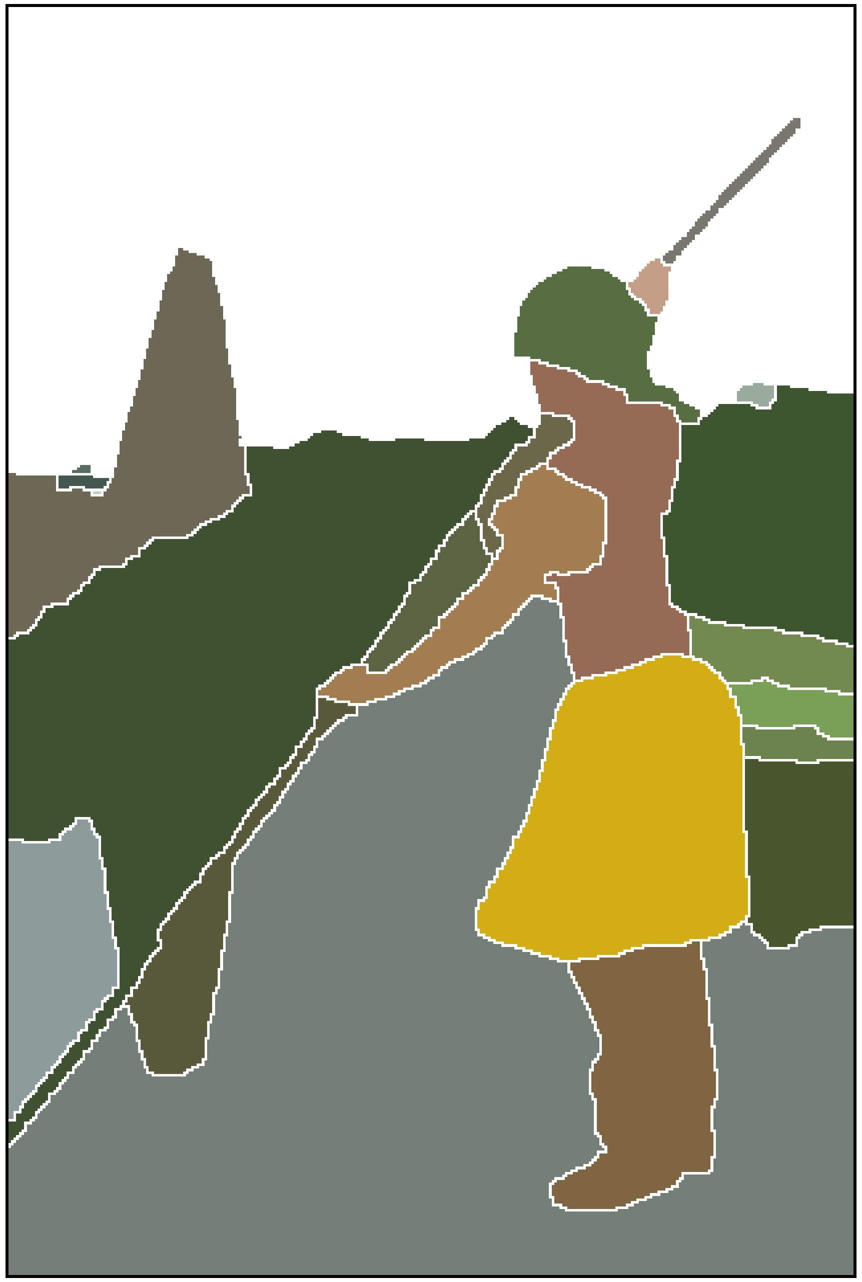
Segmentation



16-385 Computer Vision
Spring 2018, Lecture 27

http://www.cs.cmu.edu/~16385/

Course announcements

- Homework 7 is due on Sunday 6th.
  - Any questions about homework 7?
  - How many of you have looked at/started/finished homework 7?
- Yannis will have extra office hours Tuesday 4-6 pm.
- How many of you went to Vladlen Koltun's talk?

Overview of today's lecture

- Graph-cuts and GrabCut.
- Normalized cuts.
- Boundaries.
- Clustering for segmentation.

Slide credits

Most of these slides were adapted from:

- Srinivasa Narasimhan (16-385, Spring 2015).
- James Hays (Brown University).

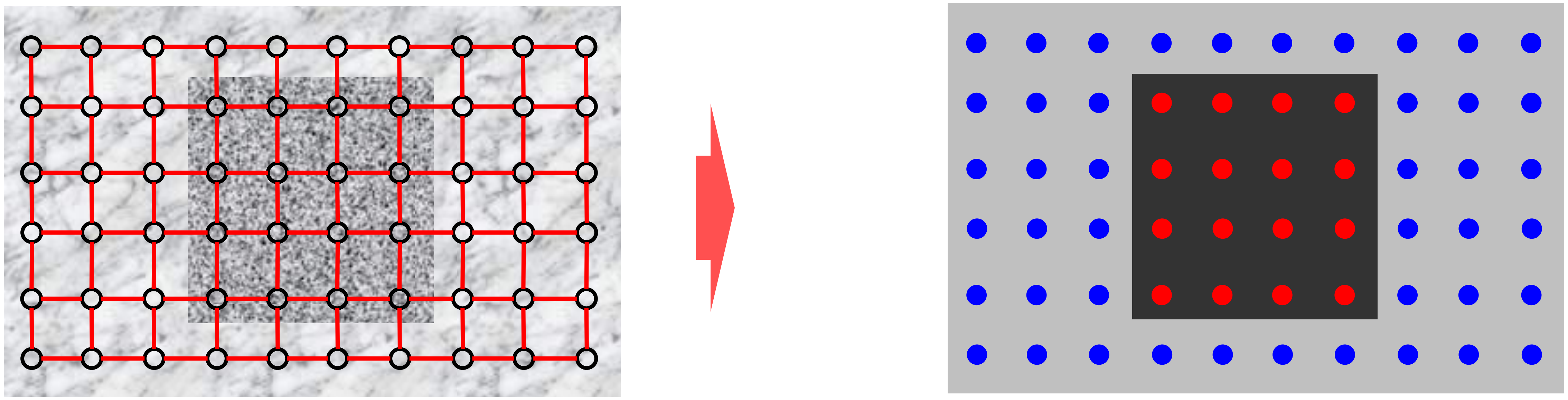
Image segmentation by pairwise similarities

- Image = { pixels }
- Segmentation = partition of image into segments
- Similarity between pixels i and j

  $S_{ij} = S_{ji} \geq 0$

- Objective: "similar pixels, with large value of S_{ij}, should be in the same segment, dissimilar pixels should be in different segments"

