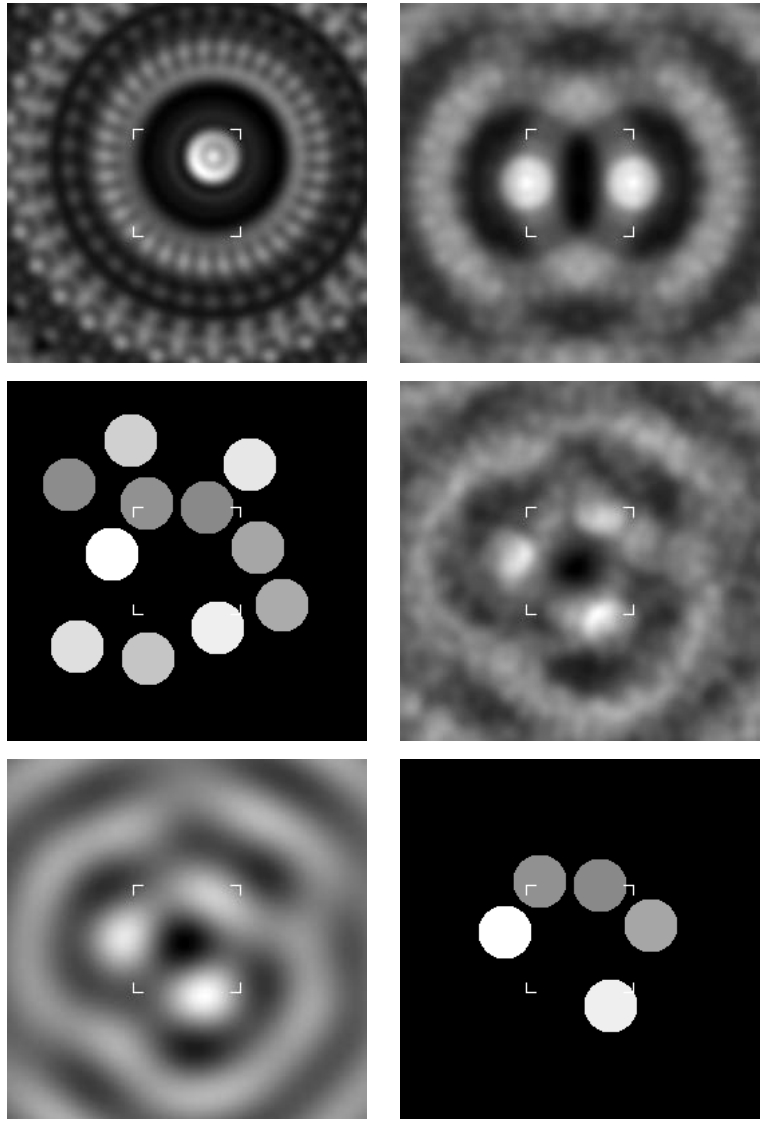
Hough Transform Calibration

Least Squares Match in FT



Polystyrene Micro Beads (1μm)





(2) CODED APERTURE IMAGING

- Can’t refract or reflect gamma rays
- Pinhole — tradeoff resolution and SNR

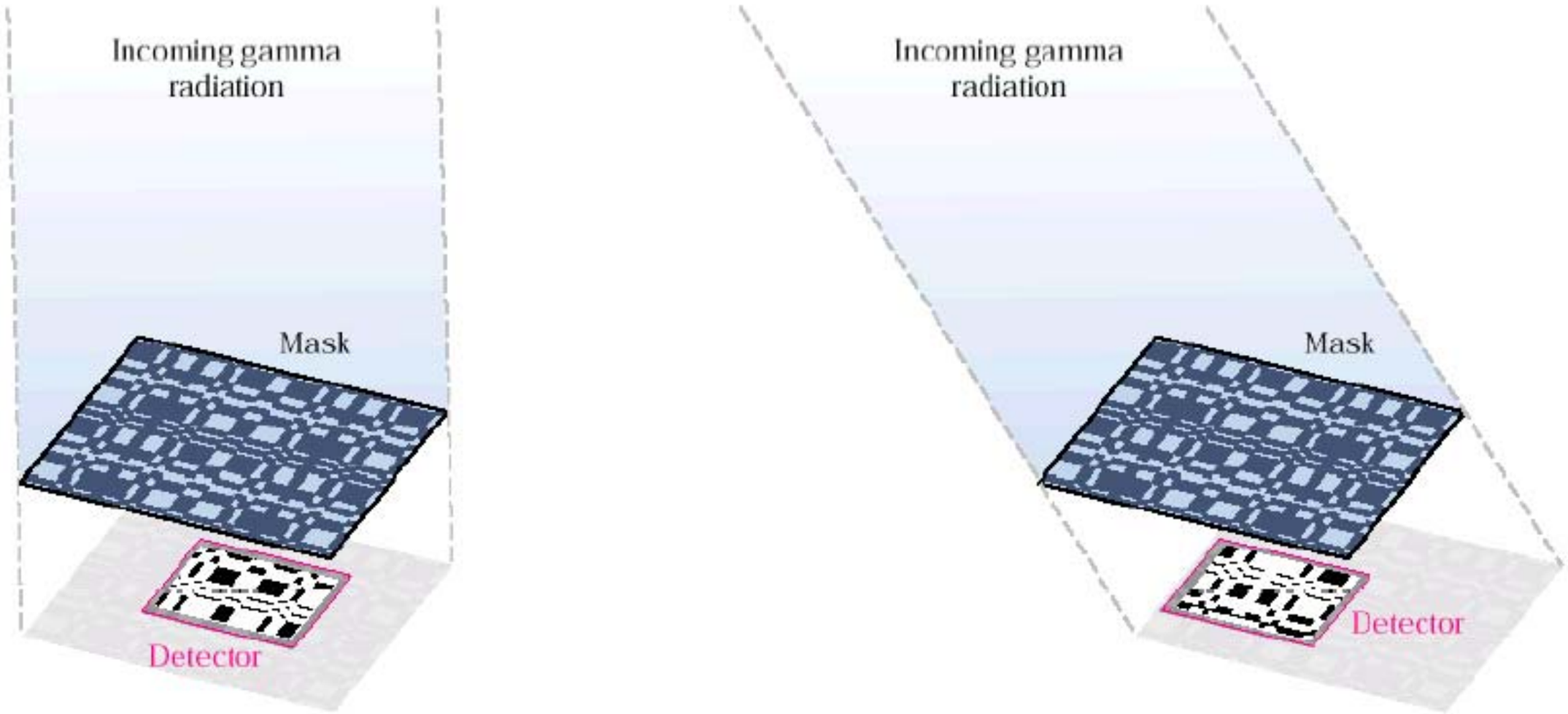
with: Richard Lanza, Roberto Accorsi,
Klaus Ziock, and Lorenzo Fabris.

Coded Aperture Imaging

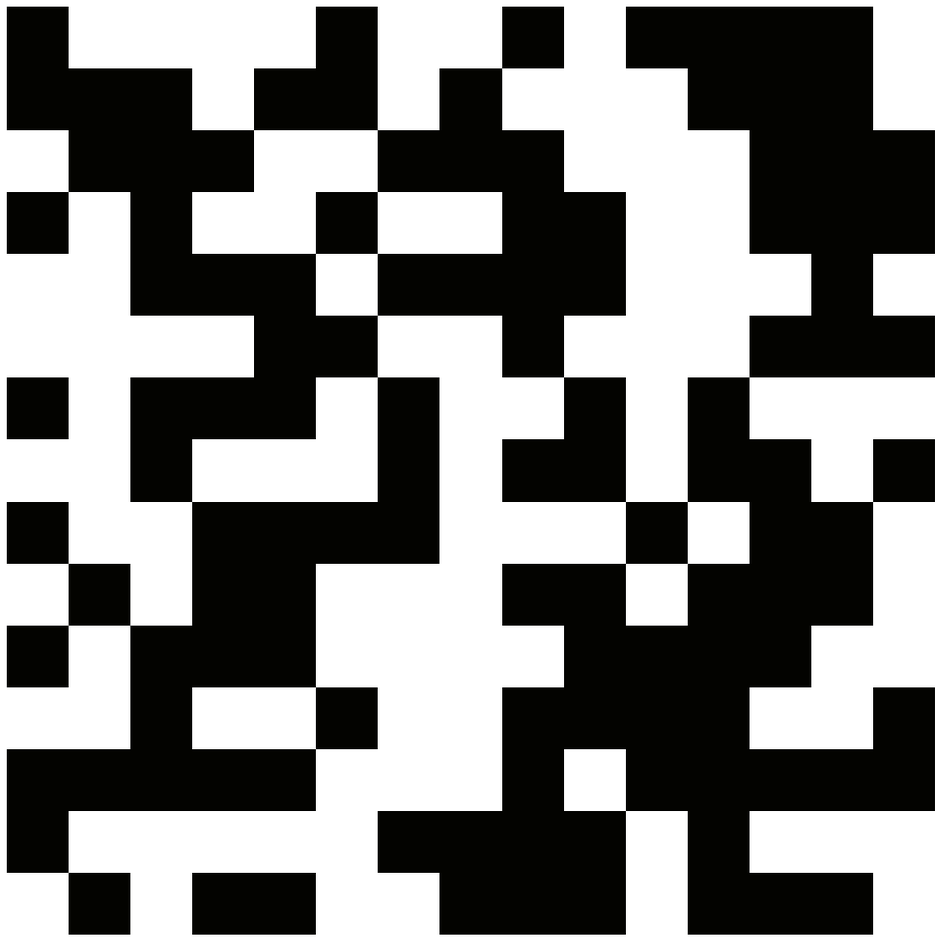
- Can’t refract or reflect gamma rays
- Pinhole — tradeoff resolution and SNR

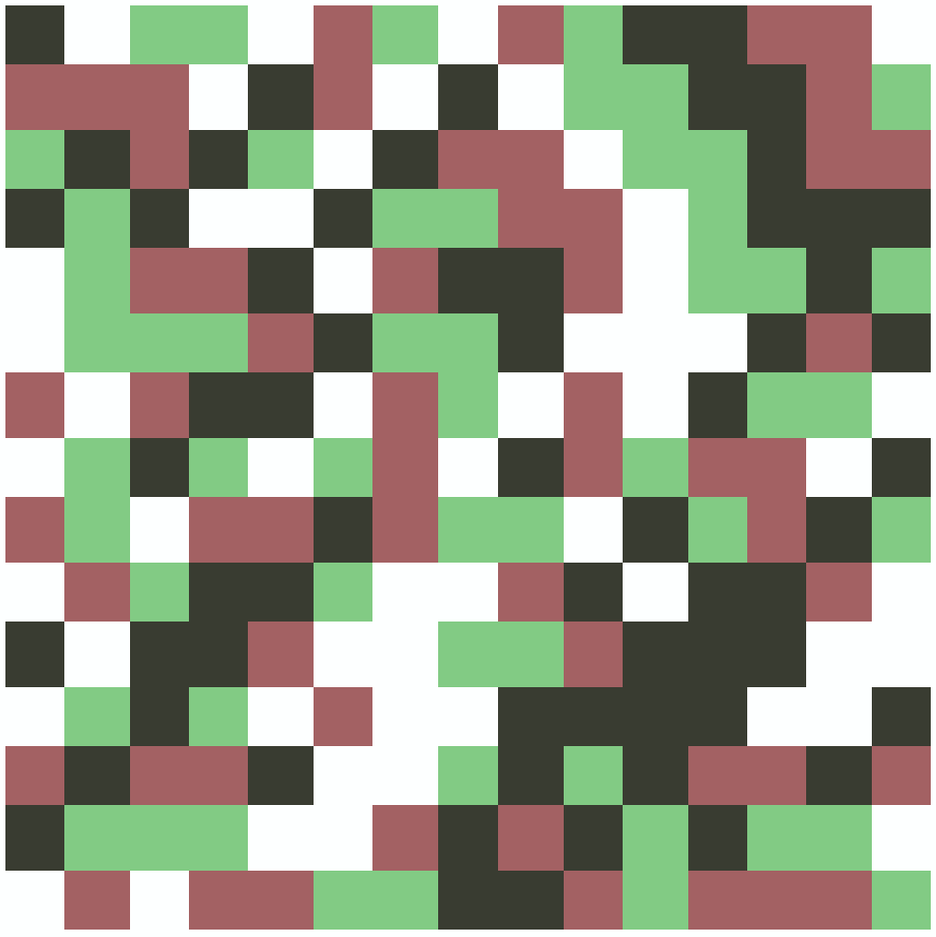
- Multiple pinholes
- Complex masks can “cast shadows”

Coded Aperture Principle



Decoding Method Rationale



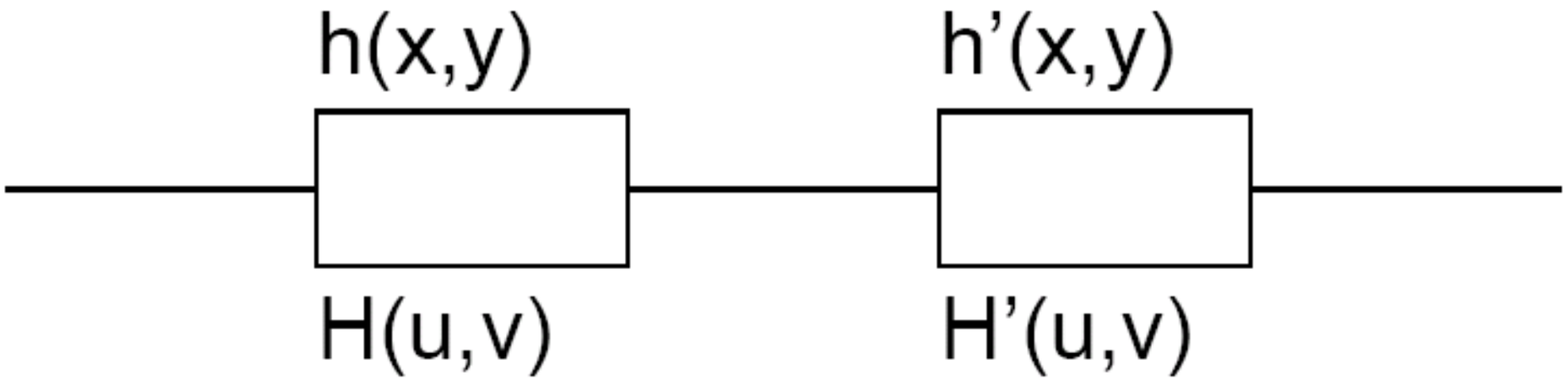


Coded Aperture Imaging

- Can't refract or reflect gamma rays
- Pinhole — tradeoff resolution and SNR
- Complex masks can "cast shadows"

- Decoding by Correlation
- Special Masks with Flat Power Spectrum

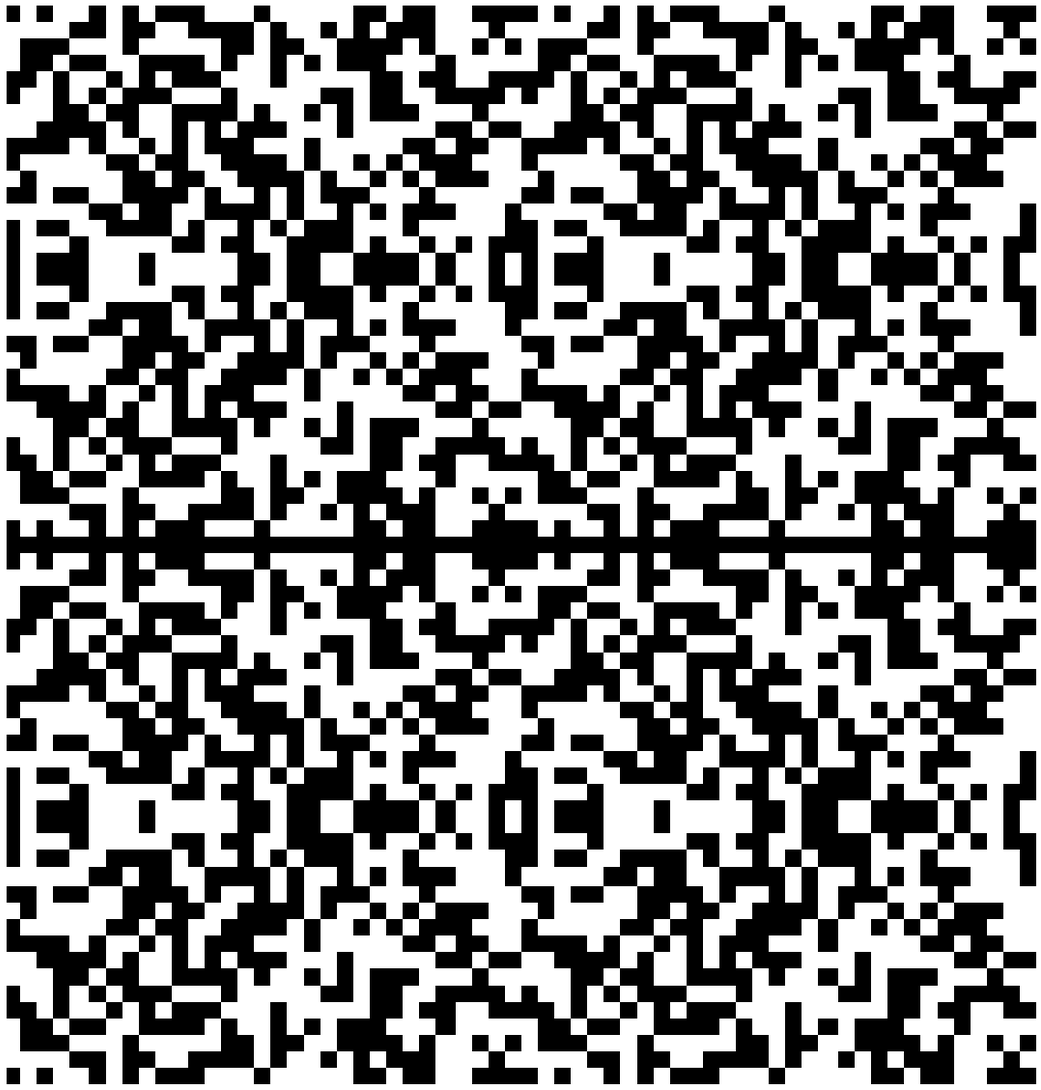
Mask Design — Inverse Systems



$$h(x,y) \otimes h'(x,y) = \delta(x,y)$$

$$H(u,v)\ \ H'(u,v) = 1$$

Masks — XRT Coarse



Mask Design — 1D

Definition: q is a quadratic residue $(\text{mod } p)$
if $\exists\, n$ s.t. $n^2 \equiv q (\text{mod } p)$

Legendre symbol

$$\left(\frac{a}{p}\right) = \begin{cases} 1 & \text{if } a \text{ is quadratic residue} \\ -1 & \text{otherwise} \end{cases}$$

Correlation with zero shift $(p-1)/2$
Correlation with non-zero shift $(p-1)/4$

Mask Design

- Auto Correlation

$$a(i) = \frac{(p-1)}{4}(1 + \delta(i))$$

- Power Spectrum

$$A(j) = \frac{(p-1)}{4}(\delta(j) + 1)$$

Coded Aperture Extensions

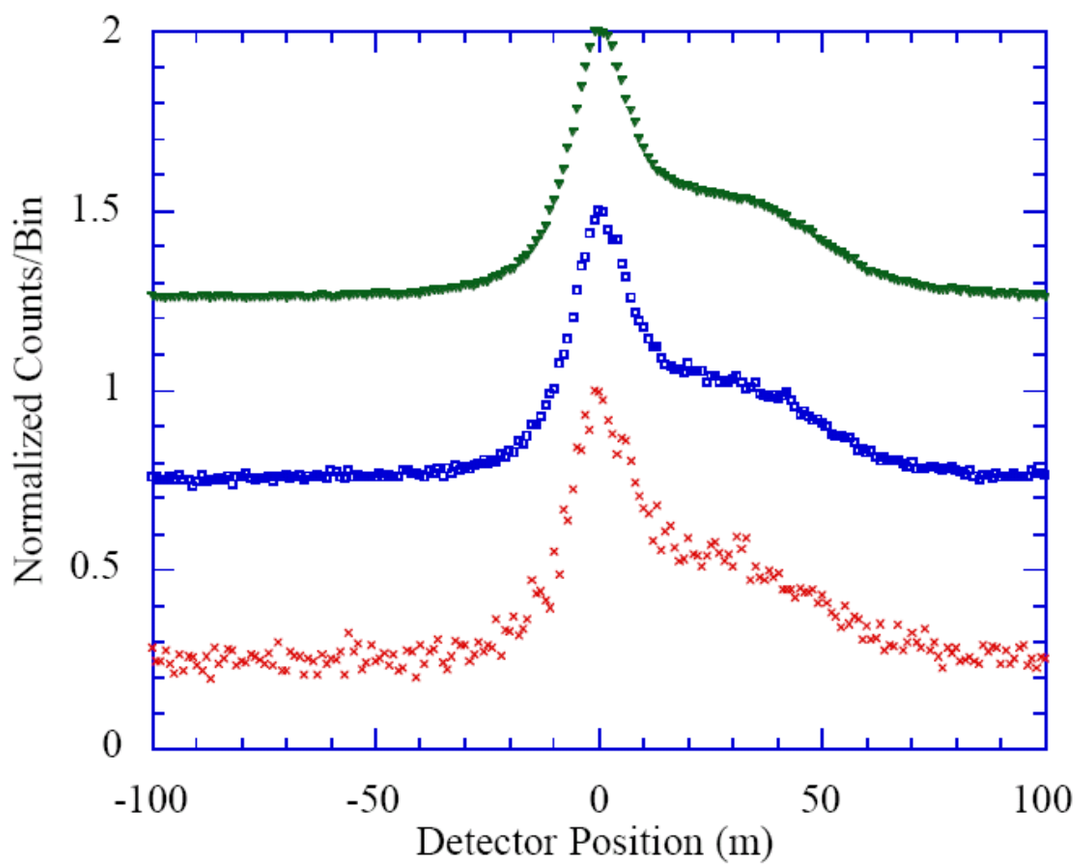
- Imaging Nearby Objects
- Mask / Countermask Combination

