Defining boundaries

16-385 Computer Vision
Where are the object boundaries?
Human annotated boundaries
edge detection
Multi-scale edge detection
Edge strength does not necessarily correspond to our perception of boundaries.
Where are the object boundaries?
Human annotated boundaries
edge detection
Defining boundaries are hard for us too
Applications

Autonomous Vehicles
(lane line detection)

tissue engineering
(blood vessel counting)

behavioral genetics
(earthworm contours)

Autonomous Vehicles
(semantic scene segmentation)

Computational Photography
(image inpainting)
Extracting Lines

16-385 Computer Vision
Where is the boundary of the mountain top?
Lines are hard to find

Original image  Edge detection  Thresholding

Noisy edge image
Incomplete boundaries
idea #1: morphology

What are some problems with the approach?
Dilation

If filter response > 0, set to 1

Erosion

If filter response is MAX, set to 1
idea #2: breaking lines

Divide and Conquer:

**Given:** Boundary lies between points A and B

**Task:** Find boundary

Connect A and B with Line

- Find strongest edge along line bisector
- Use edge point as break point

Repeat

What are some problems with the approach?
idea #3: line fitting

Given: Many \((x_i, y_i)\) pairs

Find: Parameters \((m, c)\)

Minimize: Average square distance:

\[
E = \frac{\sum_i (y_i - mx_i - c)^2}{N}
\]

What are some problems with the approach?
idea #3: line fitting

Given: Many \((x_i, y_i)\) pairs

Find: Parameters \((m, c)\)

Minimize: Average square distance:

\[
E = \frac{1}{N} \sum_i (y_i - mx_i - c)^2
\]

Using:

\[
\frac{\partial E}{\partial m} = 0 \quad \& \quad \frac{\partial E}{\partial c} = 0
\]

Note:

\[
\bar{y} = \frac{\sum_i y_i}{N} \quad \bar{x} = \frac{\sum_i x_i}{N}
\]

\[
c = \bar{y} - m \bar{x}
\]

\[
m = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}
\]

What are some problems with the approach?
Data: \((x_1, y_1), \ldots, (x_n, y_n)\)

Line equation: \(y_i = mx_i + b\)

Find \((m, b)\) to minimize

\[
E = \sum_{i=1}^{n} (y_i - mx_i - b)^2
\]

\[
Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}, \quad X = \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix}, \quad B = \begin{bmatrix} m \\ b \end{bmatrix}
\]

\[
E = \left\| Y - XB \right\|^2 = (Y - XB)^T (Y - XB) = Y^T Y - 2(XB)^T Y + (XB)^T (XB)
\]

\[
\frac{dE}{dB} = 2X^T XB - 2X^T Y = 0 \quad \Rightarrow \quad X^T XB = X^T Y
\]

Normal equations: least squares solution to \(XB=Y\)
Problems with parameterizations

Where is the line that minimizes $E$?

$$E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$
Problems with parameterizations

Where is the line that minimizes $E$?

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Problems with parameterizations

Where is the line that minimizes E?

\[ E = \sum_{i=1}^{n} (y_i - mx_i - b)^2 \]
Problems with parameterizations

Where is the line that minimizes $E$?

Line that minimizes $E$!!

(Error $E$ must be formulated carefully!)

Use this instead:

$$E = \frac{1}{N} \sum_i (\rho - x_i \cos \theta + y_i \sin \theta)^2$$

I'll explain this later …
Line fitting can be maximum likelihood
- but choice of model is important
Problems with noise

Least-squares error fit

Squared error heavily penalizes outliers
Model fitting is difficult because...

- **Extraneous data**: clutter or multiple models
  - We do not know what is part of the model?
  - Can we pull out models with a few parts from much larger amounts of background clutter?

- **Missing data**: only some parts of model are present

- **Noise**

- **Cost**:
  - It is not feasible to check all combinations of features by fitting a model to each possible subset

*So what can we do?*
Slope intercept form

\[ y = mx + b \]
Double intercept form

\[ \frac{x}{a} + \frac{y}{b} = 1 \]

x-intercept

y-intercept

Derivation:

(Similar slope)

\[ \frac{y - b}{x - 0} = \frac{0 - y}{a - x} \]

\[ ya + yx - ba + bx = -yx \]

\[ ya + bx = ba \]

\[ \frac{y}{b} + \frac{x}{a} = 1 \]
Normal Form

\[ x \cos \theta + y \sin \theta = \rho \]

**Derivation:**

\[
\cos \theta = \frac{\rho}{a} \rightarrow a = \frac{\rho}{\cos \theta}
\]

\[
\sin \theta = \frac{\rho}{b} \rightarrow b = \frac{\rho}{\sin \theta}
\]

\[
\frac{x}{a} + \frac{y}{b} = 1
\]

\[ x \cos \theta + y \sin \theta = \rho \]