Relational Graphs



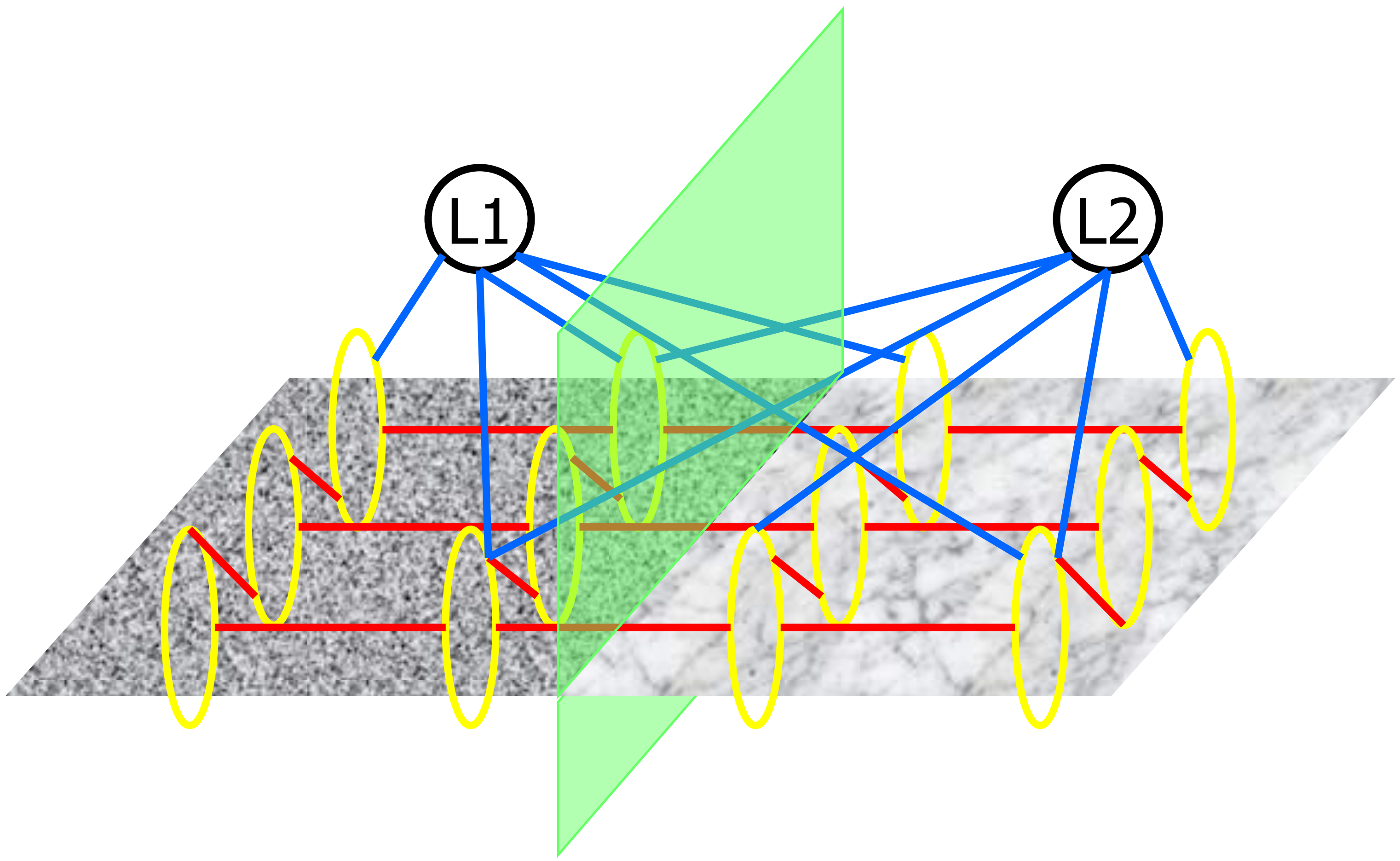
- G=(V, E, S)
  - ➔ V: each node denotes a pixel
  - ➔ E: each edge denotes a pixel-pixel relationship
  - ➔ S: each edge weight measures pairwise similarity

- Segmentation = node partitioning
  - ➔ break V into disjoint sets V_1 , V_2

Solving MRF by Graph Partitioning

Some simple MRF models can be translated into graph partitioning

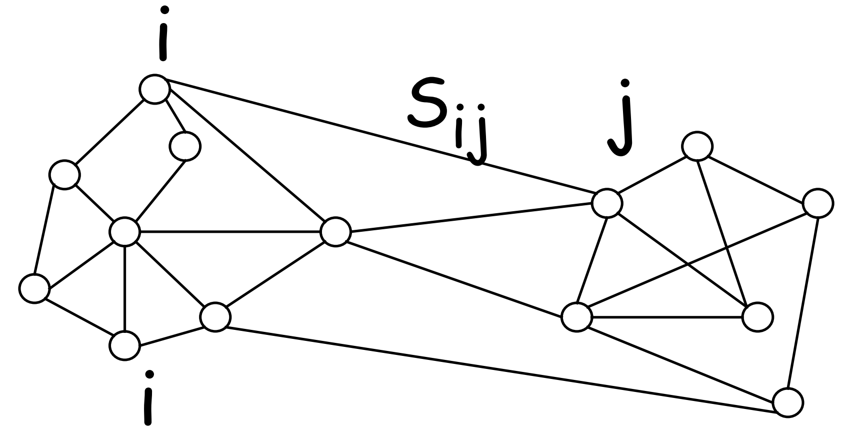
$$\min \; E(X;f) = \underbrace{\sum_{p} \sum_{q \in N(p)} W_{p,q}(X_p, X_q)}_{\text{pair relationships}} + \underbrace{\sum_{p} U_p(X_p, f_p)}_{\text{data measures}}$$



Weighted graph partitioning

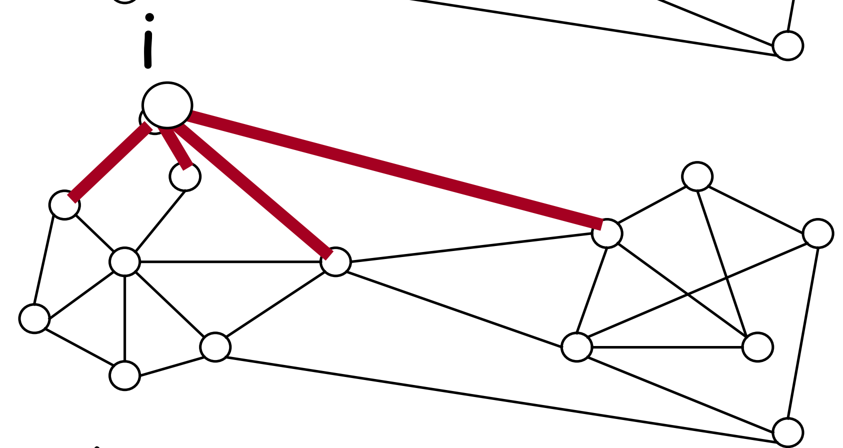
Pixels i   I = vertices of graph G

Edges ij = pixel pairs with $S_{ij} > 0$

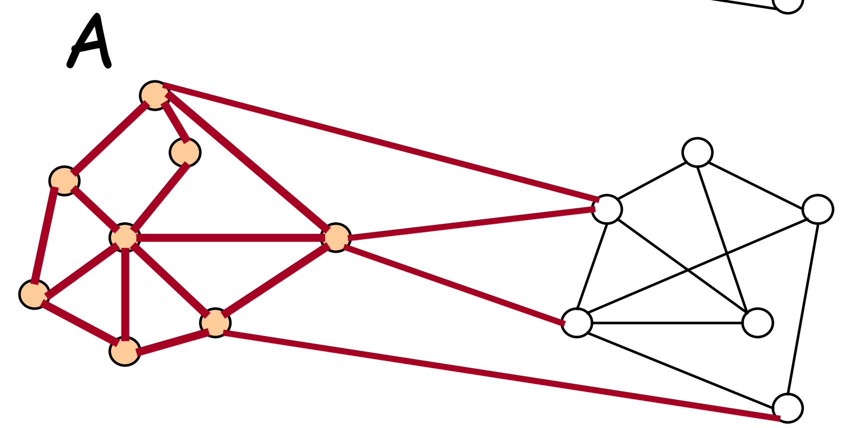


Similarity matrix $S = [\ S_{ij}\ ]$

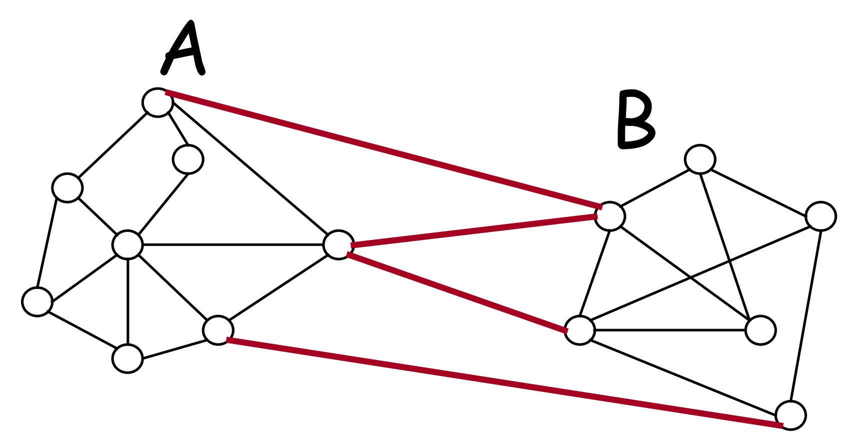
$d_i = \Sigma_{j \in G} S_{ij}$ degree of I