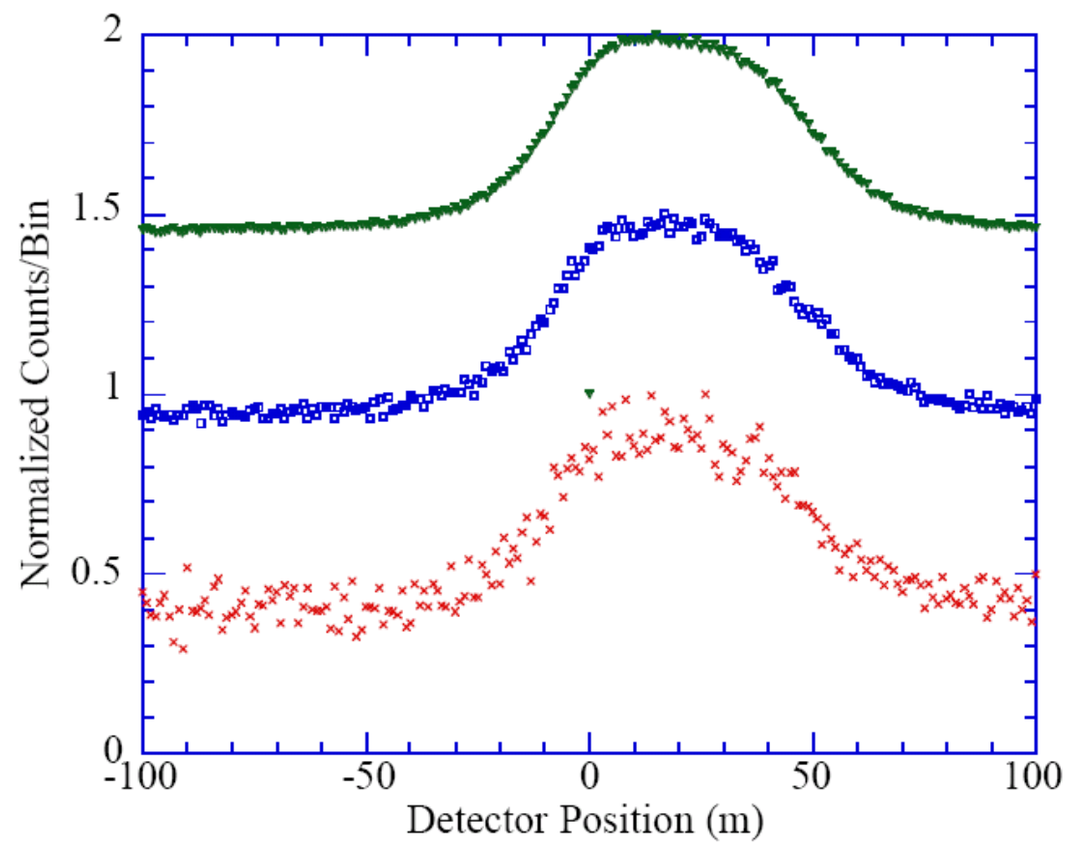
* Dynamic Reconstruction

Coded Aperture Application

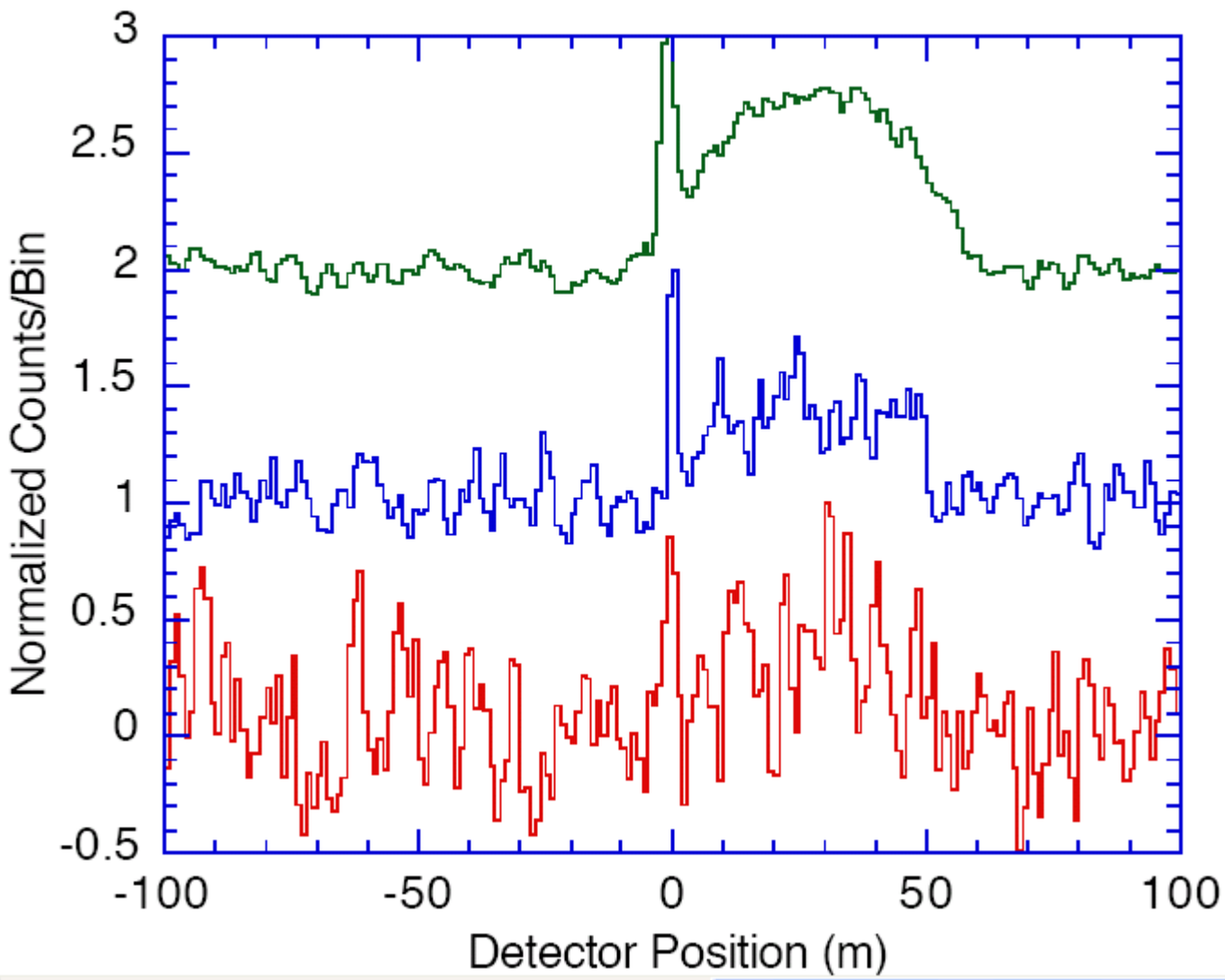
- Detection of Fissile Material
- Large Area Detector Myth
- Signal *and* Background Amplified

Large Area Alone Doesn't Help

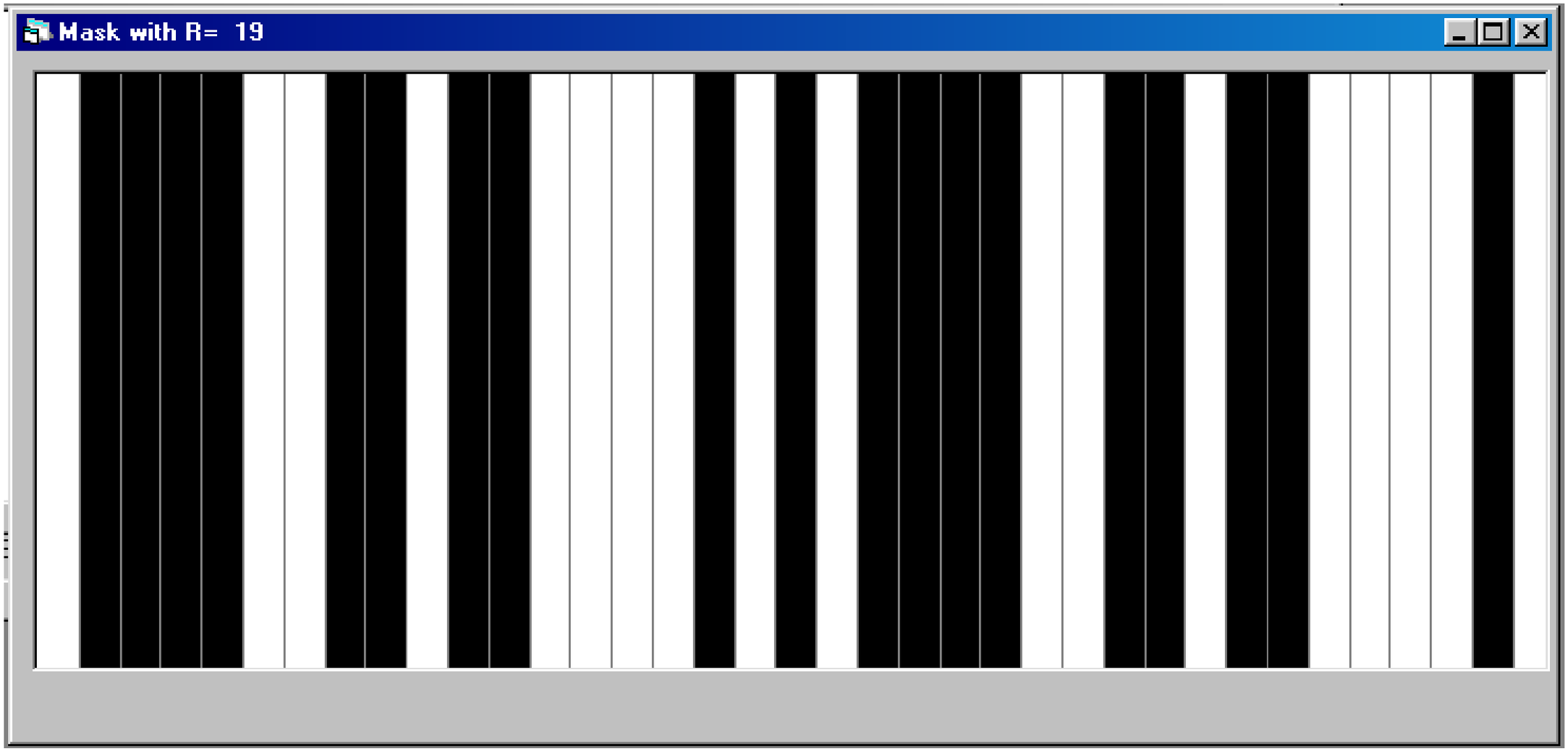




Imaging and Large Area Do!