deg A = $\Sigma_{i \in A}\, d_i$ degree of A $\subseteq$ G



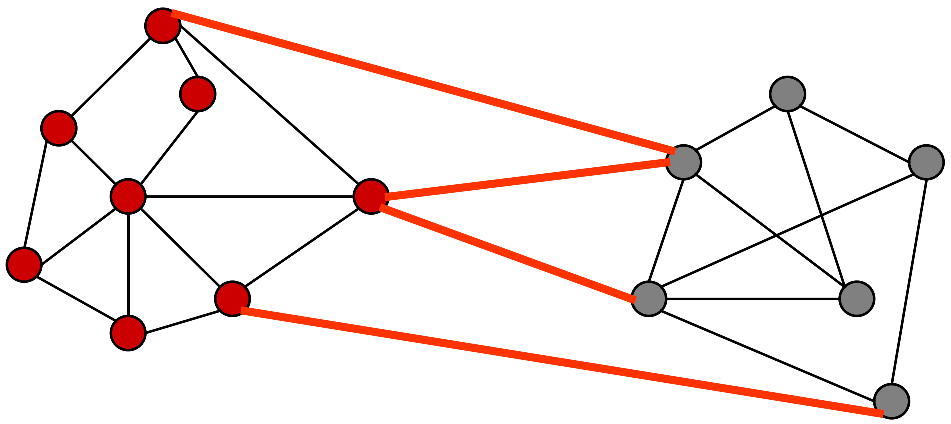
Assoc(A,B) = $\Sigma_{i \in A}\, \Sigma_{j \in B}\, S_{ij}$



Cuts in a Graph

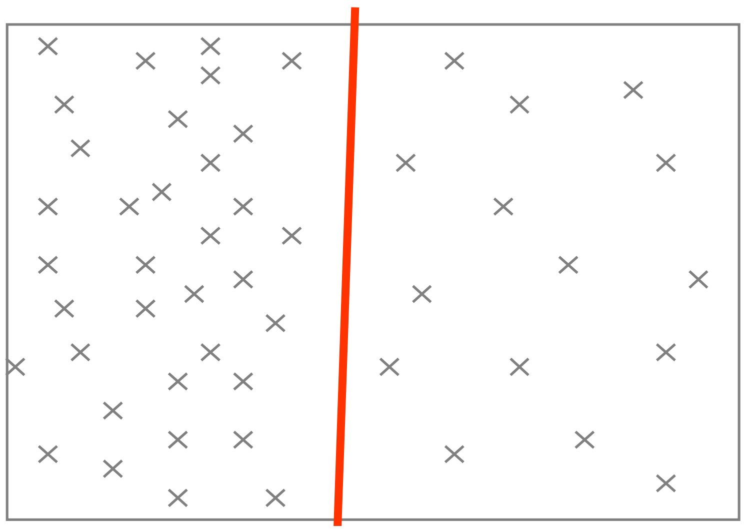
- (edge) cut = set of edges whose removal makes a graph disconnected
- weight of a cut: $\text{cut}(A, B) = \Sigma_{i \in A,} \Sigma_{j \in B} S_{ij} = \text{Assoc}(A,B)$
- the normalized cut

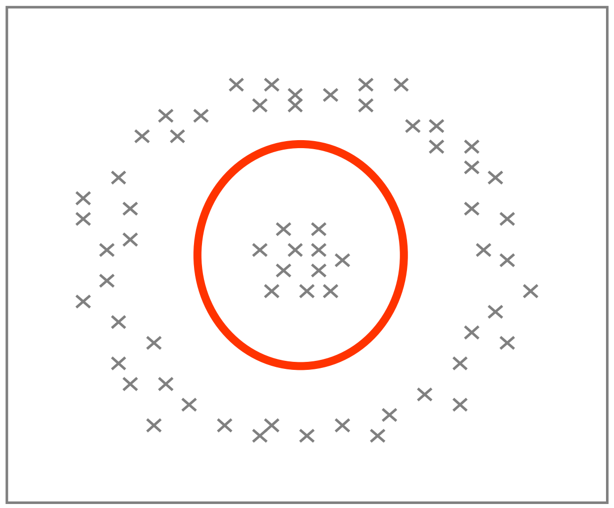
$$\text{NCut}(A,B) = \text{cut}(A, B)\left(\frac{1}{\deg A} + \frac{1}{\deg B}\right)$$



- Normalized Cut criteria: minimum $\text{cut}(A,\bar{A})$

$$S_{ij} = \frac{1}{d(x_i, x_j)} \longrightarrow$$





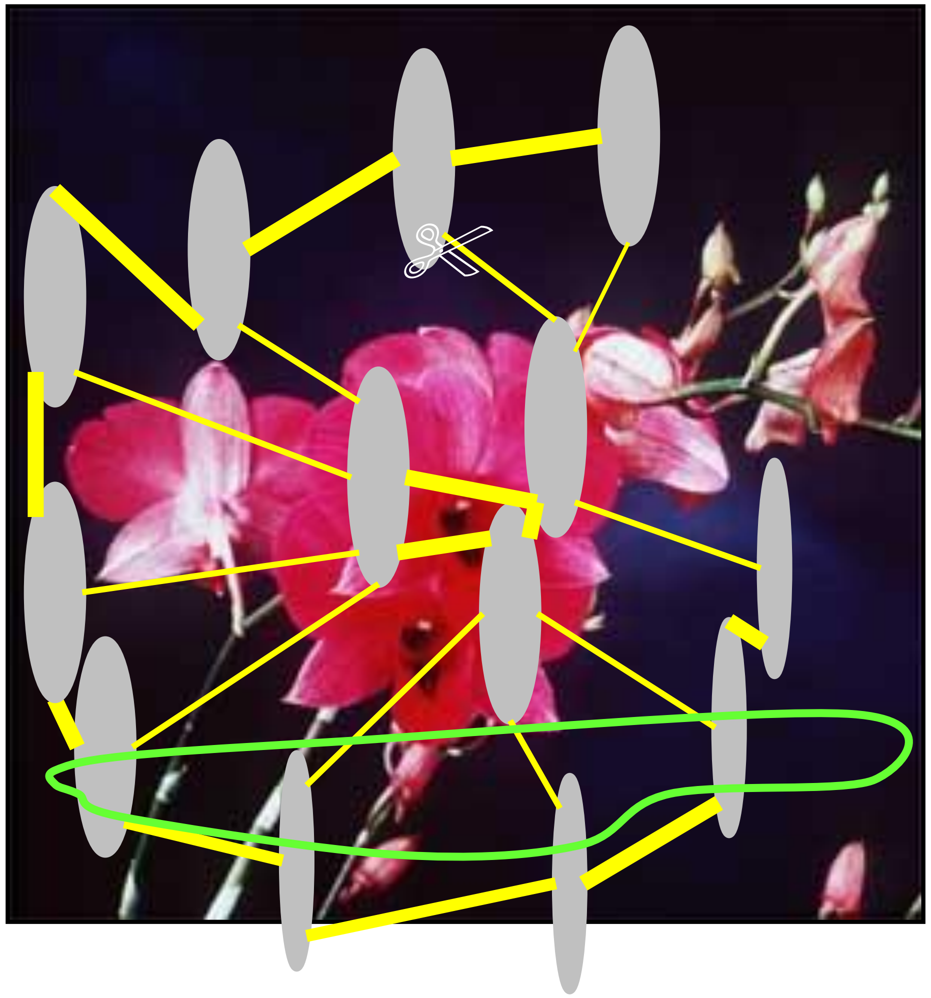
Grouping with Spectral Graph Partitioning