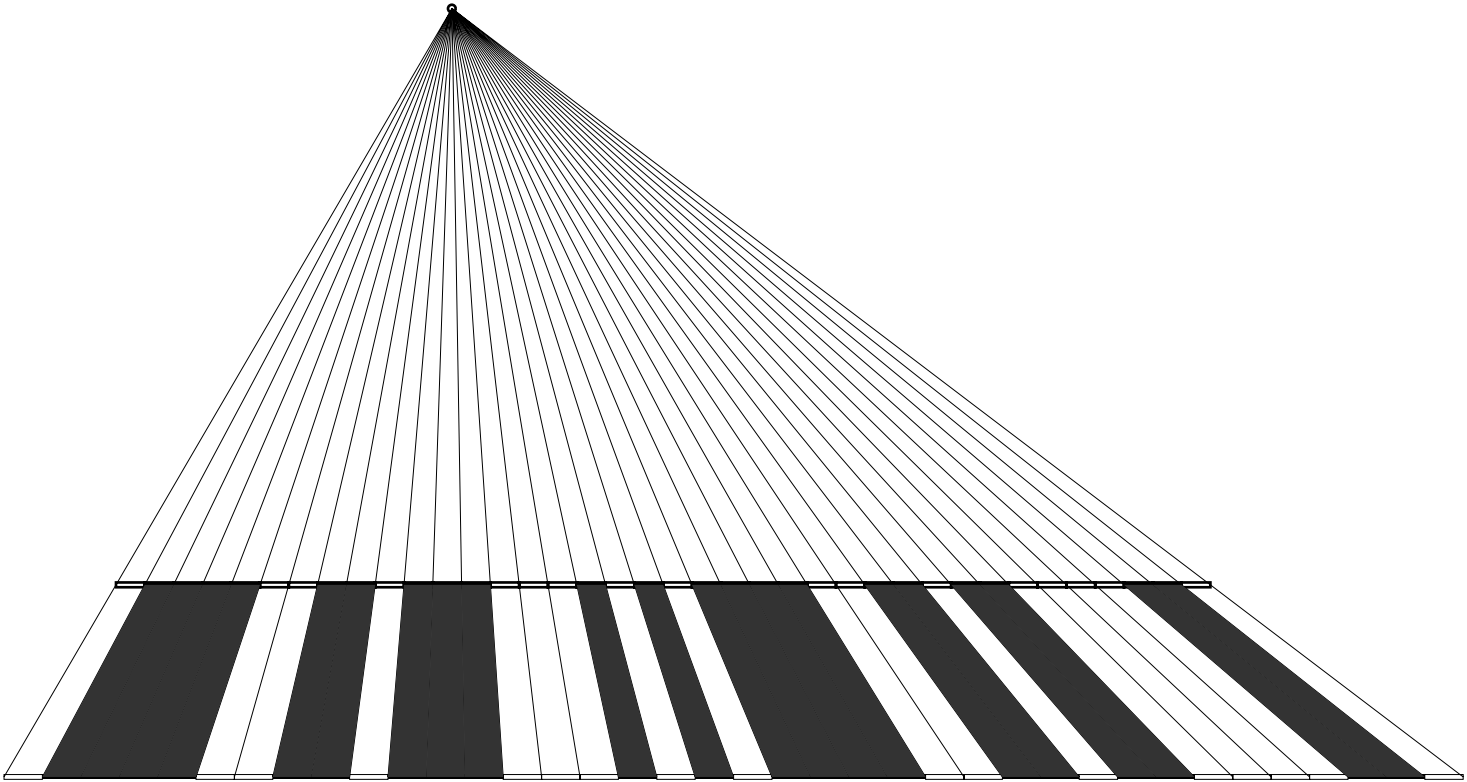
Coded Aperture Example

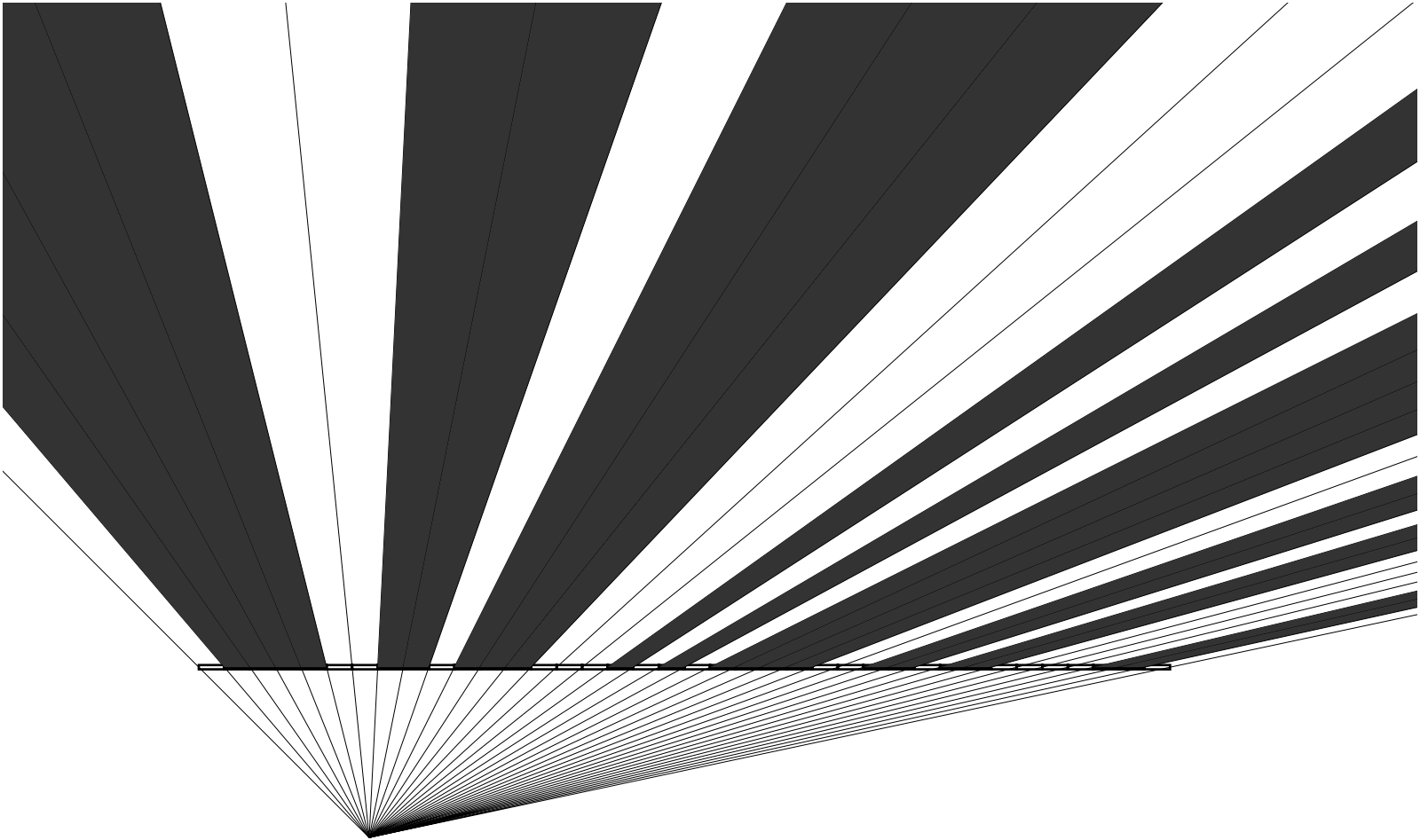


- Imaging — $1/R$ instead of $1/R^2$

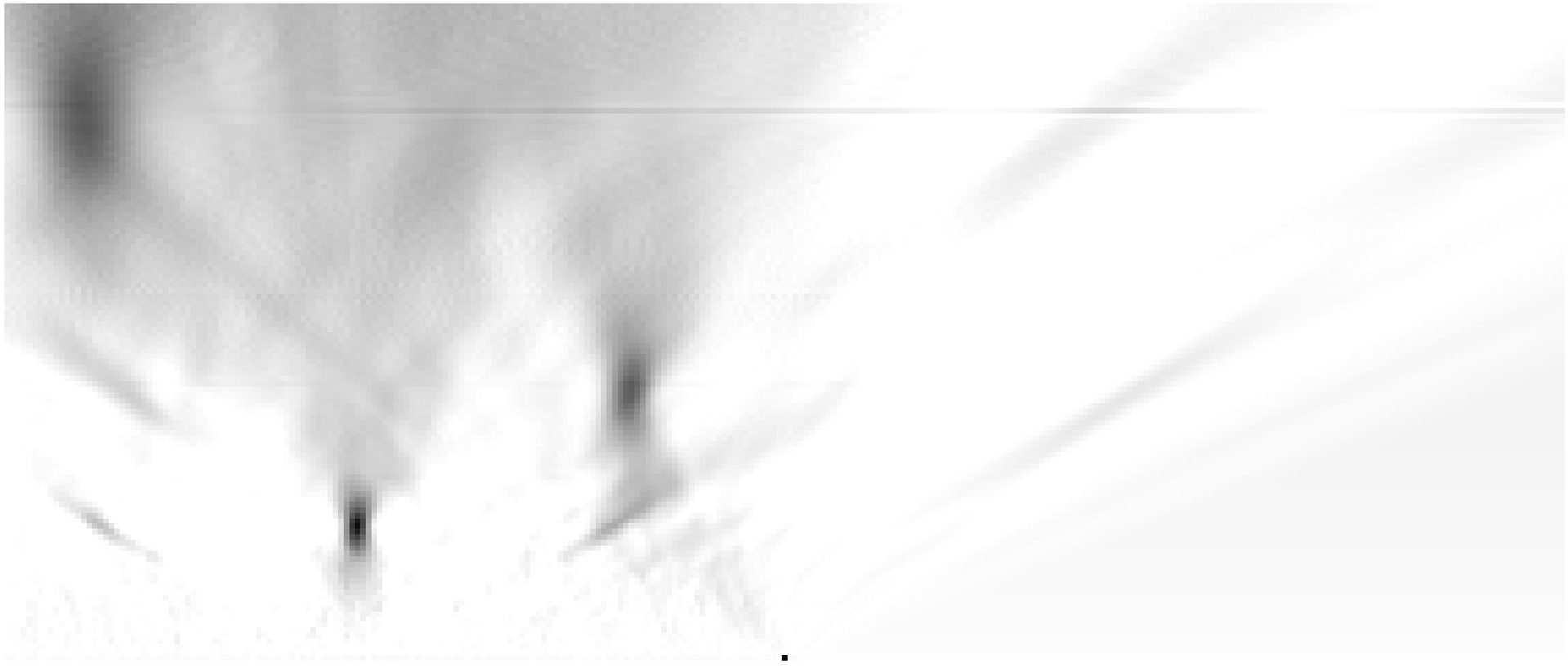
Coded Aperture Detector Array

Computational Imaging System





Dynamic Reconstruction



Three weak, distant radioactive sources

Reconstruction Animation

Coded Aperture Applications

- Detection of Fissile Material
- Imaging — $1/R$ instead of $1/R^2$

- Increasing Gamma Camera Resolution
- Replacing Rats with Mice

(4) EXACT CONE BEAM ALGORITHM

- Faster Scanning—Fewer Motion Artifacts
- Lower Exposure—Uniform Resolution

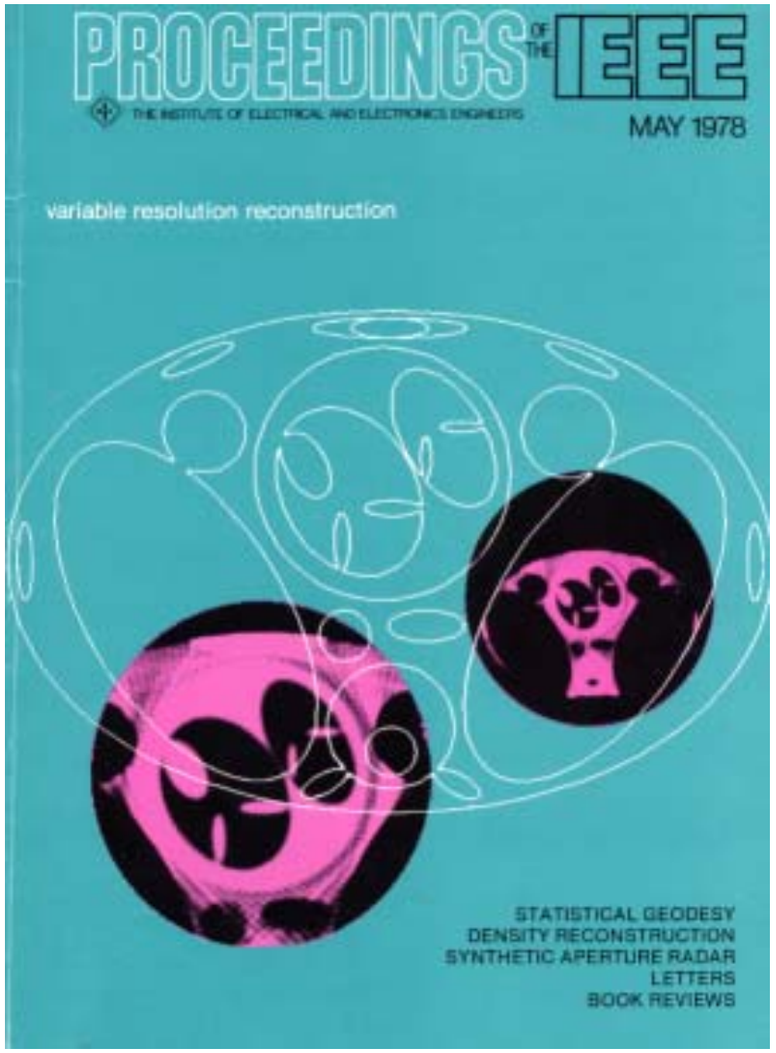
with: Xiaochun Yang

Exact Cone Beam Reconstruction

- Faster Scanning—Fewer Motion Artifacts
- Lower Exposure—Uniform Resolution

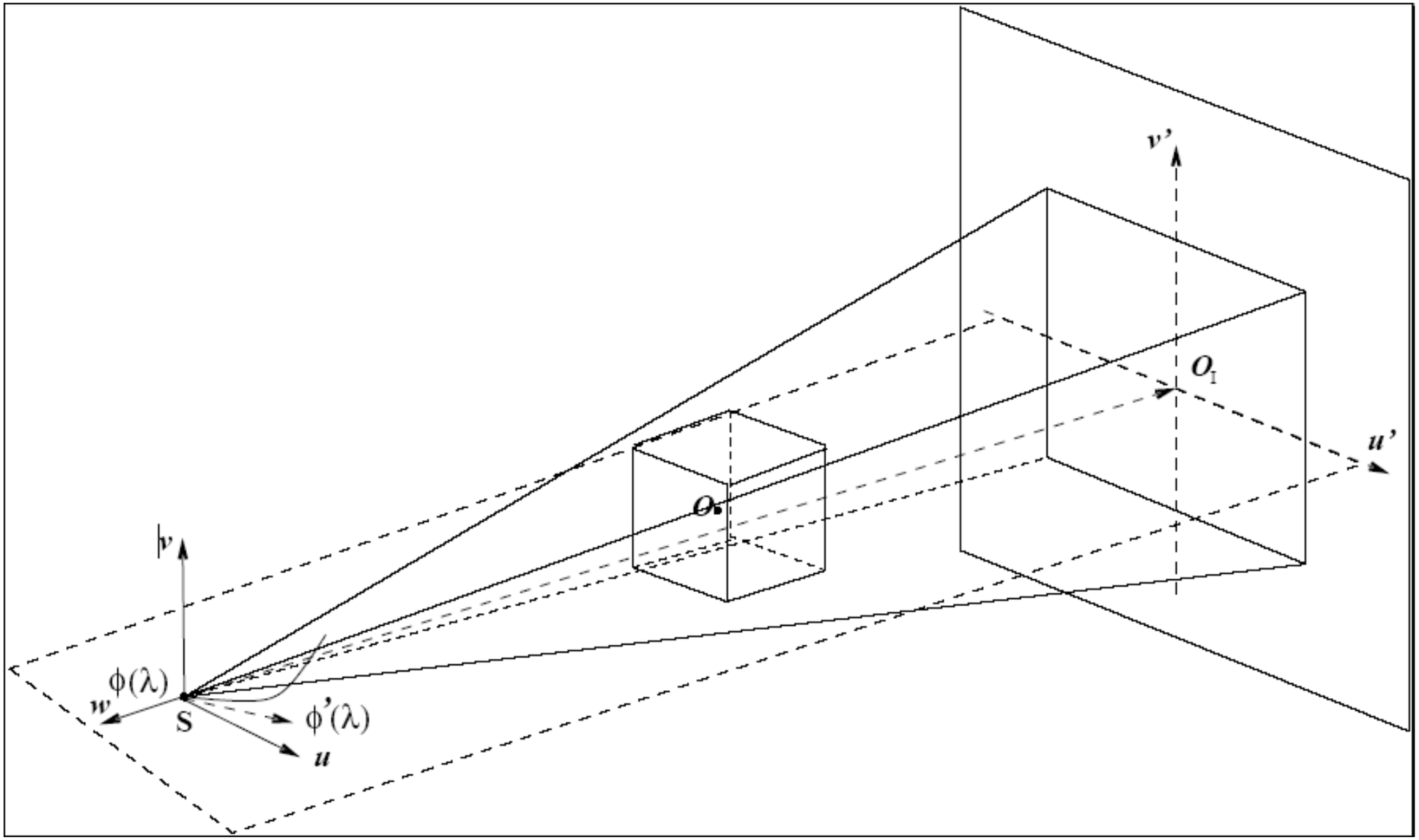
- Parallel Beam → Fan Beam
- Planar Fan → Cone Beam

Parallel Beam to Fan Beam



Coordinate Transform in 2D Radon Space

Cone Beam Geometry — 3D



Radon's Formula

- In 2D: ~ derivatives of line integrals
- In 3D: derivatives of plane integrals
- Can't get plane integrals from projections

$$\int \left(\int f(r, \theta) dr \right) d\theta$$

$$\int \int \frac{1}{r} f(x, y) \, dx \, dy$$

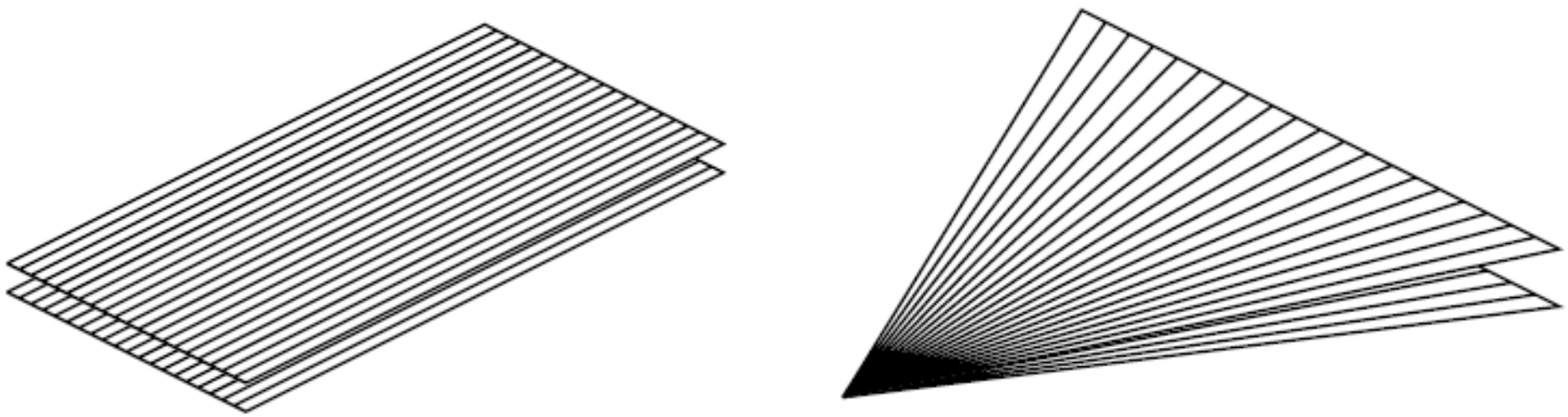
Radon's Formula in 3D

$$f(\boldsymbol{x}) = -\frac{1}{8\pi^2}\int_{S^2} \left.\frac{\partial^2 R\, f(l,\boldsymbol{\beta})}{\partial l^2}\right|_{l=\boldsymbol{x}\cdot\boldsymbol{\beta}} d\boldsymbol{\beta}$$

where

$$R\, f(l,\boldsymbol{\beta}) = \int f(\boldsymbol{x})\ \delta(\boldsymbol{x}\cdot\boldsymbol{\beta} - l)\, dV$$

Grangeat's Trick



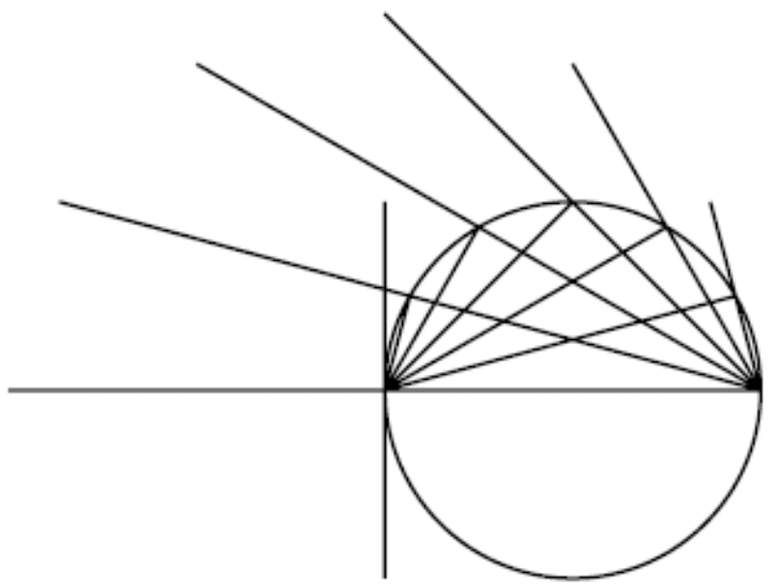
$$\frac{\partial}{\partial z}\iint f(x,y,z)\,dx\,dy =$$

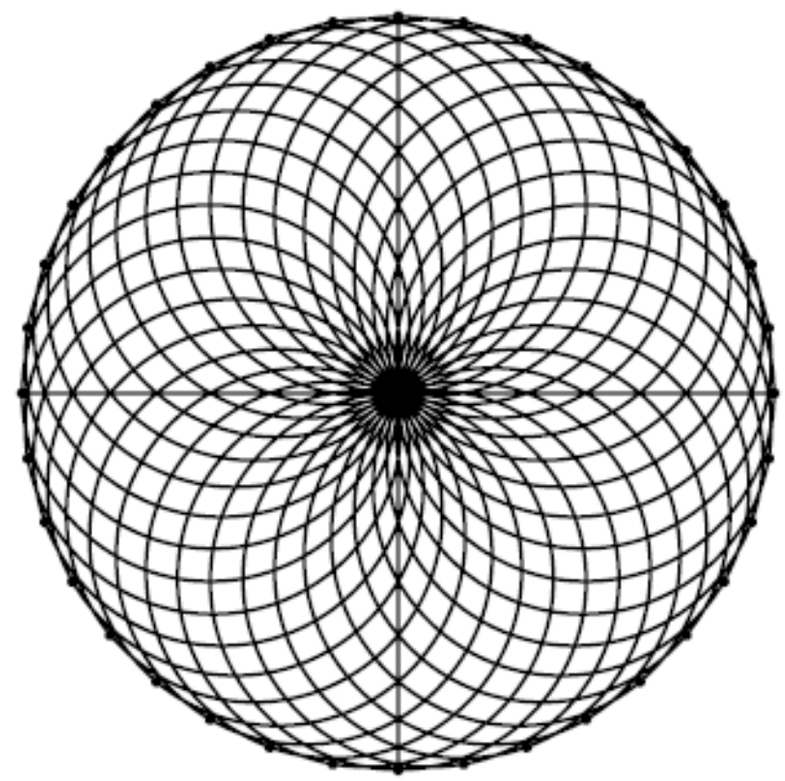
$$\frac{\partial}{\partial \theta}\iint f(r,\phi,\theta)\,dr\,d\phi$$

Exact Cone Beam Reconstruction

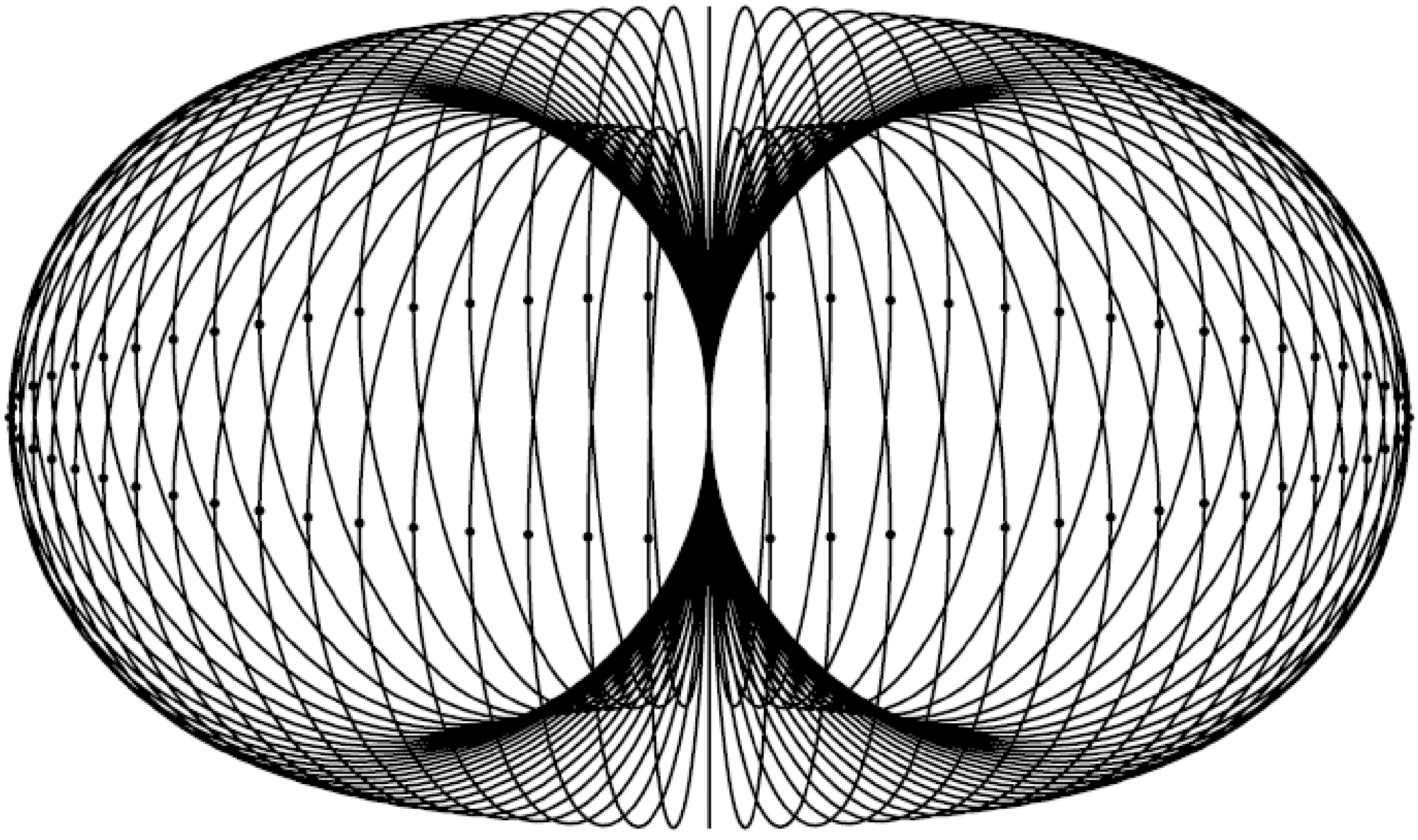
- Data Sufficiency Condition
- Good “Orbit” for Radiation Source

Radon Space — 2D

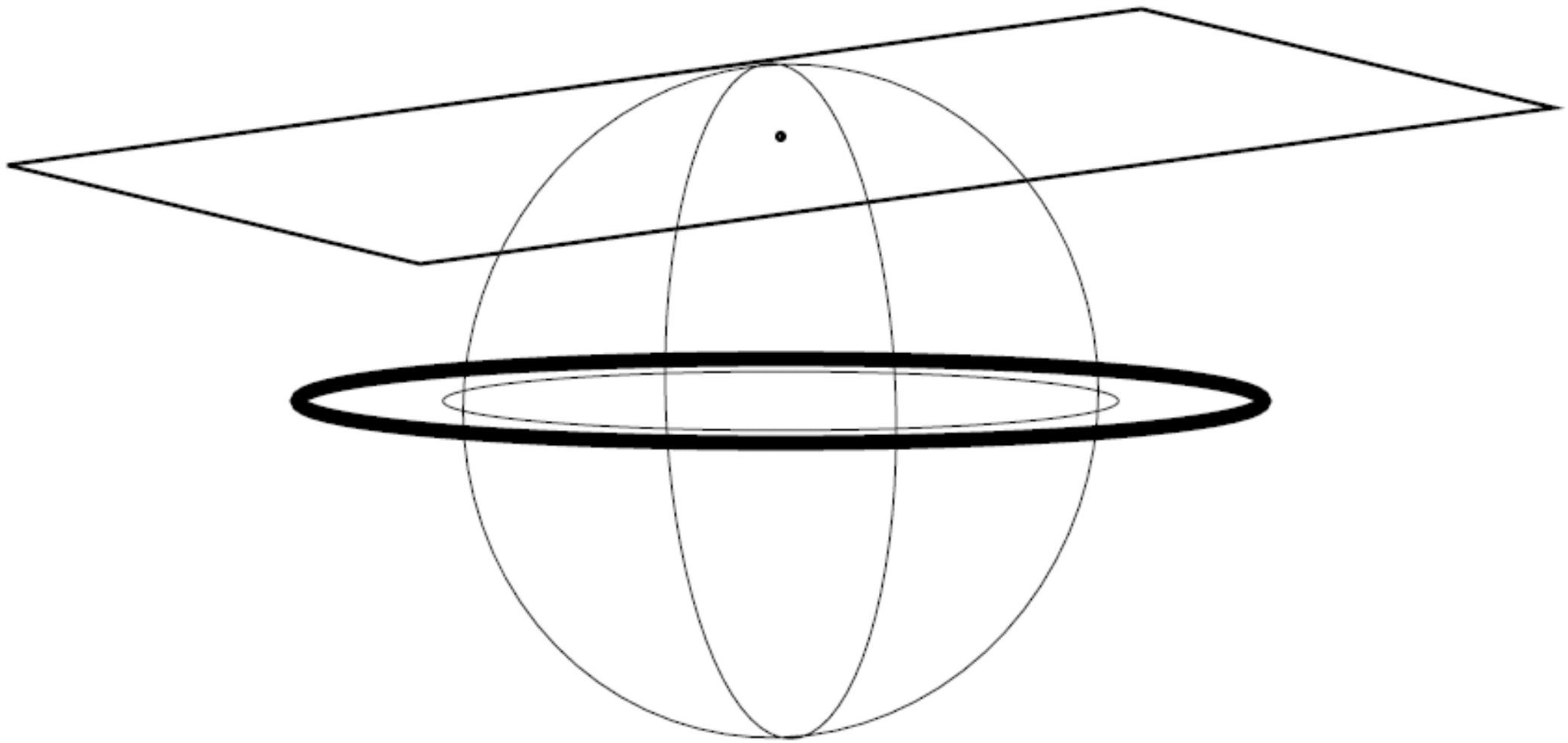




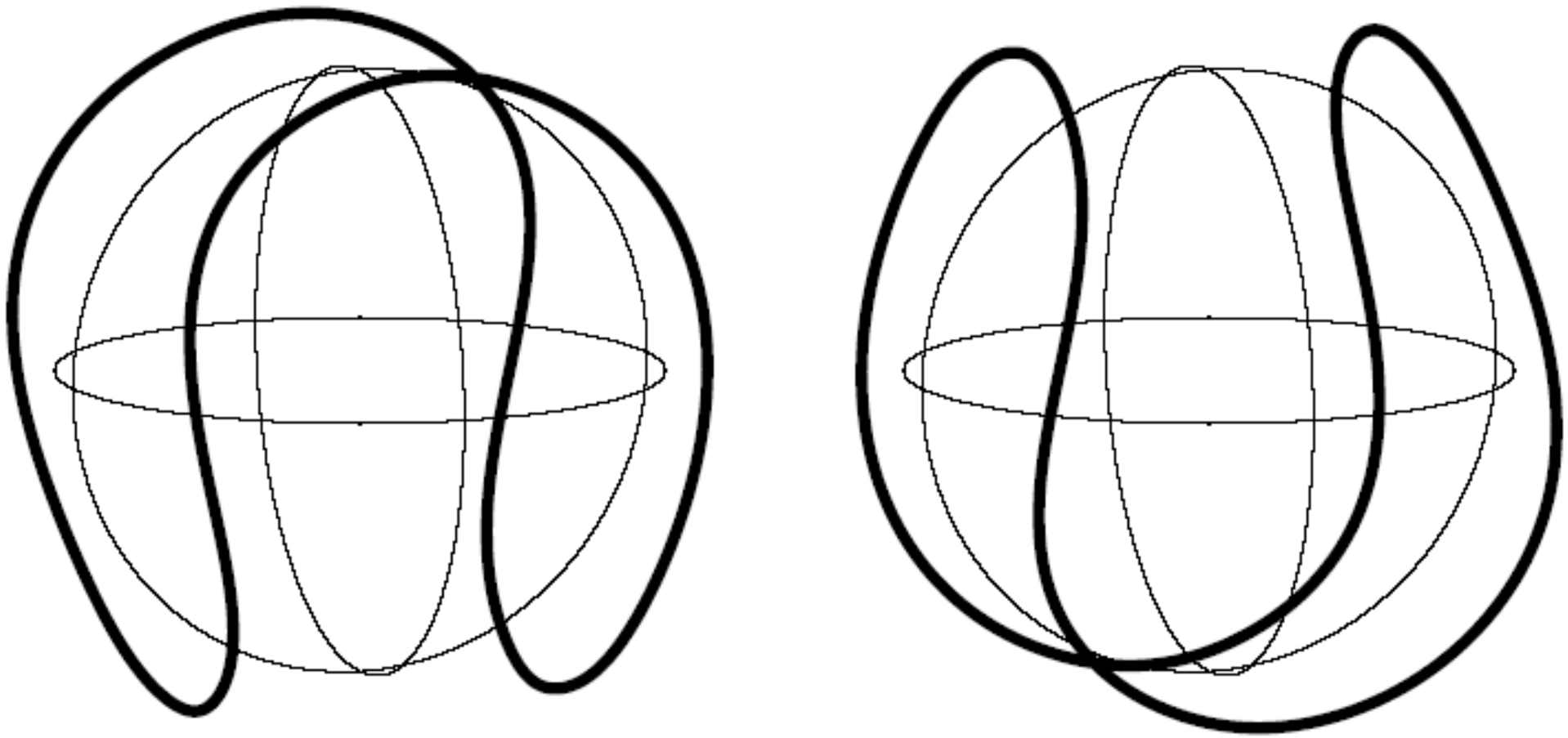
Circular Orbit is Inadequate (3D)



Data Insufficiency



Good Source Orbit



(3) DIAPHANOGRAPHY

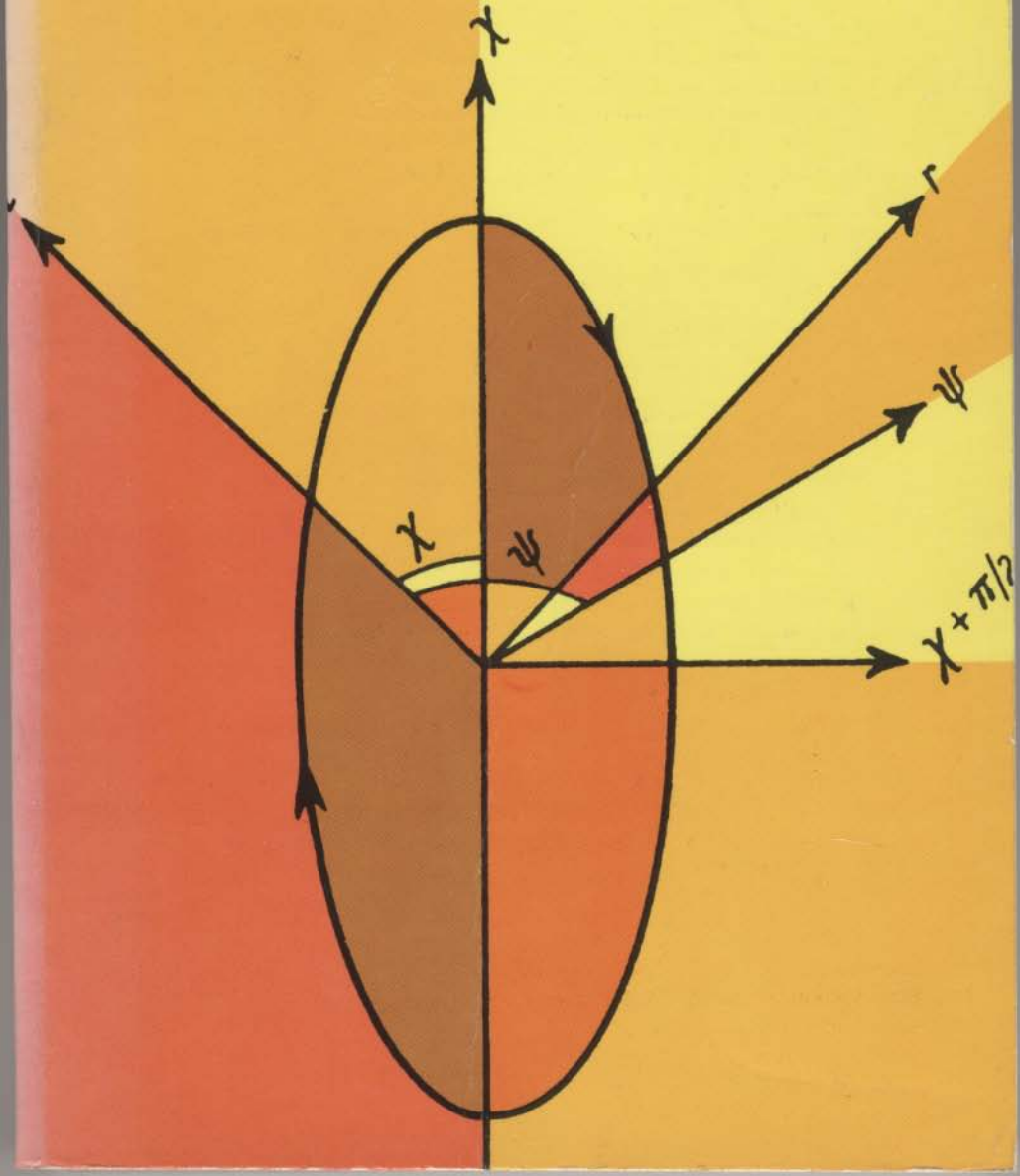
(Diffuse Optical Imaging)

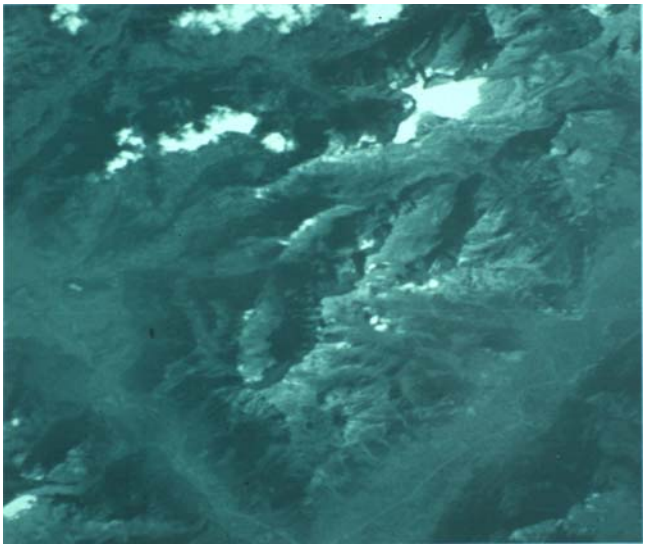
- Highly Scattering — Low Absorption
- Many Sources — Many Detectors

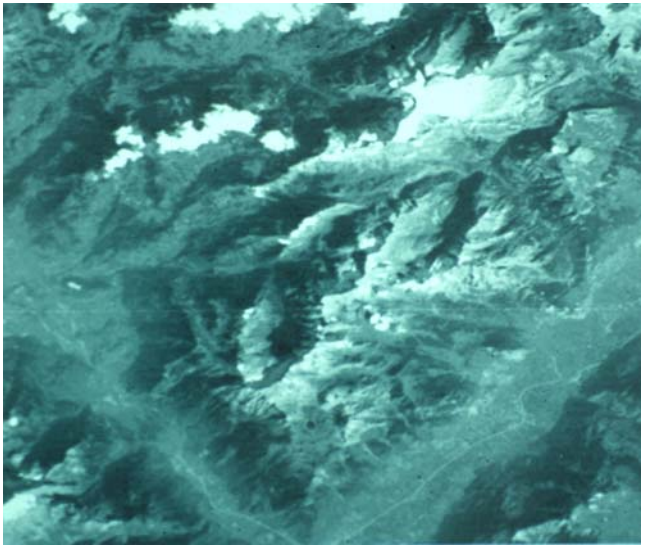
with: Xiaochun Yang, Richard Lanza,
David Boas and Anna Custo.

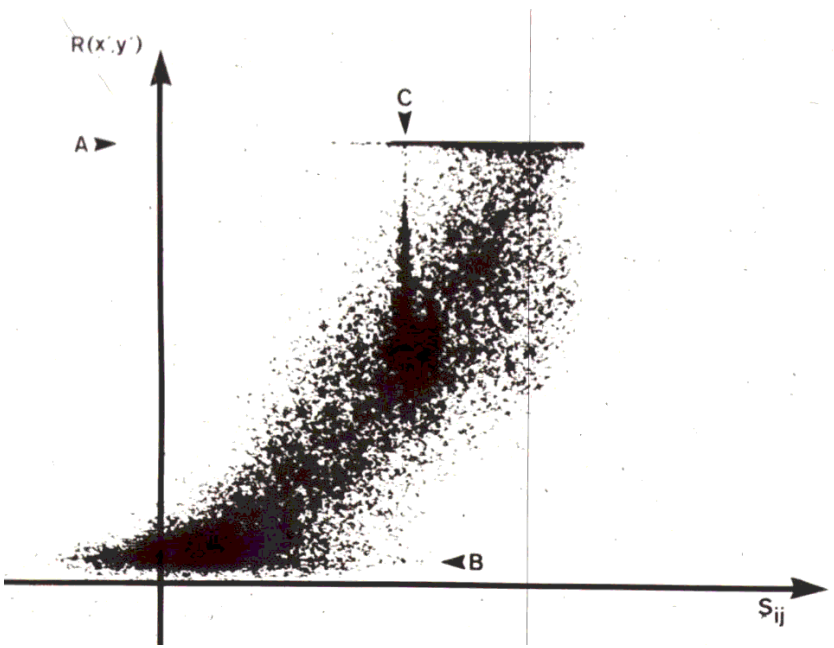
S. Chandrasekhar

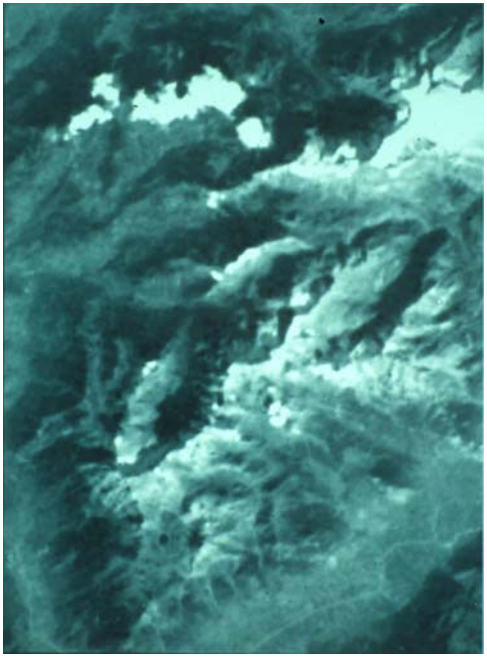
RADIATIVE TRANSFER

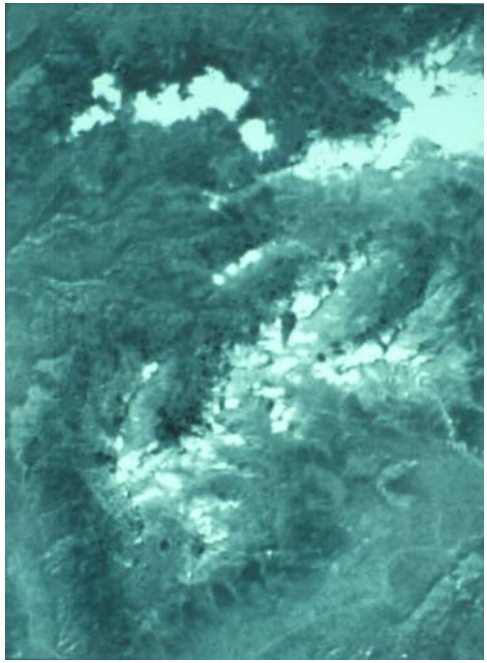


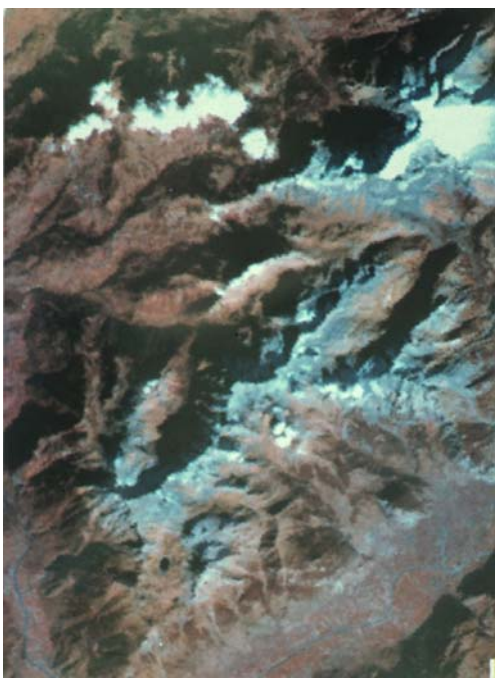


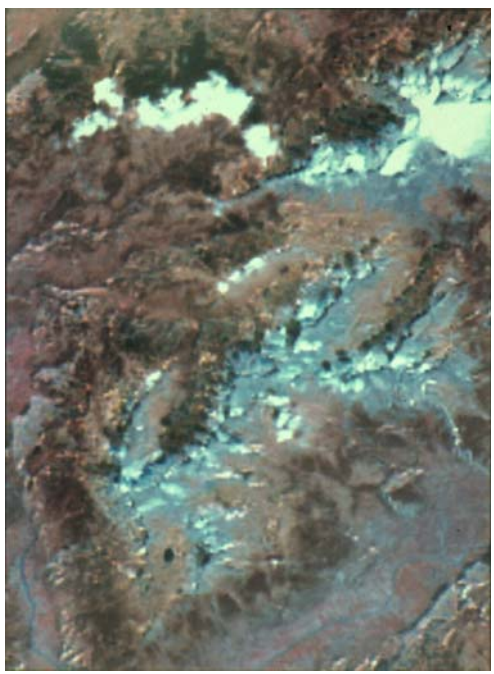



R(x',y')
A
C
B
S_{ij}



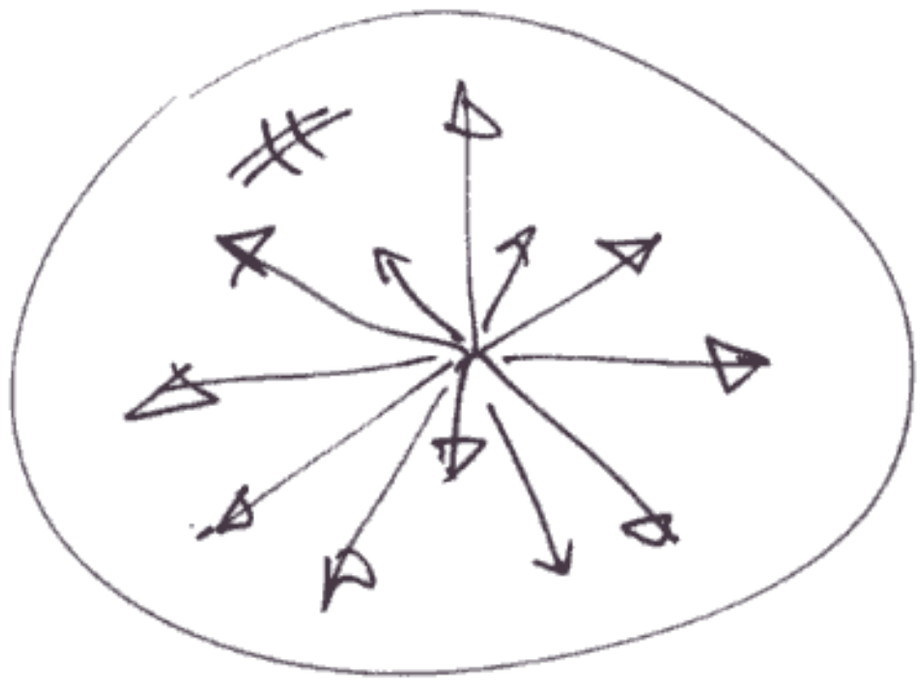






Diaphanography

- Randomization of Direction



- *Scalar* Flux Density

Diffusion Approximation

$$-\nabla \cdot (\nu_d \nabla \Phi) + \mu_a c \Phi = q_0$$

where

$$\nu_d = \frac{c}{3(\mu_a + (1-f)\mu_s)}$$

μ_a is the absorption coefficient
μ_s is the scattering coefficient
f is the anistropy factor