SGP: data structure = a weighted graph, weights describing data affinity

$$\min \quad Ncut(A,B) = \frac{cut(A,B)}{\deg(A)} + \frac{cut(A,B)}{\deg(B)}$$

$$cut(A,B) = \sum_{i \in A} \sum_{j \in B} S(i,j)$$

$$\deg(A) = \sum_{i \in A} \sum_{j \in G} S(i,j)$$



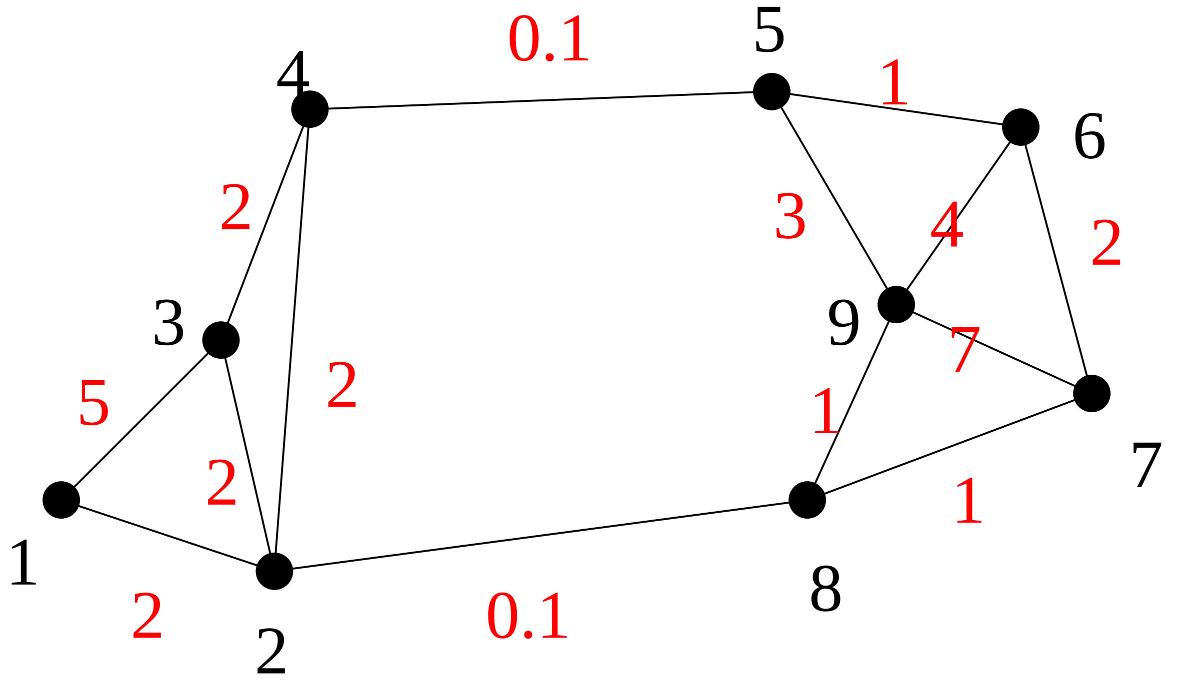
Segmentation is to find a node partitioning of a relational graph, with minimum total cut-off affinity.

Discriminative models are used to evaluate the weights between nodes.

The solution sought is the cuts of the minimum energy.

NP-Hard!

Matrix representation of the graph problem:



$$\mathbf{M} = \begin{bmatrix} 0 & 2 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 2 & 2 & 0 & 0 & 0 & 0.1 & 0 \\ 5 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 7 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 3 & 4 & 7 & 1 & 0 \end{bmatrix}$$

affinity matrix

Eigenvector approach to segmentation

Represent a connected component **(**or cluster *C***)**
By a weight vector **w** such that (*indicator vector*):

$$w_i = \begin{cases} 1 \text{ if } i \in C \\ 0 \text{ if } i \notin C \end{cases}$$

$\mathbf{w}^t\mathbf{M}\mathbf{w}$ is the association of *C* because:

$$\mathbf{w}^t\mathbf{M}\mathbf{w} = \sum_{i,j \in C} m_{ij}$$

If *C* is a good cluster, then the average association between features
In *C* should be large. Therefore, we want:

$\mathbf{w}^t\mathbf{M}\mathbf{w}$ is large

Suggests algorithm:

- Build matrix **M**
- Find **w** such that $\mathbf{w}^t\mathbf{M}\mathbf{w}$ is maximum.

Problem: **w** is a binary vector

Replace binary vector with continuous weight vector. Interpretation:

w_i large if i belongs to C.

Problem becomes:

- Find $\mathbf{w}$ such that $\mathbf{w}^t\mathbf{M}\mathbf{w}$ is maximum
- Construct the corresponding component C by:
  i belongs to C if w_i is large.

Problem with scale:

The relative values of the w_i's are important, the total magnitude of $\mathbf{w}$ is not.

Normalization:

$$\mathop{Max}_{\mathbf{w}} \frac{\mathbf{w}^t\mathbf{M}\mathbf{w}}{\mathbf{w}^t\mathbf{w}}$$

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Normalization:

$$\underset{\mathbf{w}}{Max}\ \frac{\mathbf{w}^t\mathbf{M}\mathbf{w}}{\mathbf{w}^t\mathbf{w}}$$

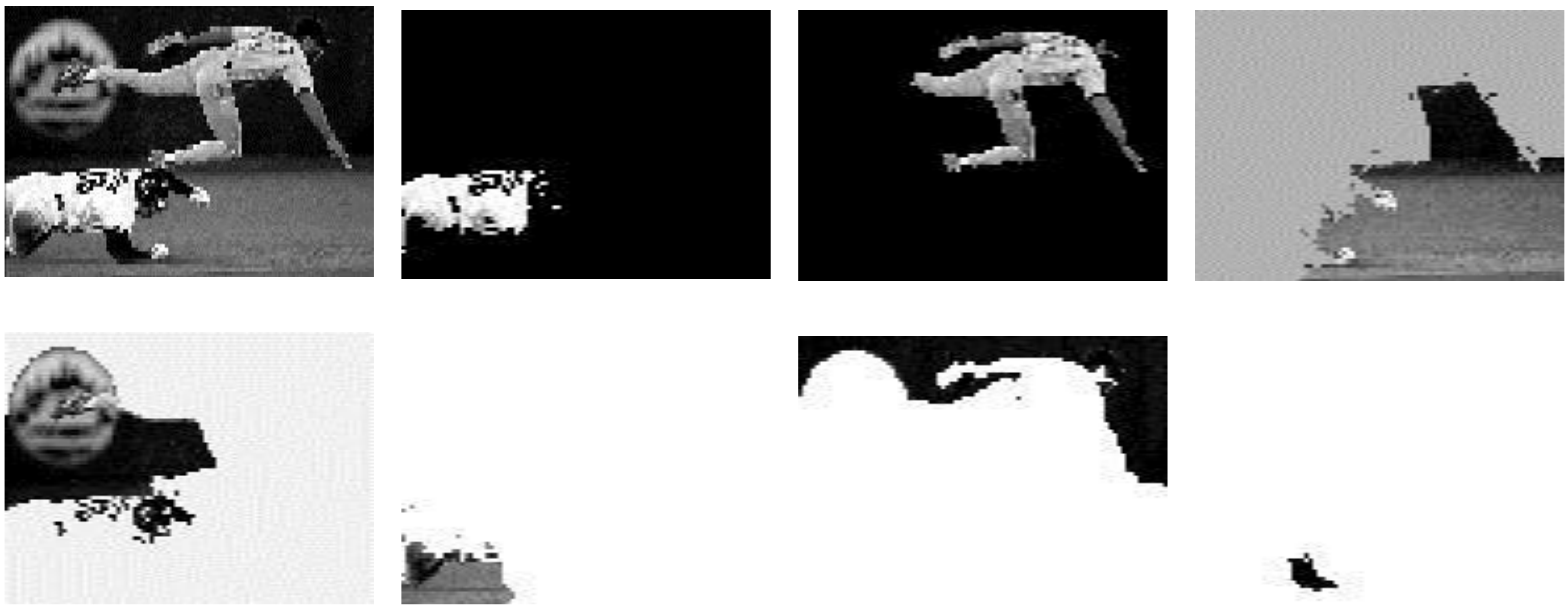
Rayleigh's ratio theorem:

Given a symmetric matrix $\mathbf{M}$, the maximum of the ratio

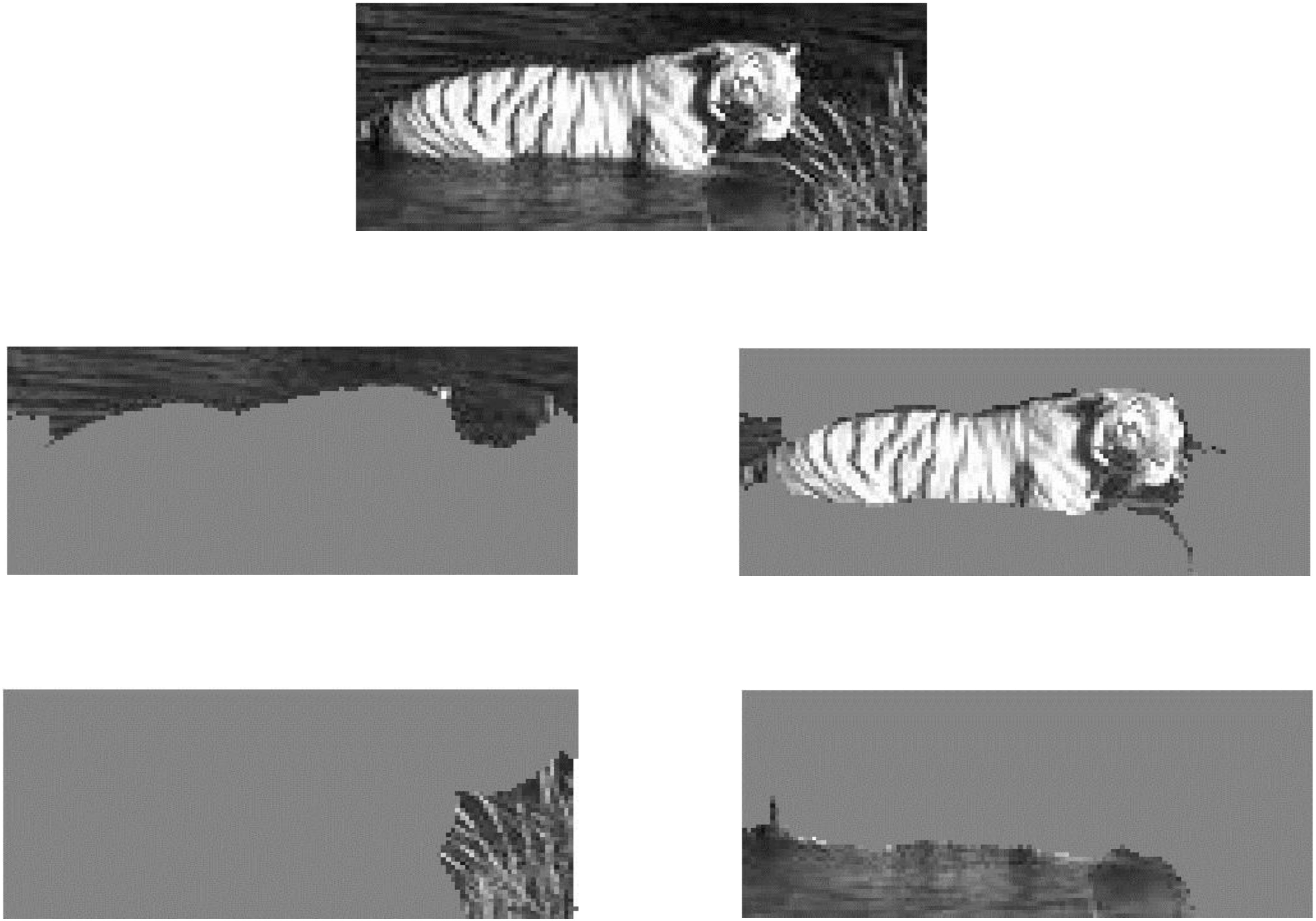
$$\frac{\mathbf{w}^t\mathbf{M}\mathbf{w}}{\mathbf{w}^t\mathbf{w}}$$

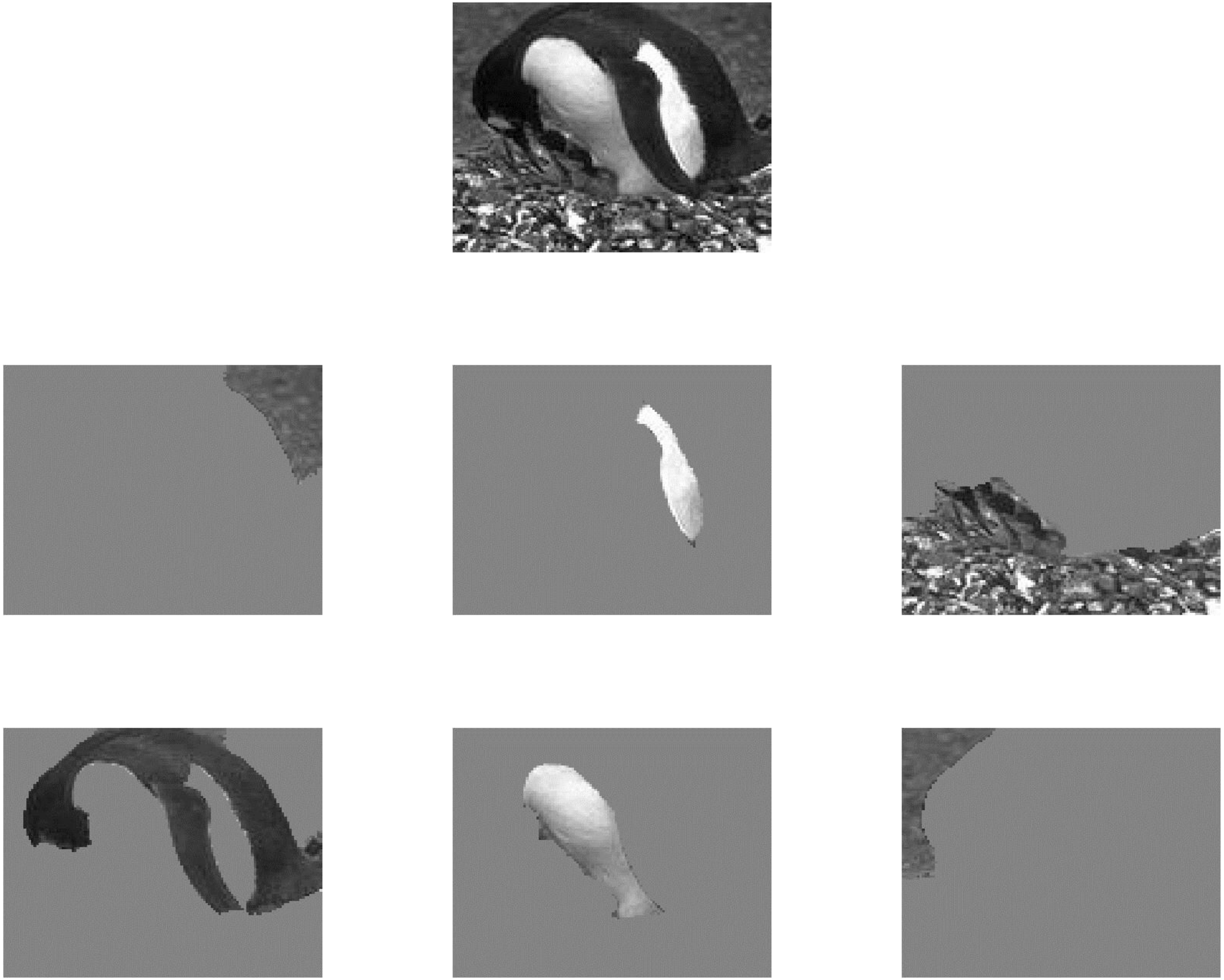
is obtained for the eigenvector $\mathbf{w}_{max}$ corresponding to the largest eigenvalue λ_{max} of $\mathbf{M}$.