Diffusion Approximation

$$\nabla^2 \Phi - k\Phi = -\frac{1}{\nu_d} q_0$$

where $k = 3\mu_a(\mu_a + (1-f)\mu_s)$

Green's function (k constant, no boundary):

$$\Phi = \frac{1}{r} e^{-\sqrt{k}r}$$

Diaphanography

- Approximation: Diffusion Equation

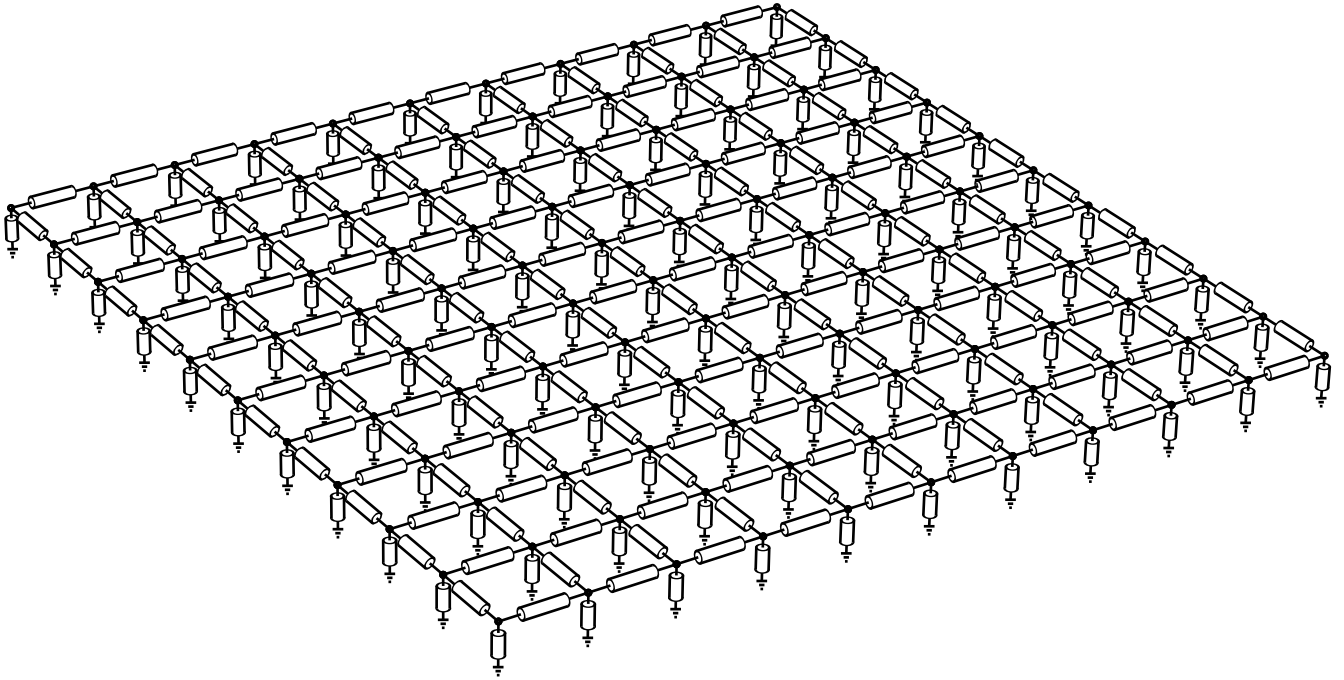
$$\Delta v(x, y) + \rho(x, y)c(x, y) = 0$$

$v(x, y)$ flux density

$\rho(x, y)$ scattering coefficient

$c(x, y)$ absorption coefficient

- Forward: given $c(x, y)$ find $v(x, y)$



3×3 node grid example

Here we have three input nodes (1, 2, 3), three output nodes (7, 8, 9), and three interior nodes (4, 5, 6). In three experiments we apply currents to each of the input nodes in turn, each time reading out all of the output nodes, yielding a total of nine measurements. We try and recover the nine leakage conductances to ground from each of the nine nodes.

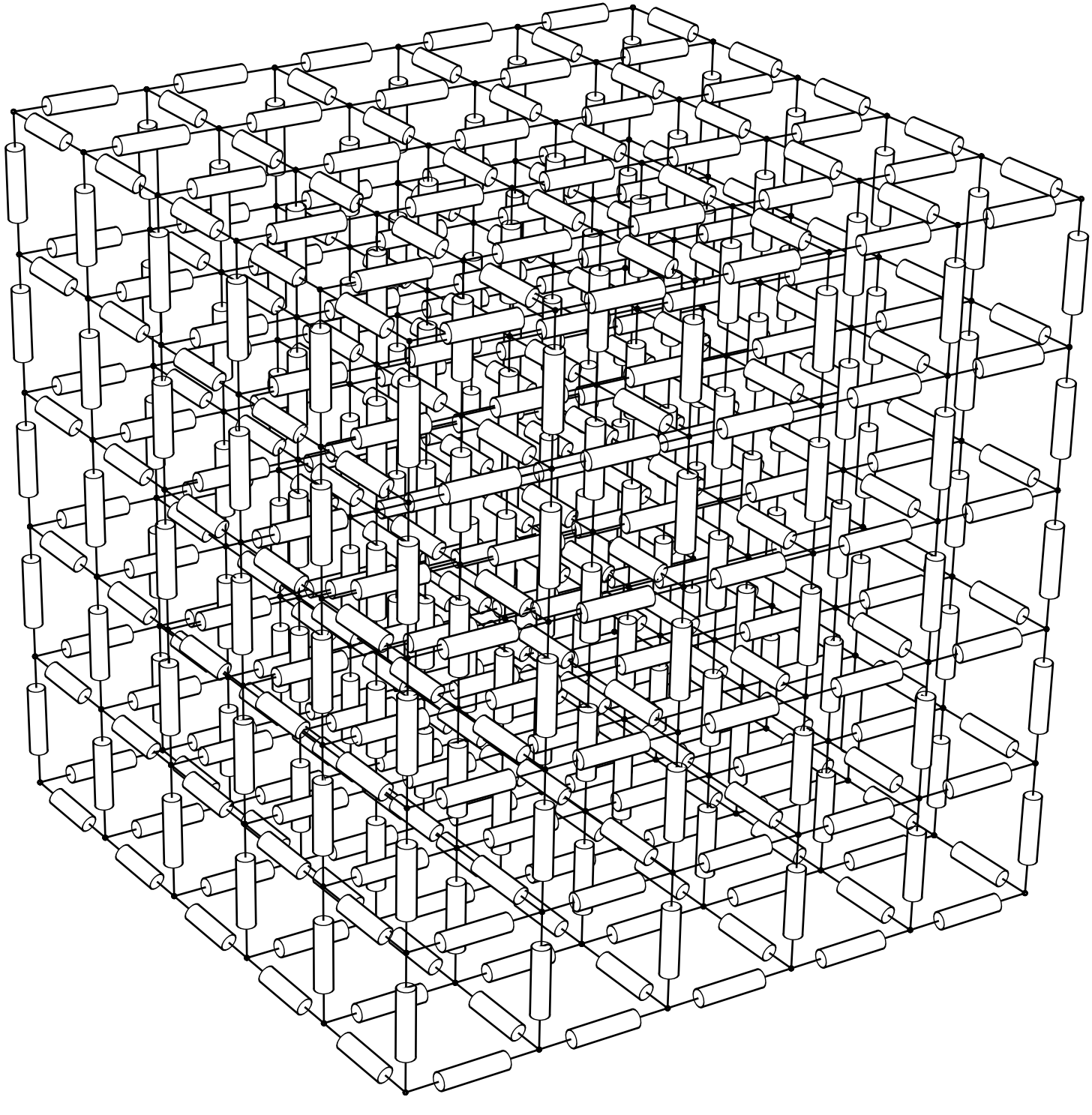
It is natural to partition the conductance matrix as follows given that I_4, I_5, I_6, I_7, I_8, and I_9 are always zero, and that we do not measure V_1, V_2, V_3, V_4, V_5, and V_6.

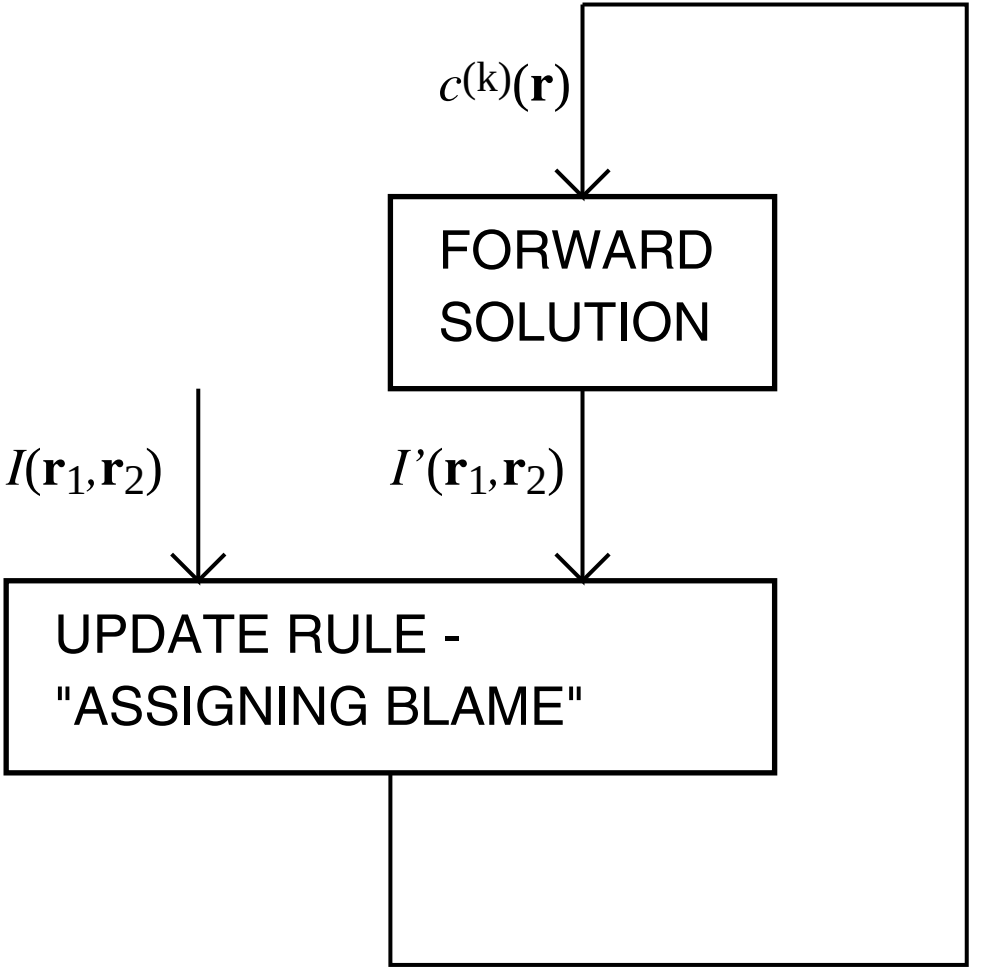
$$\begin{pmatrix} G_{11} & \vdots & G_{12} & \vdots & G_{13} \\ \cdots & \cdot & \cdots & \cdot & \cdots \\ G_{21} & \vdots & G_{22} & \vdots & G_{23} \\ \cdots & \cdot & \cdots & \cdot & \cdots \\ G_{31} & \vdots & G_{32} & \vdots & G_{33} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ \cdots \\ V_4 \\ V_5 \\ V_6 \\ \cdots \\ V_7 \\ V_8 \\ V_9 \end{pmatrix} = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ \cdots \\ I_4 \\ I_5 \\ I_6 \\ \cdots \\ I_7 \\ I_8 \\ I_9 \end{pmatrix}$$

$$
\begin{pmatrix}
g_1' & -g_{12} & -g_{13} & \vdots & -g_{14} & 0 & 0 & \vdots & 0 & 0 & 0 \\
-g_{12} & g_2' & -g_{23} & \vdots & 0 & -g_{25} & 0 & \vdots & 0 & 0 & 0 \\
-g_{13} & -g_{23} & g_3' & \vdots & 0 & 0 & g_{36} & \vdots & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdot & \cdots & \cdots & \cdots & \cdot & \cdots & \cdots & \cdots \\
-g_{14} & -0 & 0 & \vdots & g_4' & -g_{45} & -g_{46} & \vdots & -g_{47} & 0 & 0 \\
0 & -g_{25} & 0 & \vdots & -g_{45} & g_5' & -g_{56} & \vdots & 0 & -g_{58} & 0 \\
0 & 0 & -g_{36} & \vdots & -g_{46} & -g_{56} & g_6' & \vdots & 0 & -0 & -g_{69} \\
\cdots & \cdots & \cdots & \cdot & \cdots & \cdots & \cdots & \cdot & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \vdots & -g_{47} & 0 & 0 & \vdots & g_7' & -g_{78} & -g_{79} \\
0 & 0 & 0 & \vdots & 0 & -g_{58} & 0 & \vdots & -g_{78} & g_8' & -g_{89} \\
0 & 0 & 0 & \vdots & 0 & 0 & -g_{69} & \vdots & -g_{79} & -g_{89} & g_9'
\end{pmatrix}
\begin{pmatrix}
V_1 \\ V_2 \\ V_3 \\ \cdots \\ V_4 \\ V_5 \\ V_6 \\ \cdots \\ V_7 \\ V_8 \\ V_9
\end{pmatrix}
=
\begin{pmatrix}
I_1 \\ I_2 \\ I_3 \\ \cdots \\ I_4 \\ I_5 \\ I_6 \\ \cdots \\ I_7 \\ I_8 \\ I_9
\end{pmatrix}
$$

We can write the inverse as follows:

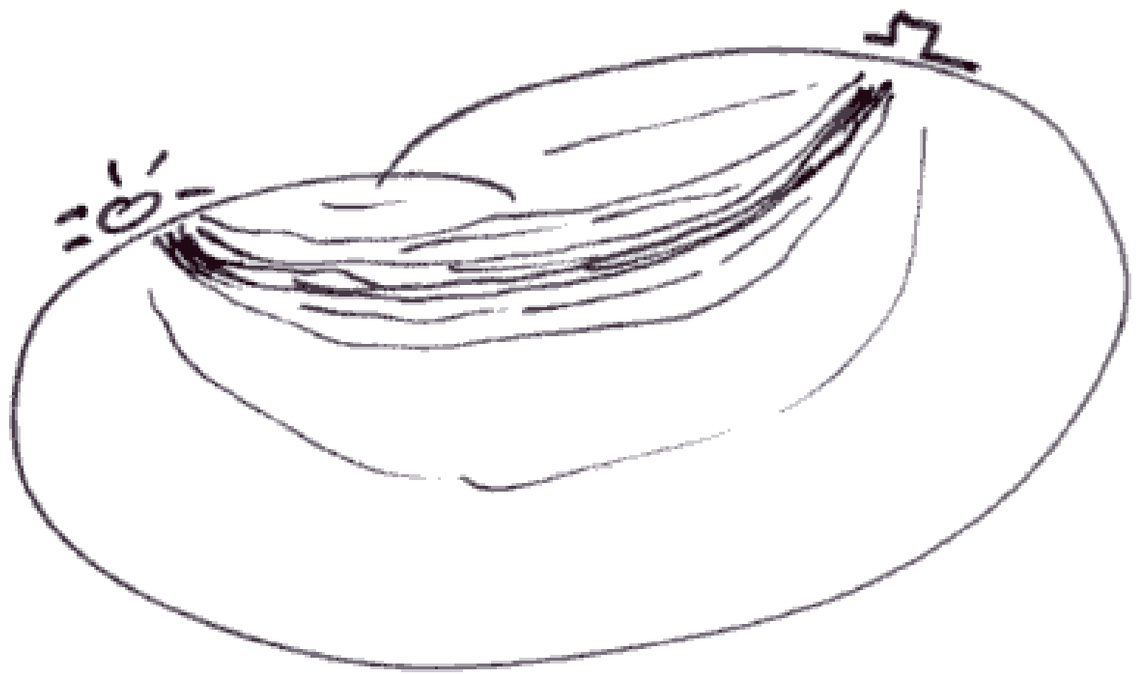
$$\begin{pmatrix} C_{11} & \vdots & C_{12} & \vdots & C_{13} \\ \cdots & \cdot & \cdots & \cdot & \cdots \\ C_{21} & \vdots & C_{22} & \vdots & C_{23} \\ \cdots & \cdot & \cdots & \cdot & \cdots \\ C_{31} & \vdots & C_{32} & \vdots & C_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ \cdots \\ I_4 \\ I_5 \\ I_6 \\ \cdots \\ I_7 \\ I_8 \\ I_9 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ \cdots \\ V_4 \\ V_5 \\ V_6 \\ \cdots \\ V_7 \\ V_8 \\ V_9 \end{pmatrix}$$



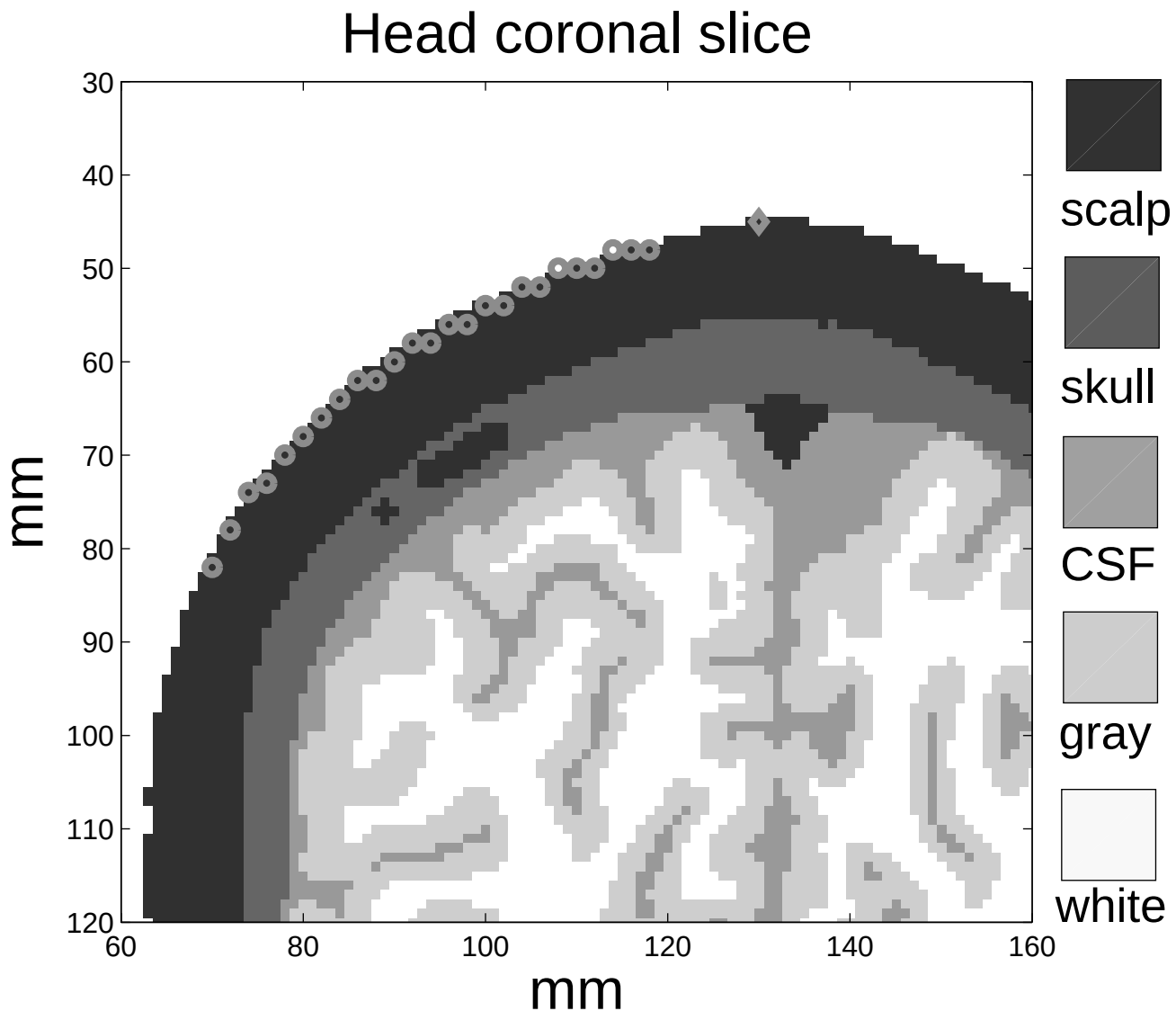

$c^{(k)}(\mathbf{r})$
FORWARD SOLUTION
$I(\mathbf{r}_1,\mathbf{r}_2)$
$I'(\mathbf{r}_1,\mathbf{r}_2)$
UPDATE RULE - "ASSIGNING BLAME"

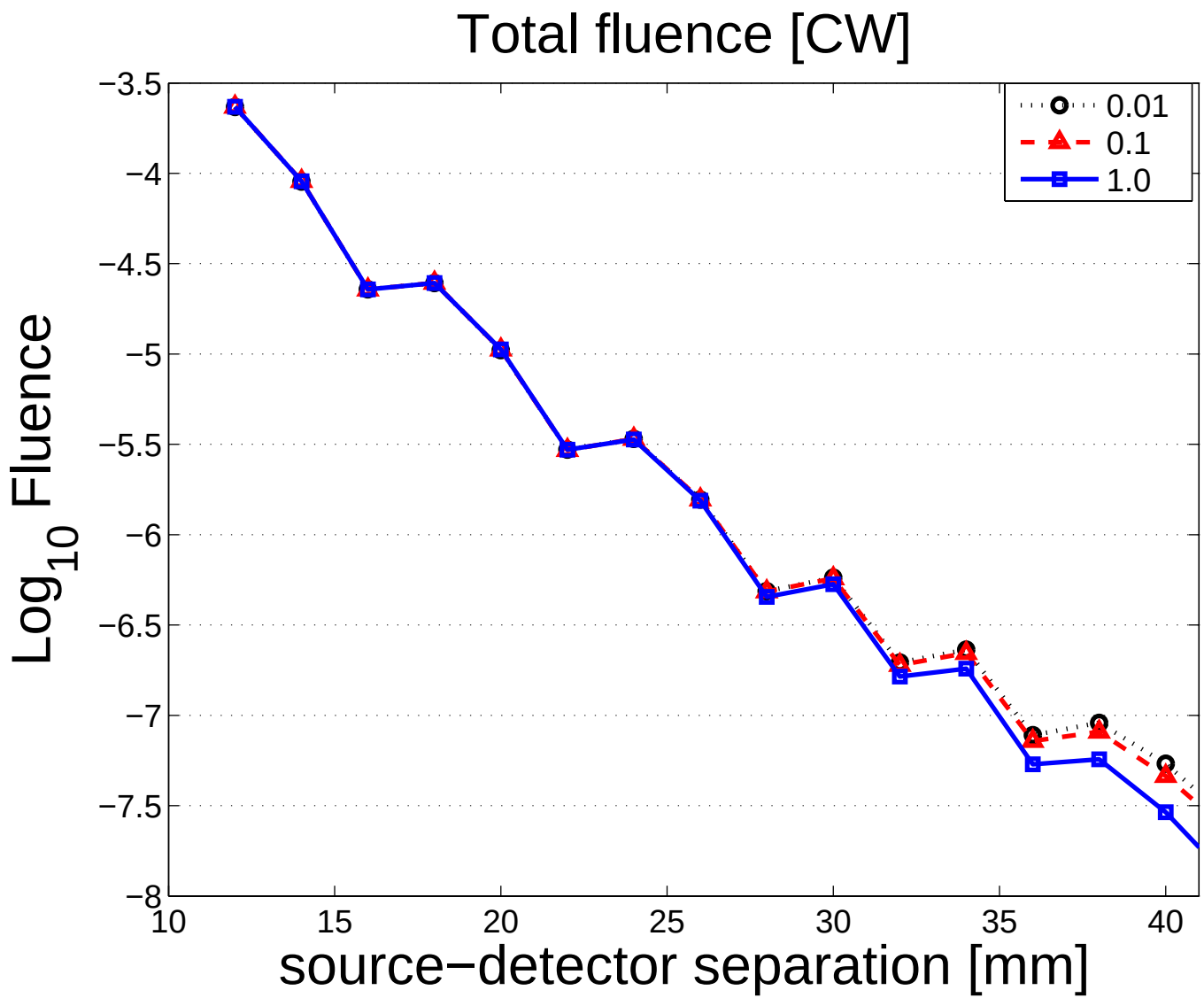
Diaphanography

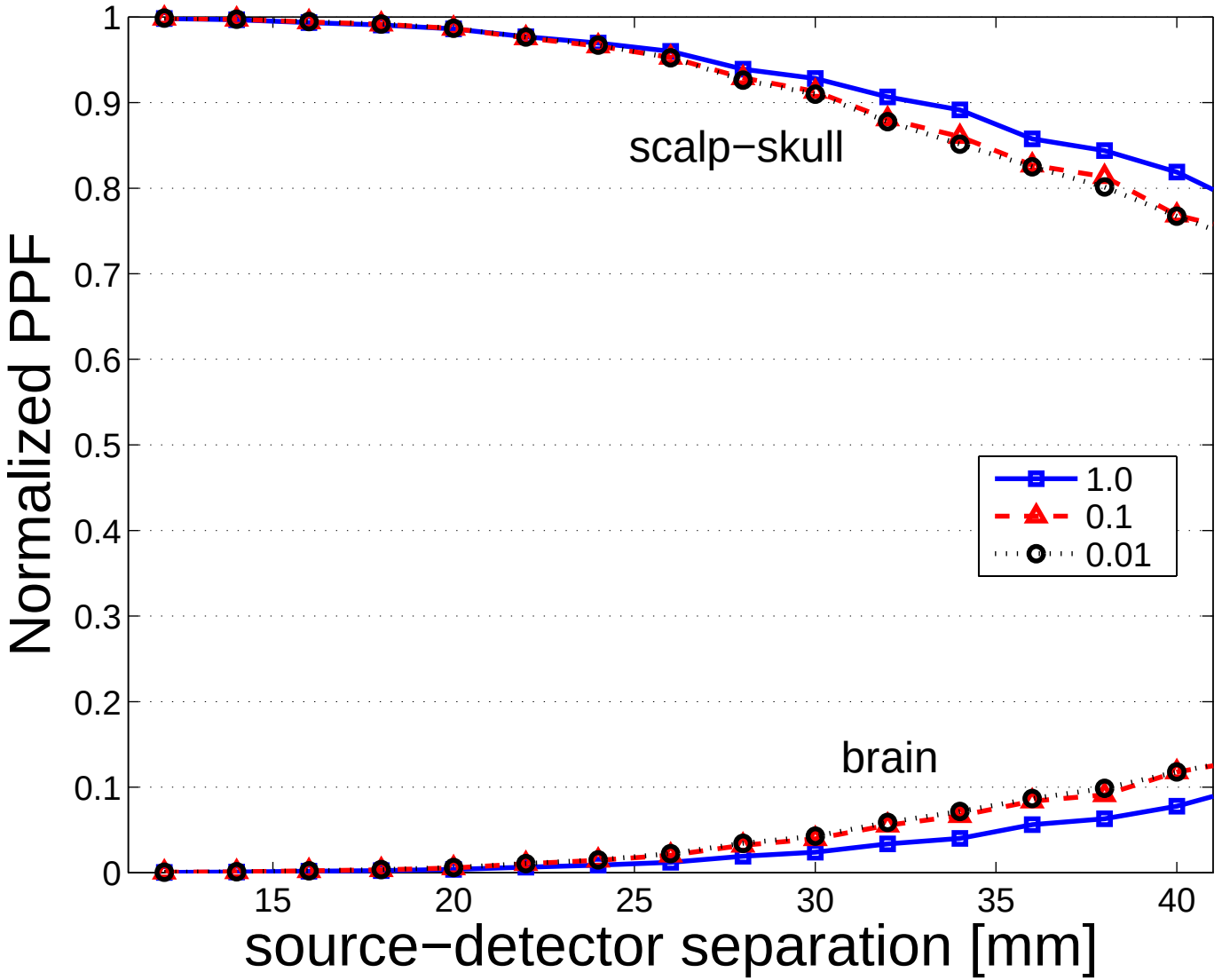
- "Invert" Diffusion Equation

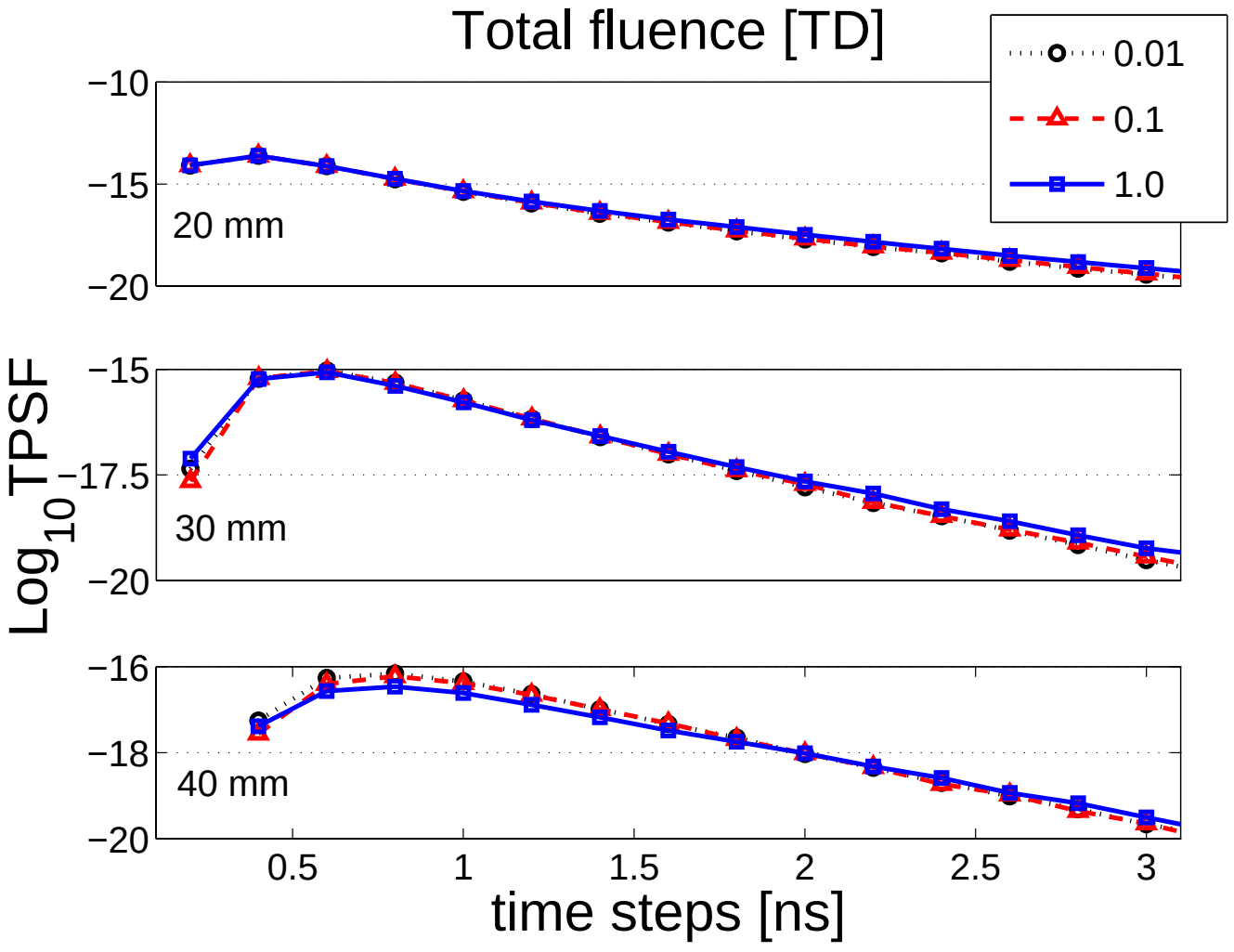


- Regions of Influence


Head coronal slice
30
40
50
60
70
80
90
100
110
120
mm
60
80
100
120
140
160
mm
scalp
skull
CSF
gray
white


Total fluence [CW]
0.01
0.1
1.0
−3.5
−4
−4.5
−5
−5.5
−6
−6.5
−7
−7.5
−8
10
15
20
25
30
35
40
Log10 Fluence
source−detector separation [mm]


Normalized PPF
source−detector separation [mm]
scalp−skull
brain
1.0
0.1
0.01
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
15
20
25
30
35
40


Total fluence [TD]
0.01
0.1
1.0
−10
−15
−20
20 mm
−15
−17.5
−20
30 mm
−16
−18
−20
40 mm
Log10 TPSF
0.5
1
1.5
2
2.5
3
time steps [ns]