Brightness Image Segmentation

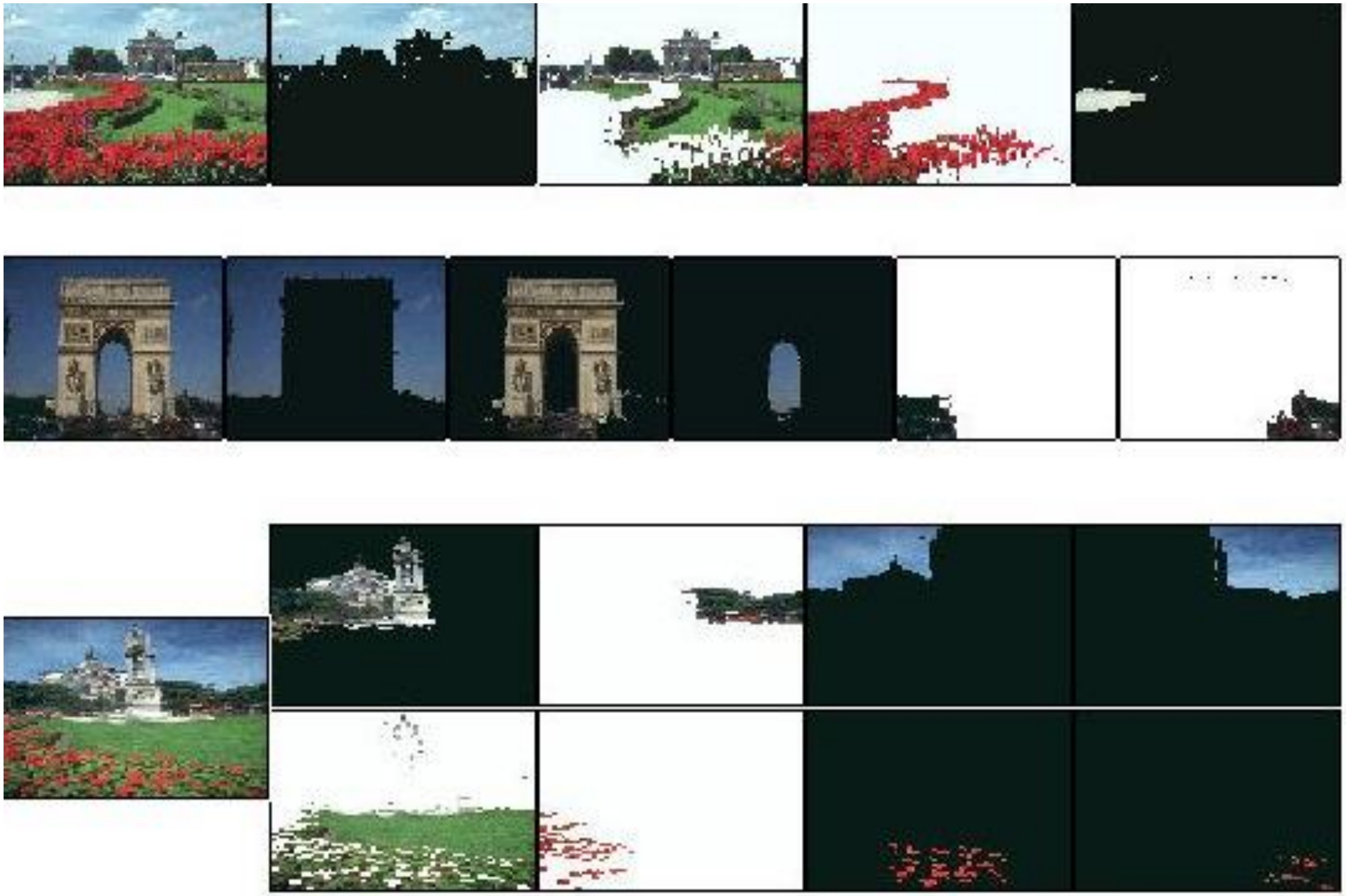


Brightness Image Segmentation

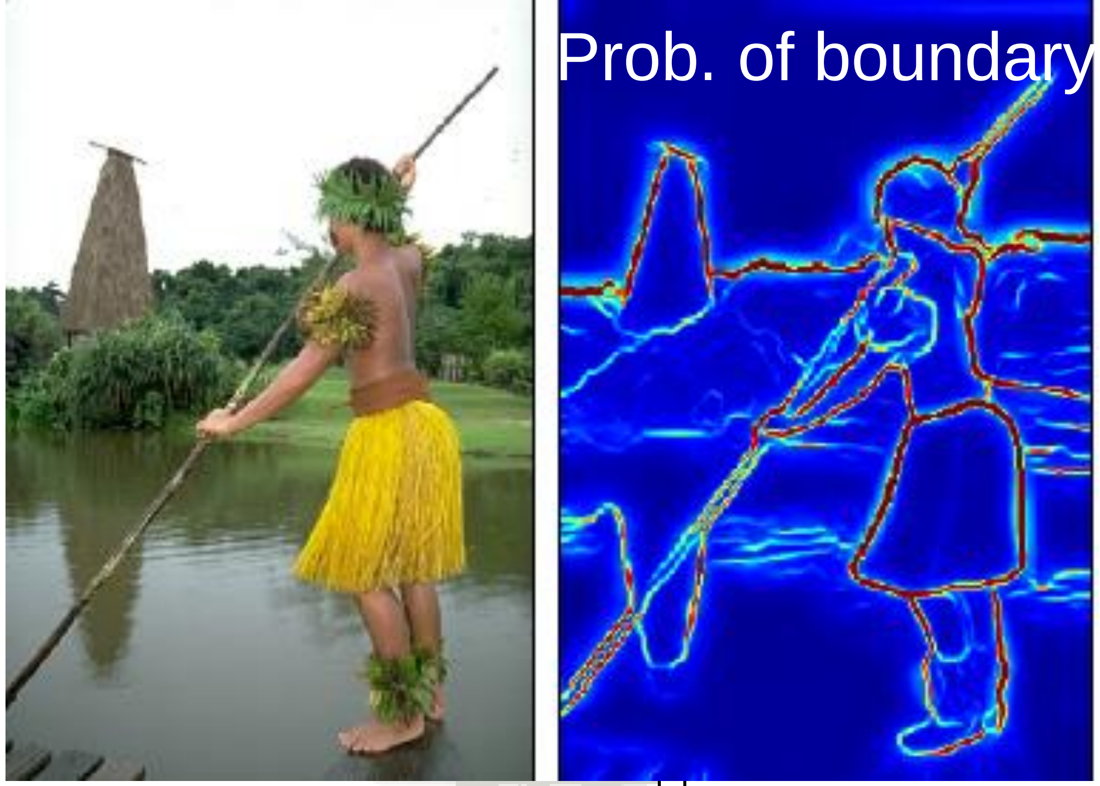




Results on color segmentation

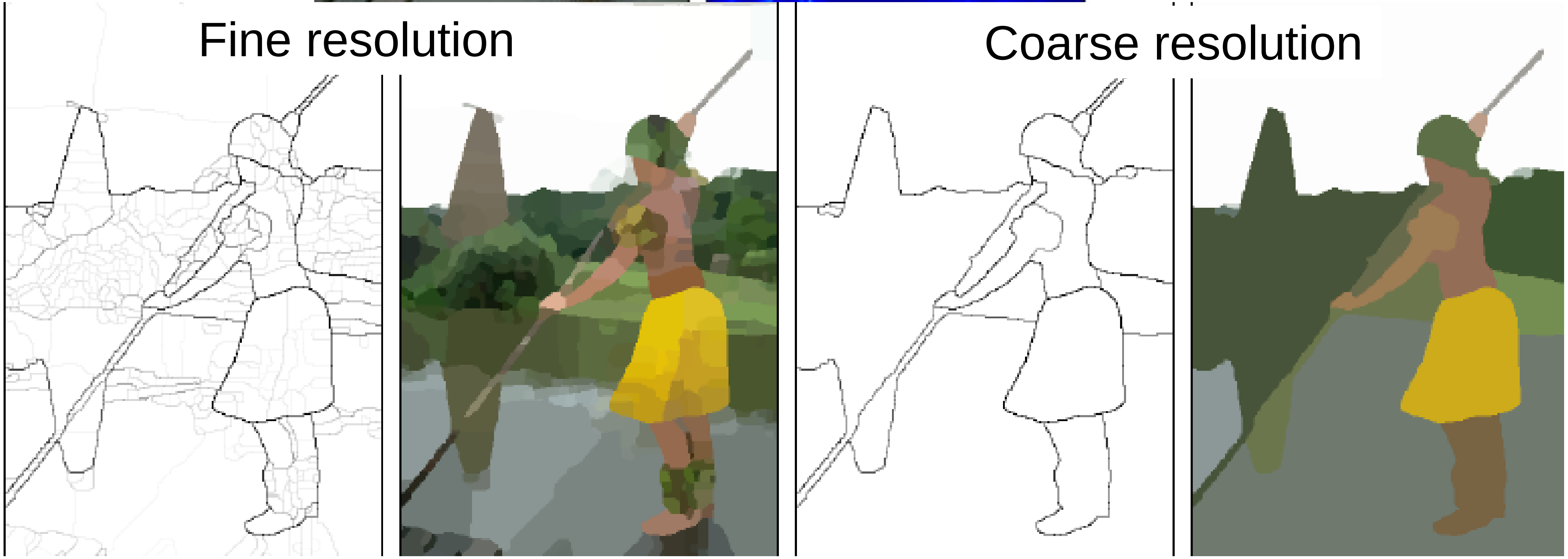


Segmentation from boundaries



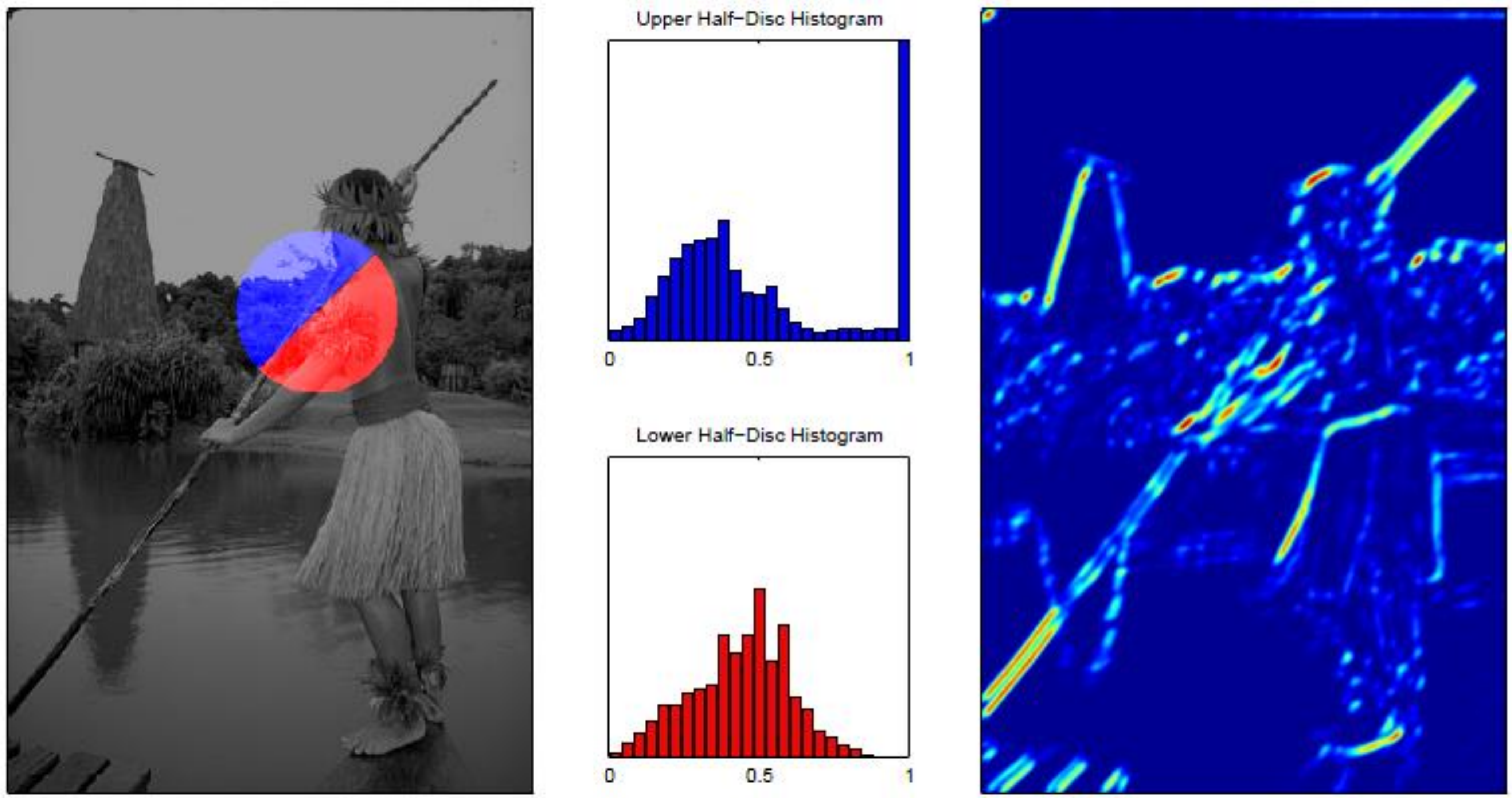
Intuition:

- Duality between regions and boundaries
- Maybe “easier” to estimate boundaries first
- Then use the boundaries to generate segmentations at different levels of granularity



- All examples from: P. Arbelaez, M. Maire, C. Fowlkes and J. Malik. Contour Detection and Hierarchical Image Segmentation. IEEE TPAMI, Vol. 33, No. 5, pp. 898-916, May 2011.
- Complete package: http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/resources.html