COMPUTATIONAL IMAGING

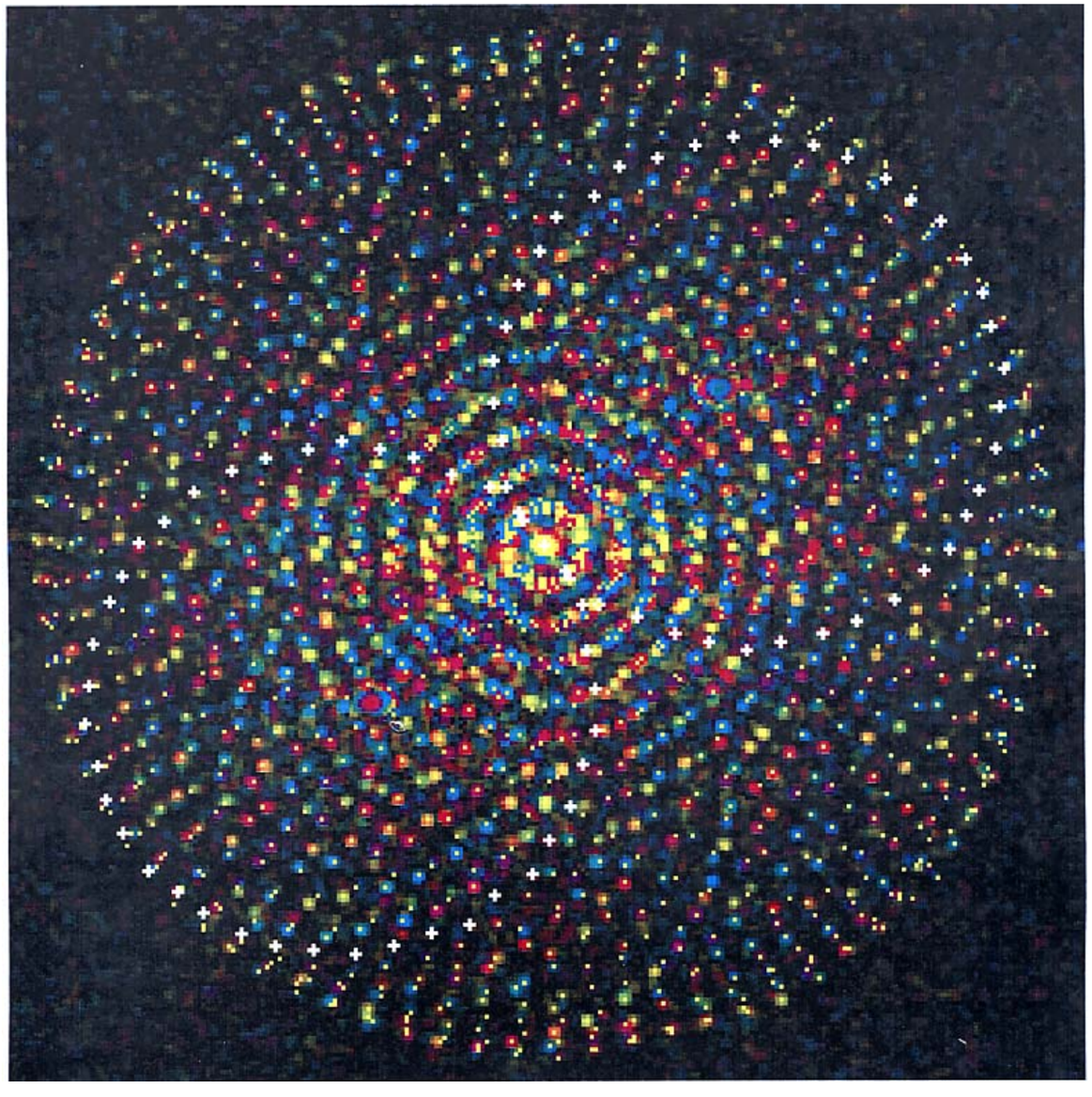
(1) Synthetic Aperture Imaging

(2) Coded Aperture Imaging

(3) Exact Cone Beam Reconstruction

(4) Diaphanography—DiffuseTomography

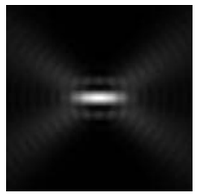
COMPUTATIONAL IMAGING



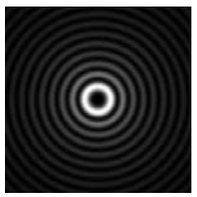
Synthetic Aperture Lithography

- Create pattern — controlled interference

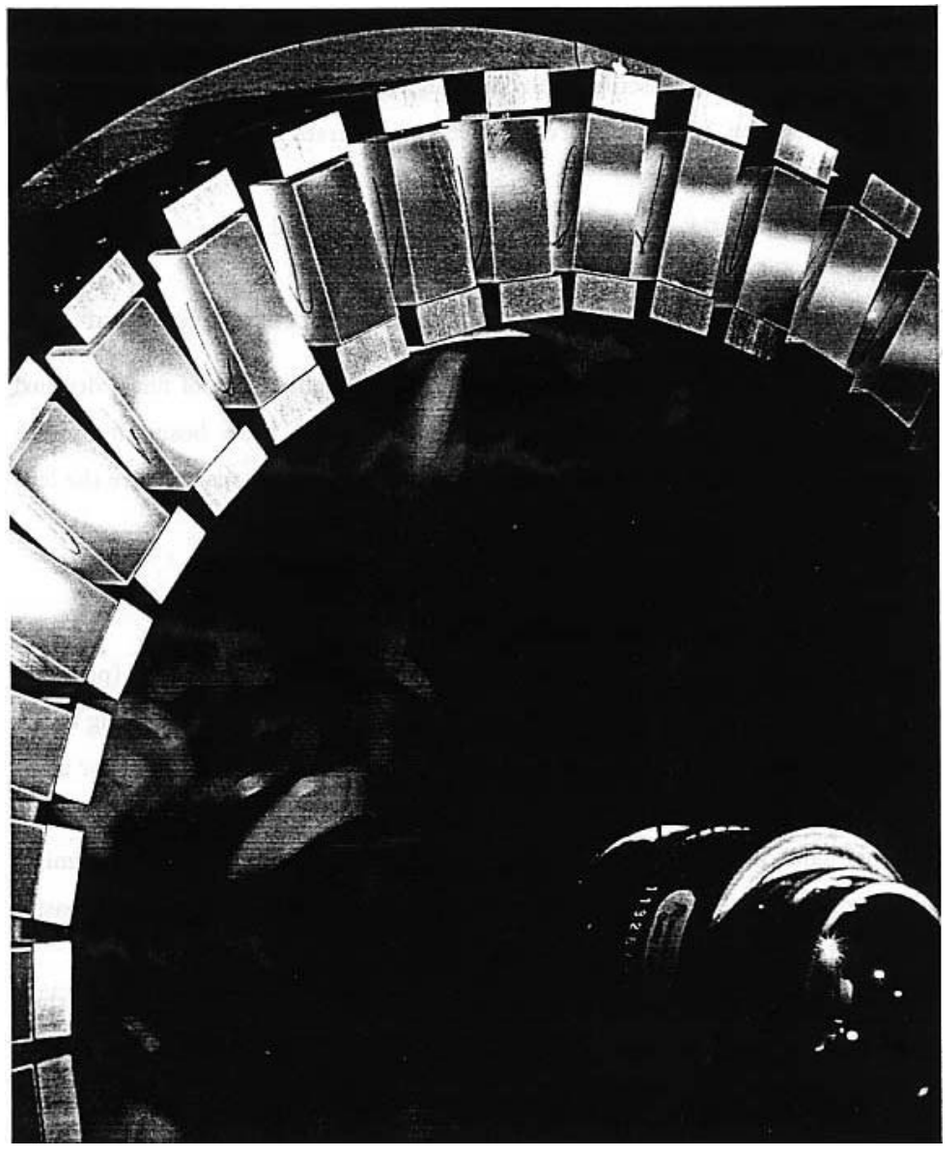
  Example: Two Dots 

  Example: Straight Line 

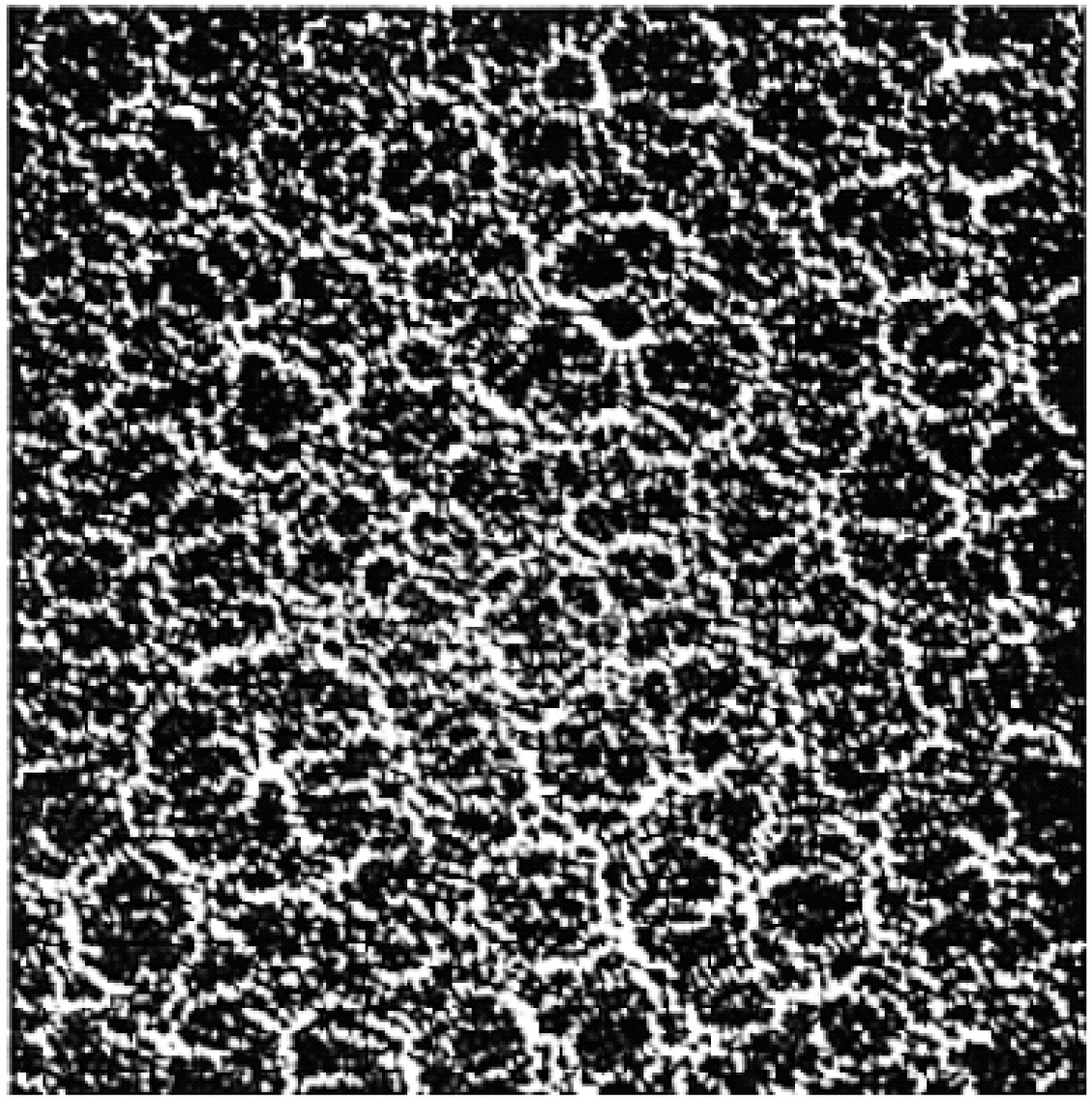
- Destructive interference “safe zone”

  Example: Bessel Ring 

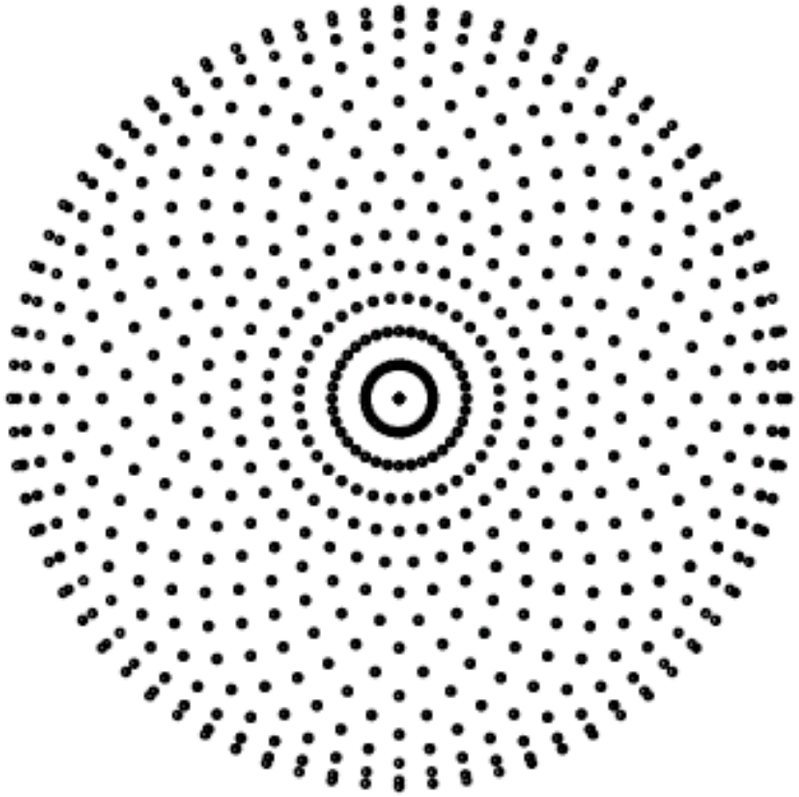
SAM M4



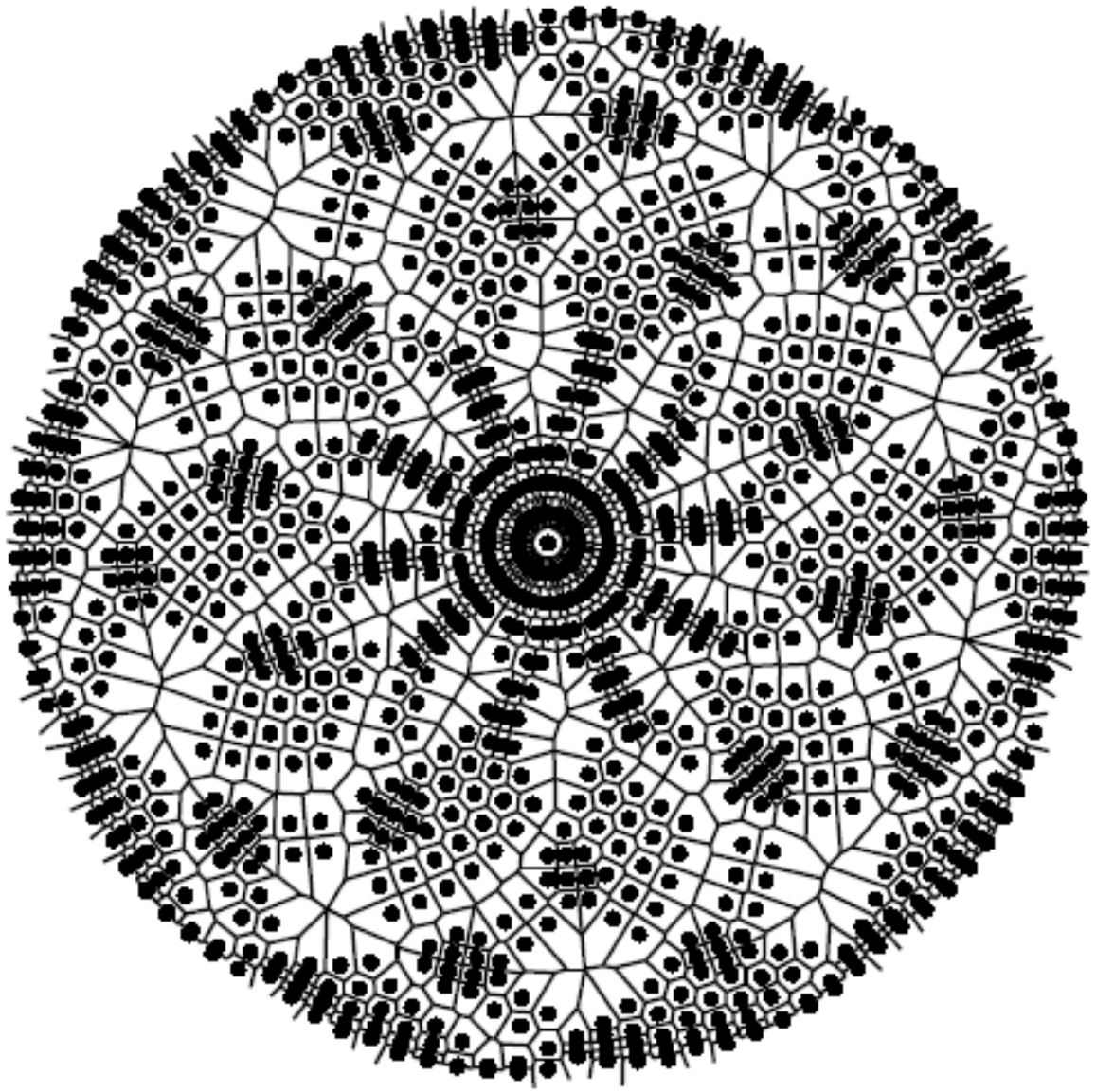
Interference Pattern Texture



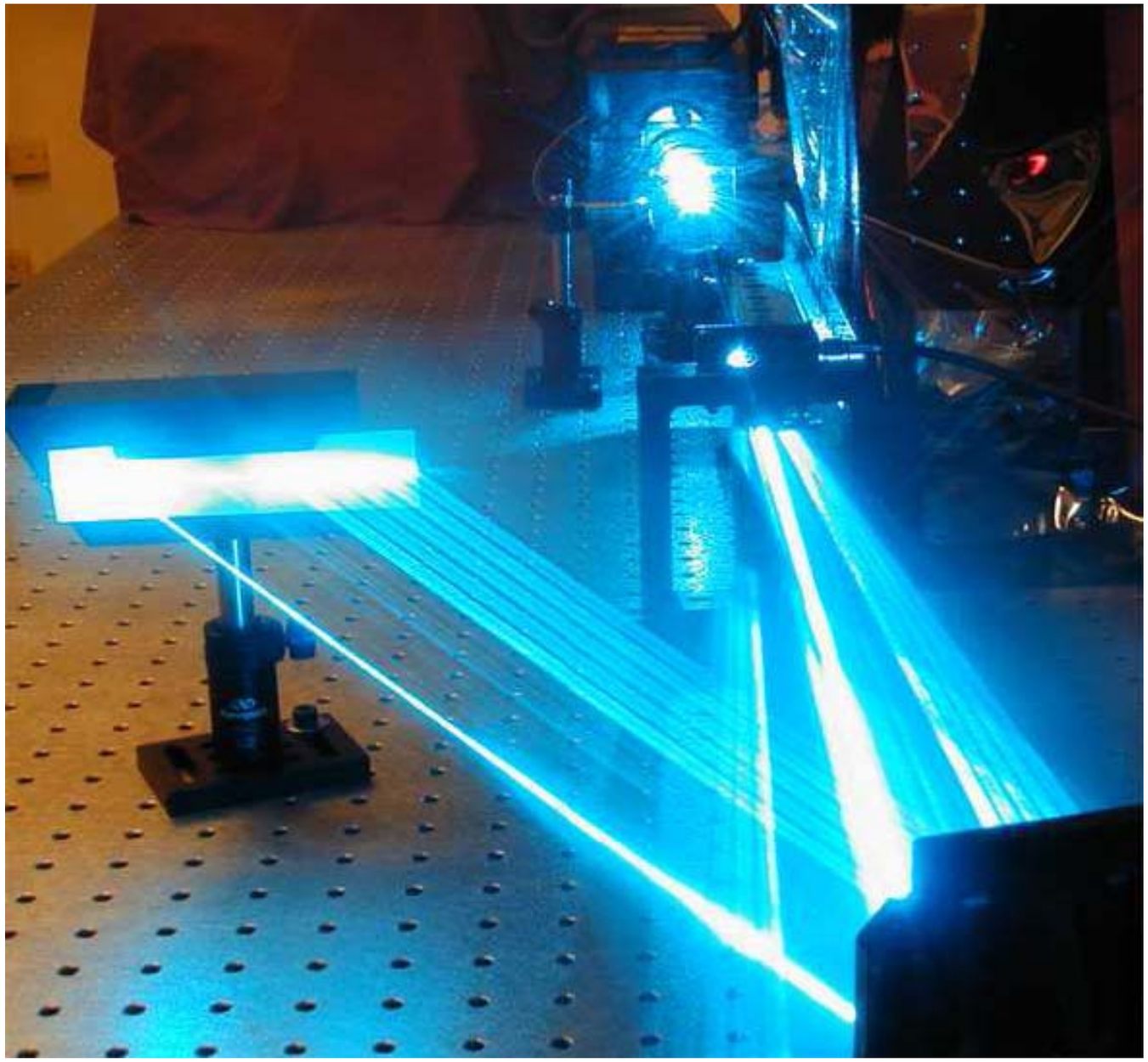
Fourier Transform of Texture Pattern

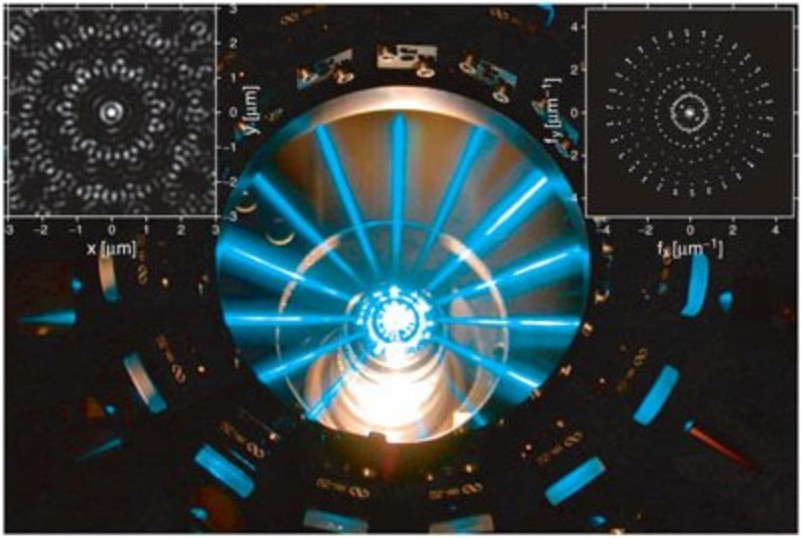


Uneven Fourier Sampling



Amplitude and Phase Control





Cover image

Stanley S. Hong, Berthold K. P. Horn, Dennis M. Freeman, and Michael S. Mermelstein, Appl. Phys. Lett. ***88****, 261107 (2006).*

Read the Article

Back

Resolution Enhancement

- Reflective Optics Illumination

  Vaccum UV — Short Wavelength

Resolution Enhancement

- Reflective Optics Illumination

  Vaccum UV — Short Wavelength

- Fluorescence Mode

  Resolution Determined by Illumination

Masks — Fresnel Camera

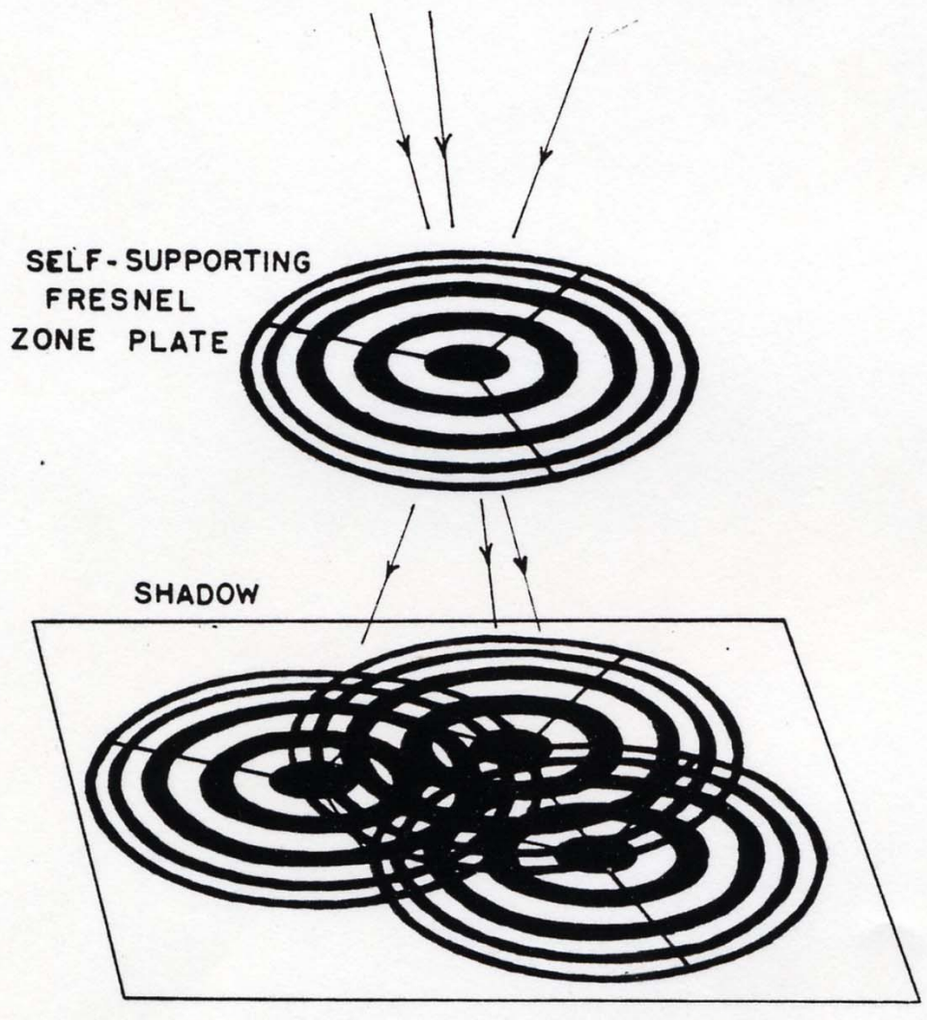
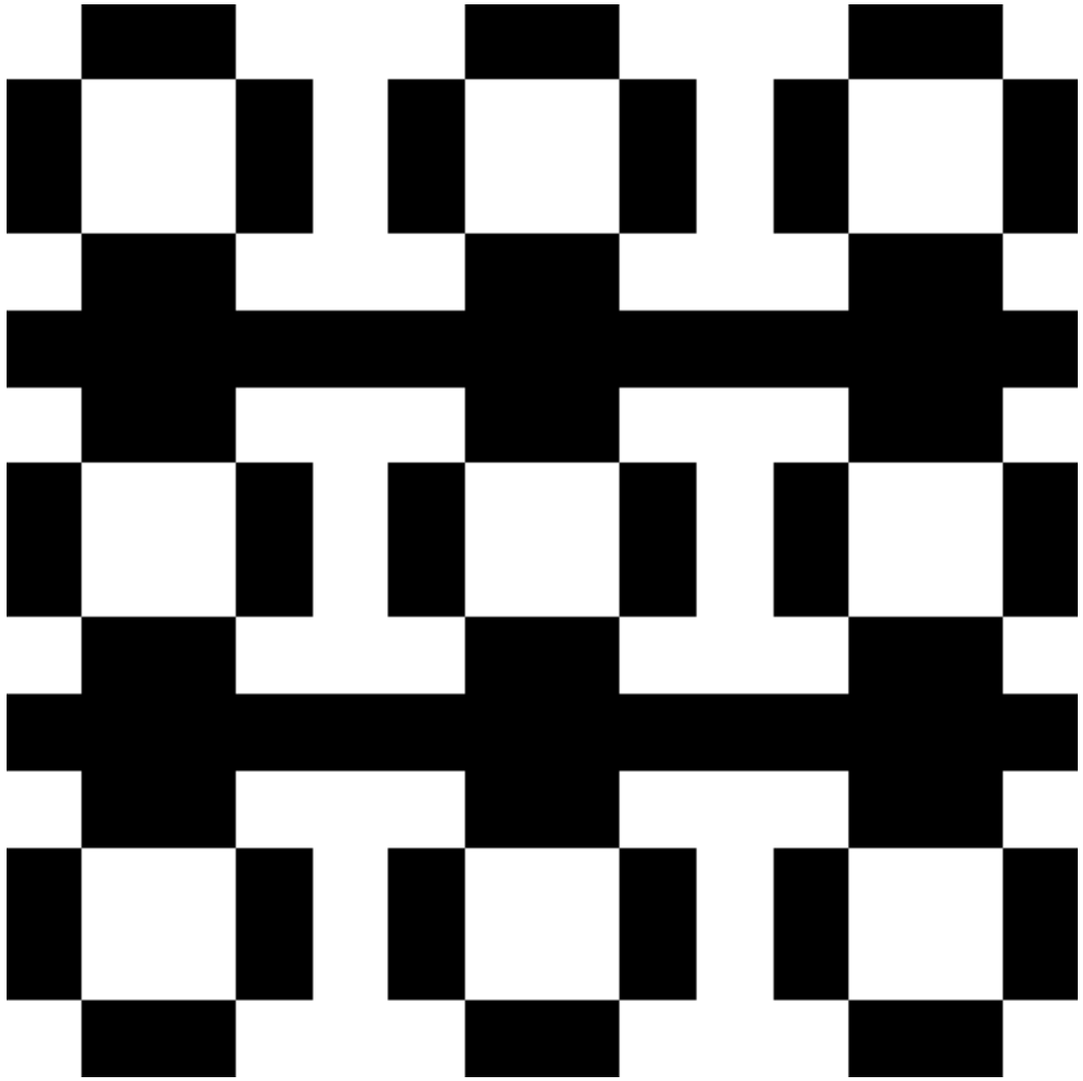
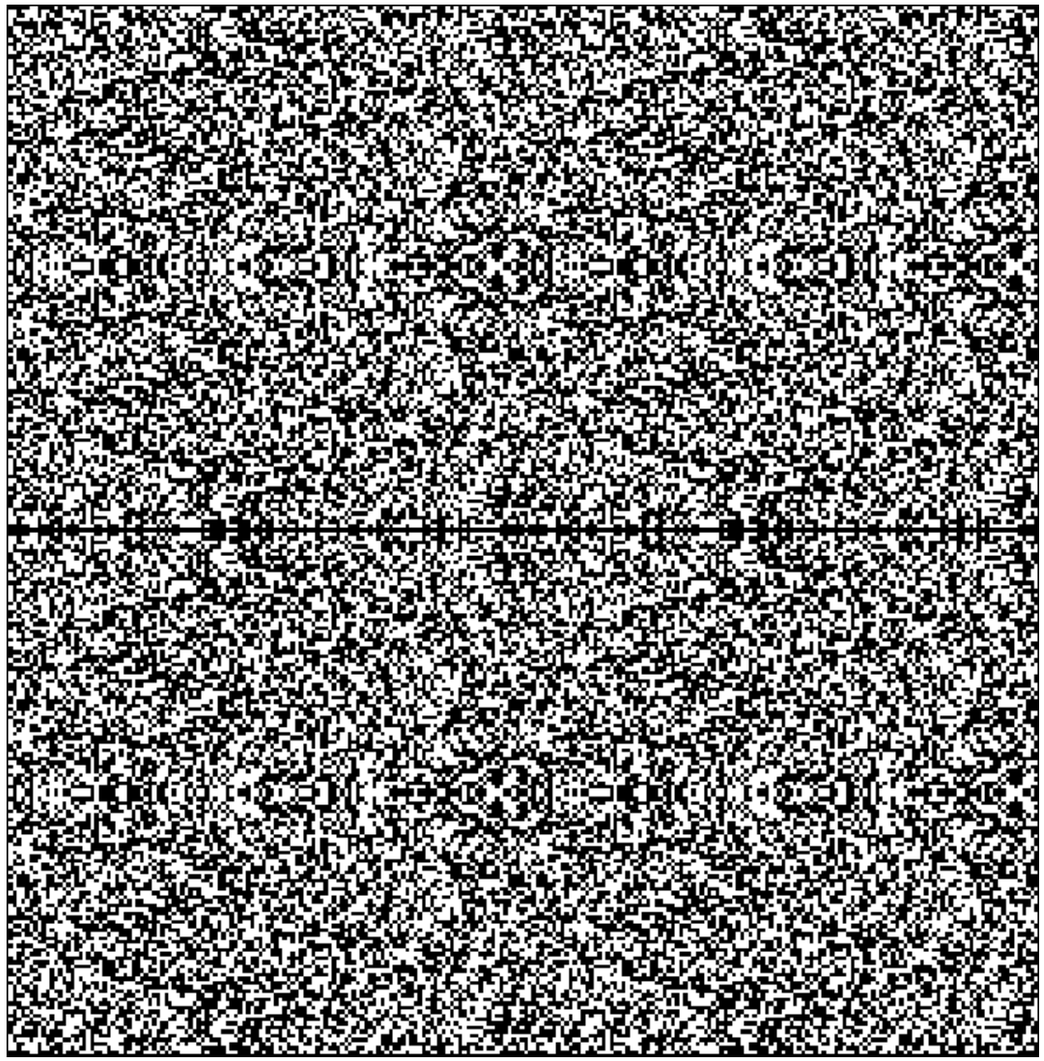


FIG. 1. X-ray star camera.

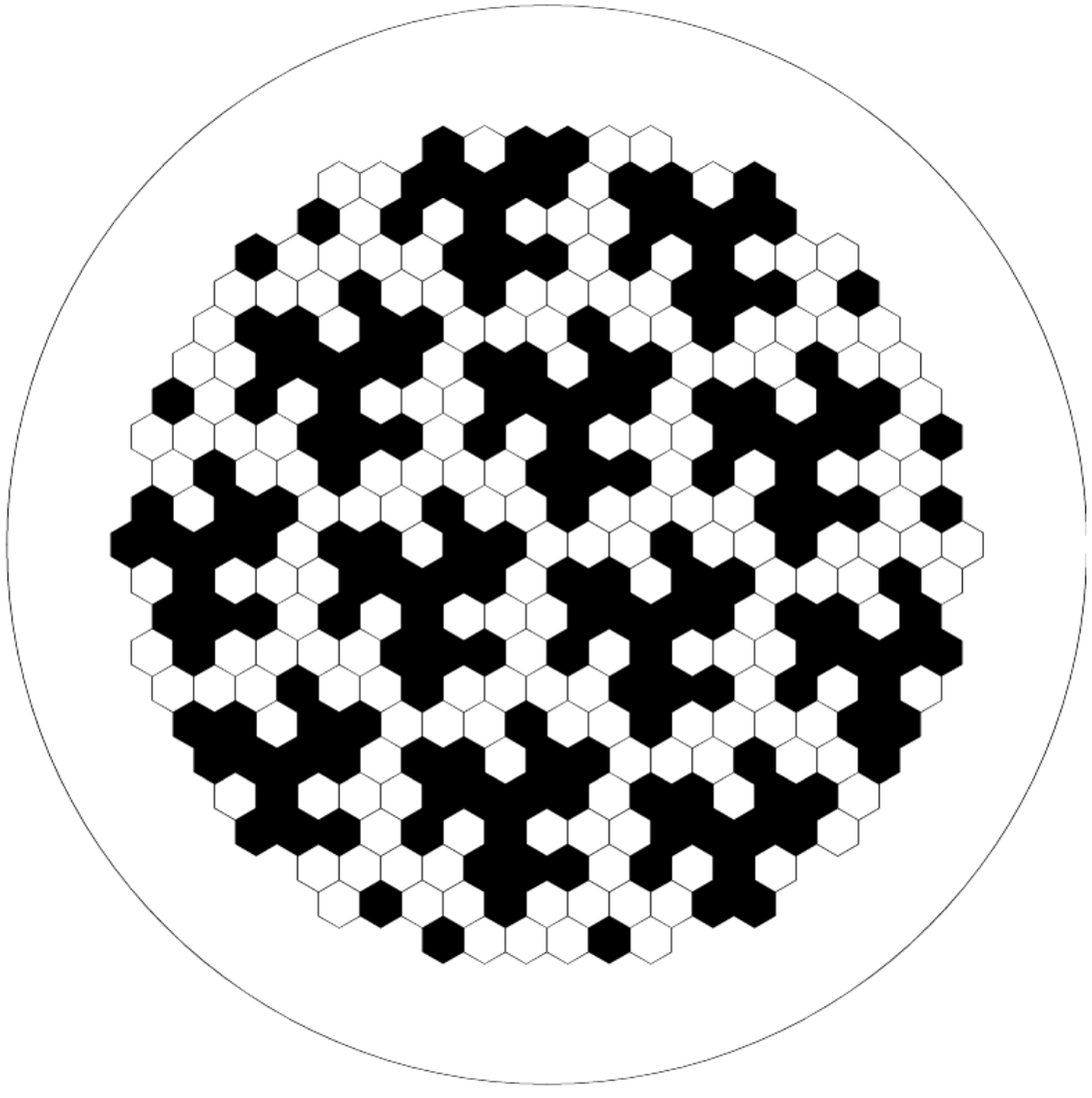
Masks — Legri URA



Masks — XRT Fine



Masks — Hexagonal

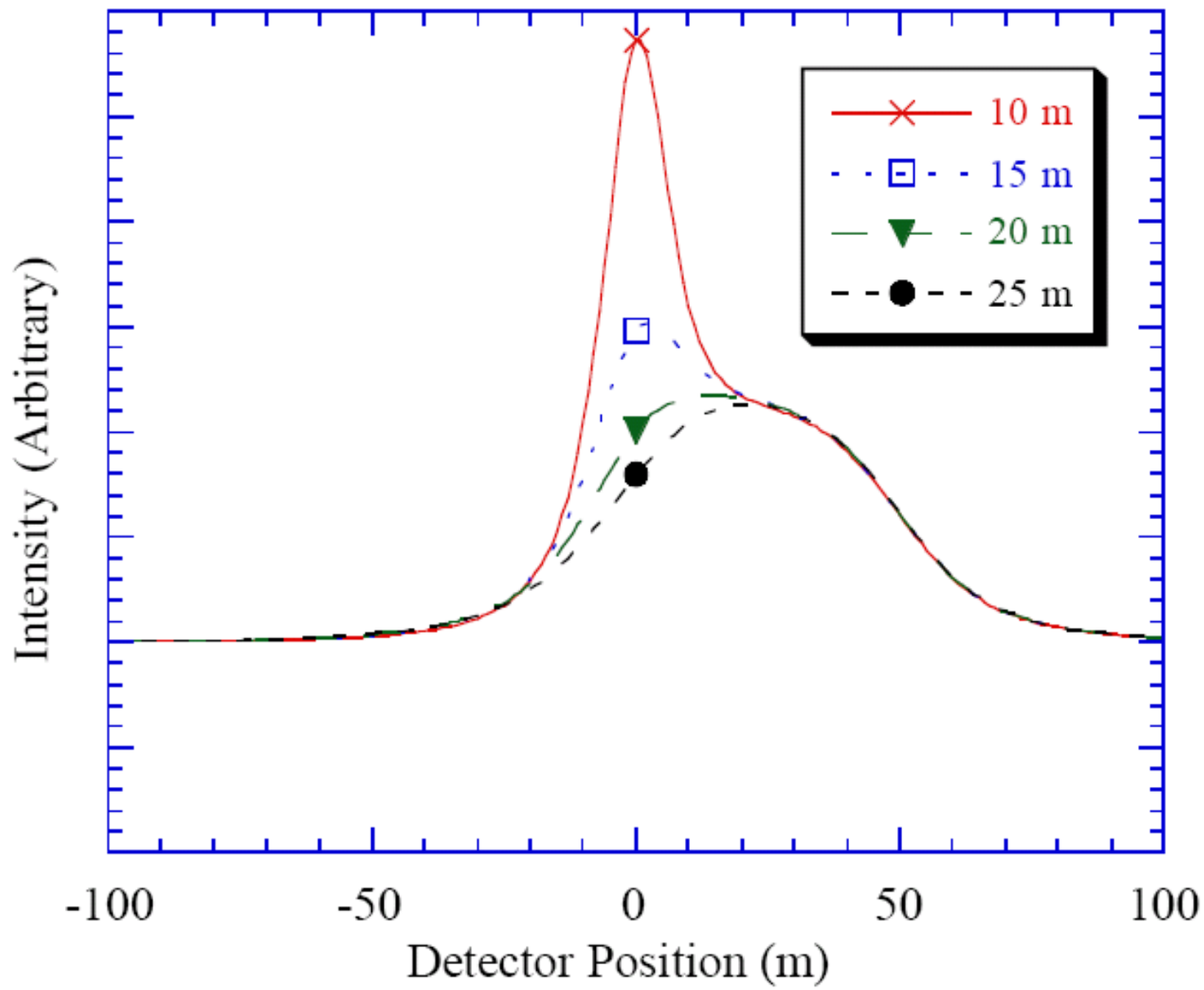


Maximizing SNR

$$\min \sum_{i=1}^{n} w_i^2 \qquad \text{subject to} \qquad \sum_{i=1}^{n} w_i = 1$$

$$\text{yields} \qquad w_i = \frac{1}{n}$$

Spatially Varying Background

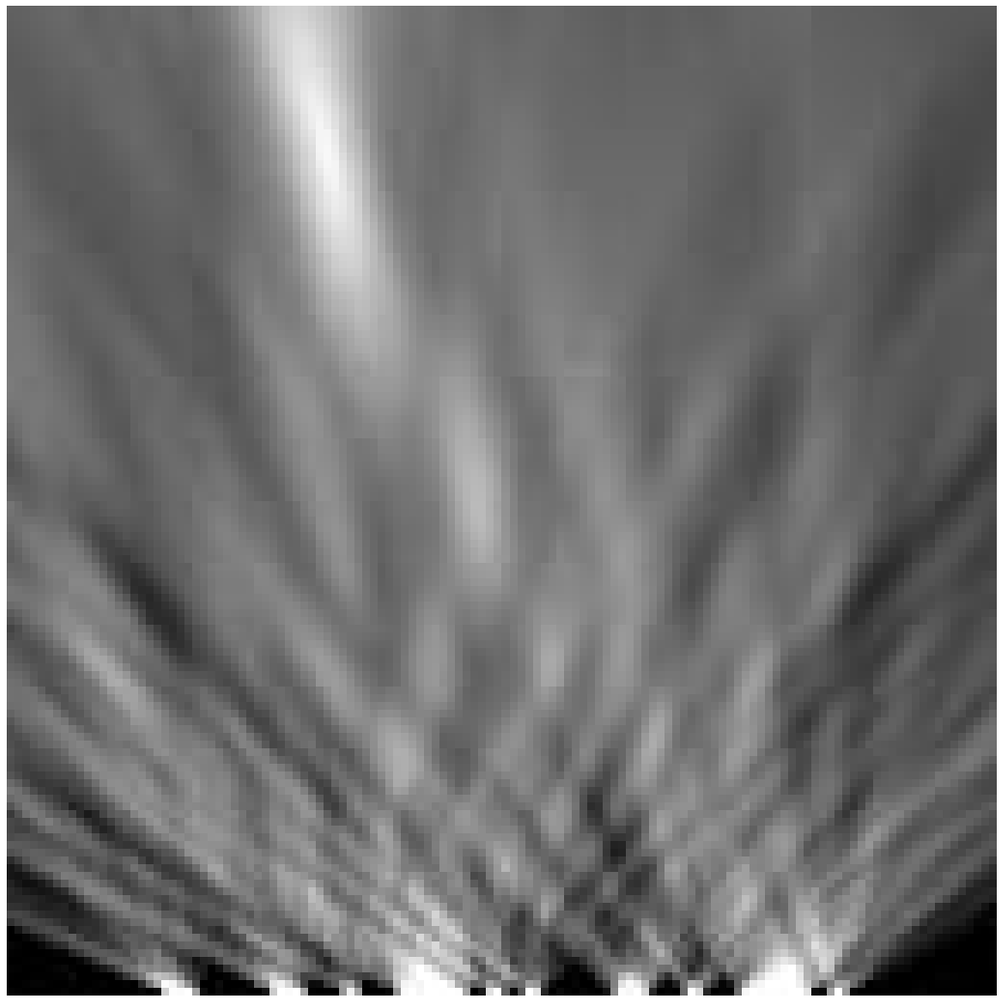


Coded Aperture Extensions

- Artifacts due to Finite Distance
- Mask / Countermask Combination

- Multiple Detector Array Positions
- “Synthetic Aperture” Radiography

Dynamic Reconstruction



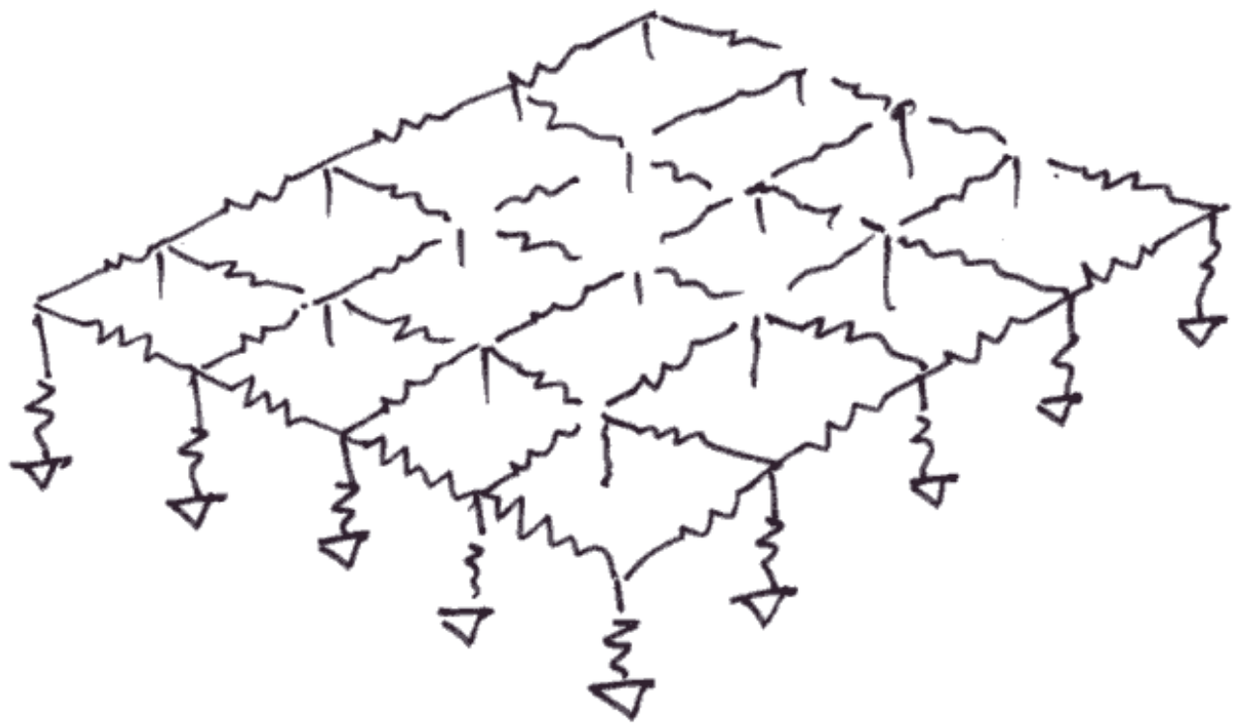
Exact Cone Beam Reconstruction

- Data Sufficiency Condition
- Good “Orbit” for Radiation Source

- Practical Issue: Spiral CT Scanners
- Practical Issue: “Long Body” Problem

Diaphanography

- Approximation: Diffusion Equation



- Leaky Resistive Sheet Analog (2D)