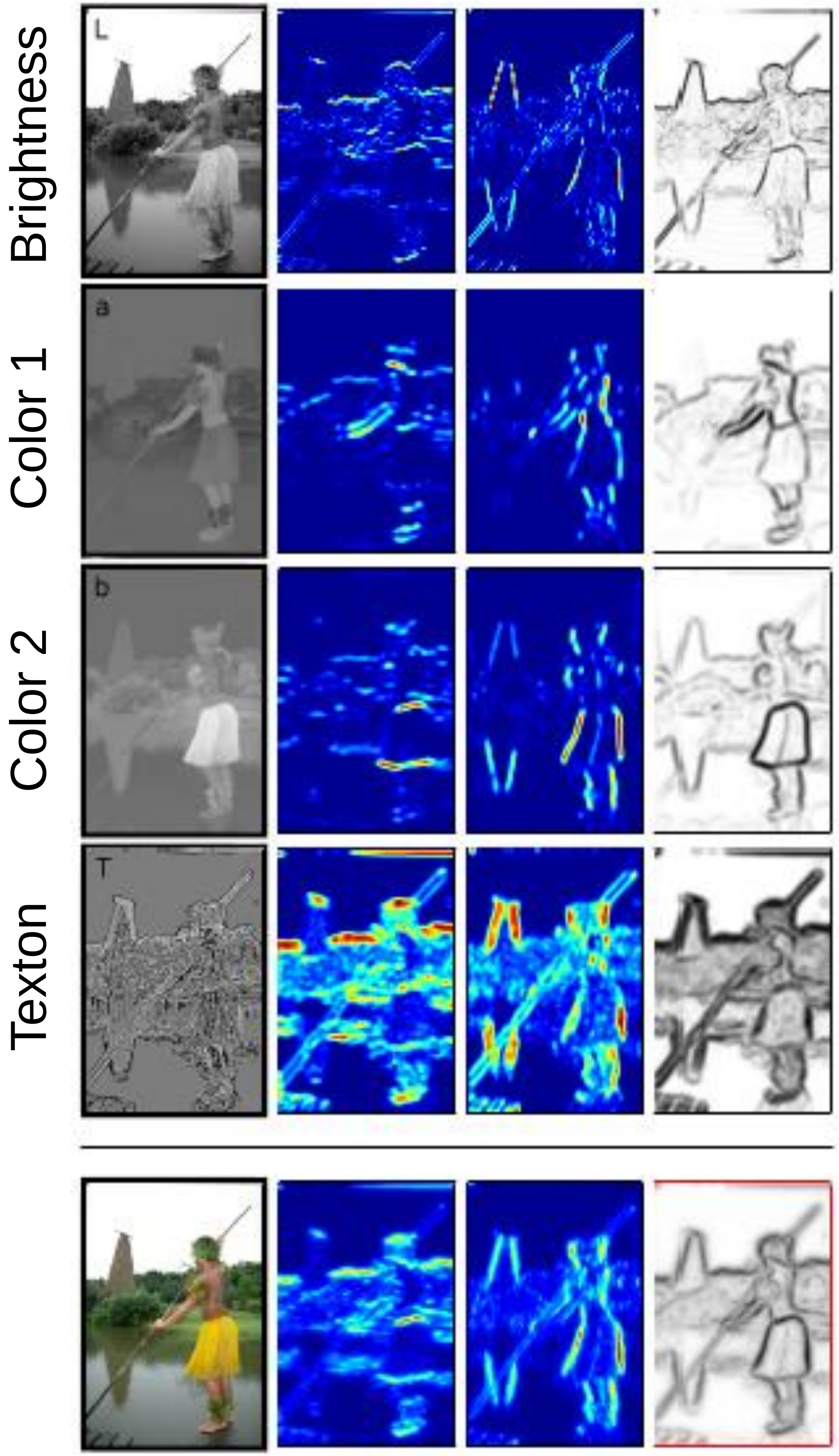
Finding boundaries

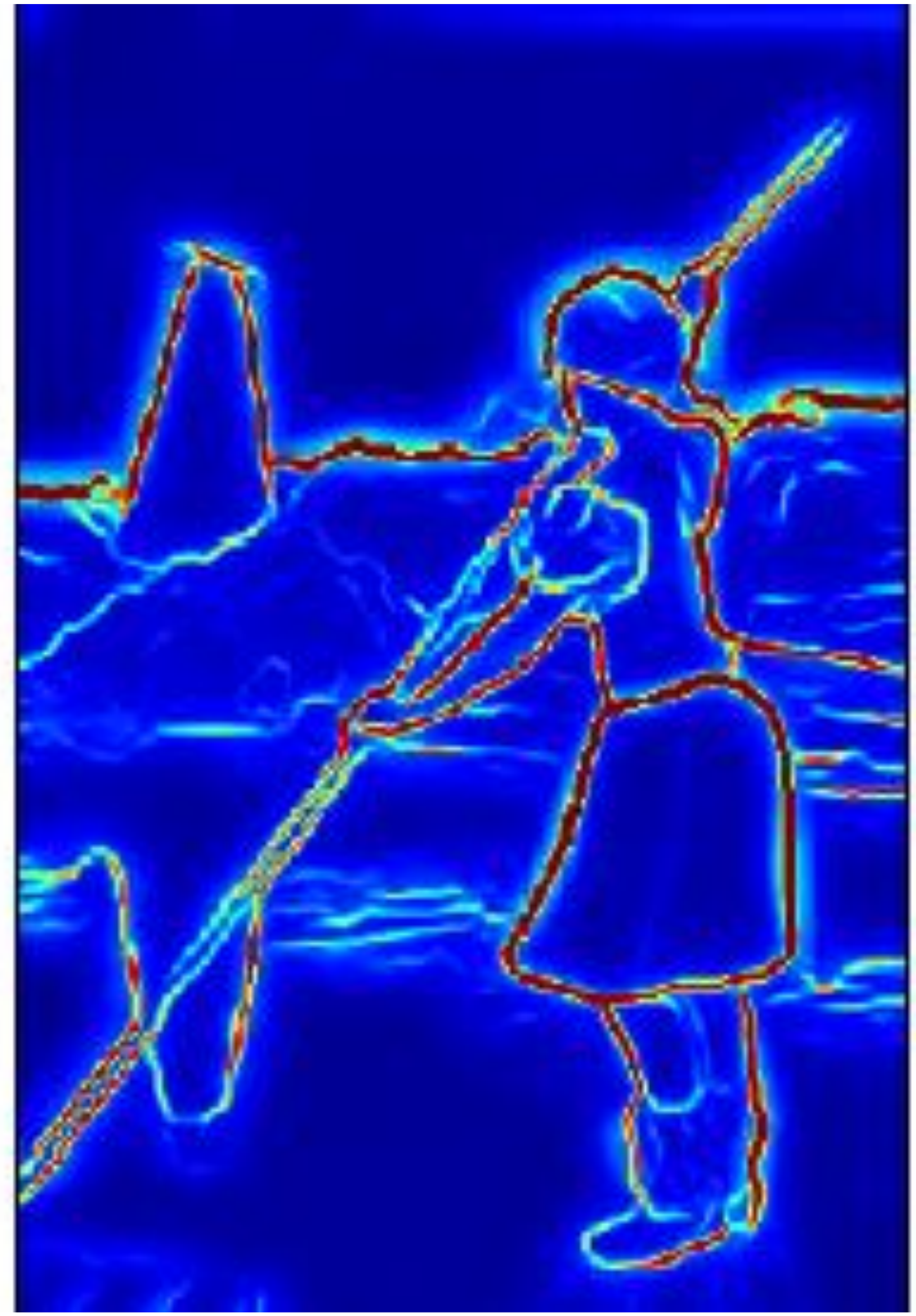


Pb

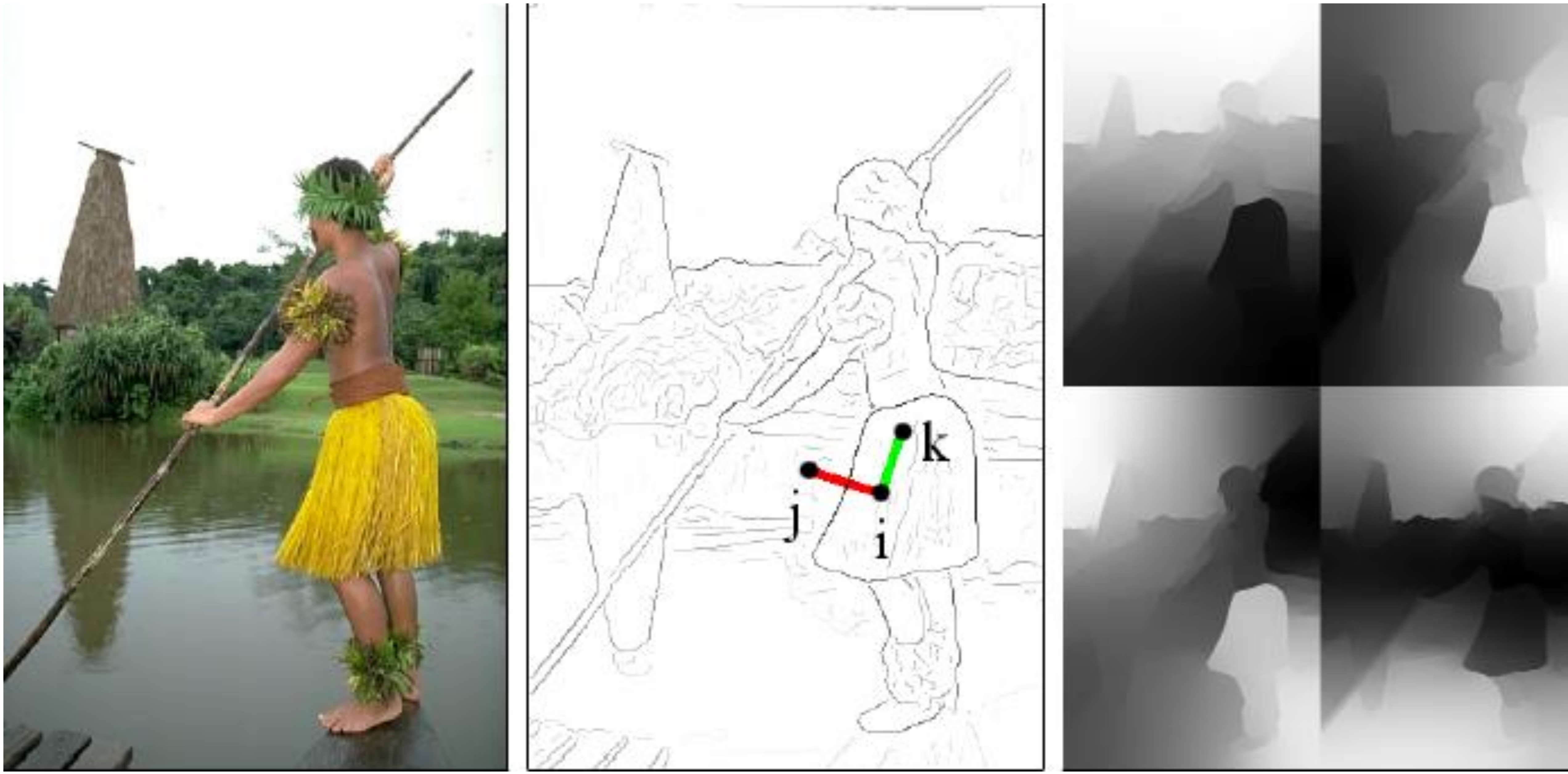
- *Pb* = Probability of boundary
- χ^2 difference between histograms at different orientations → Classifier → Probability of boundary

Combining multiple cues





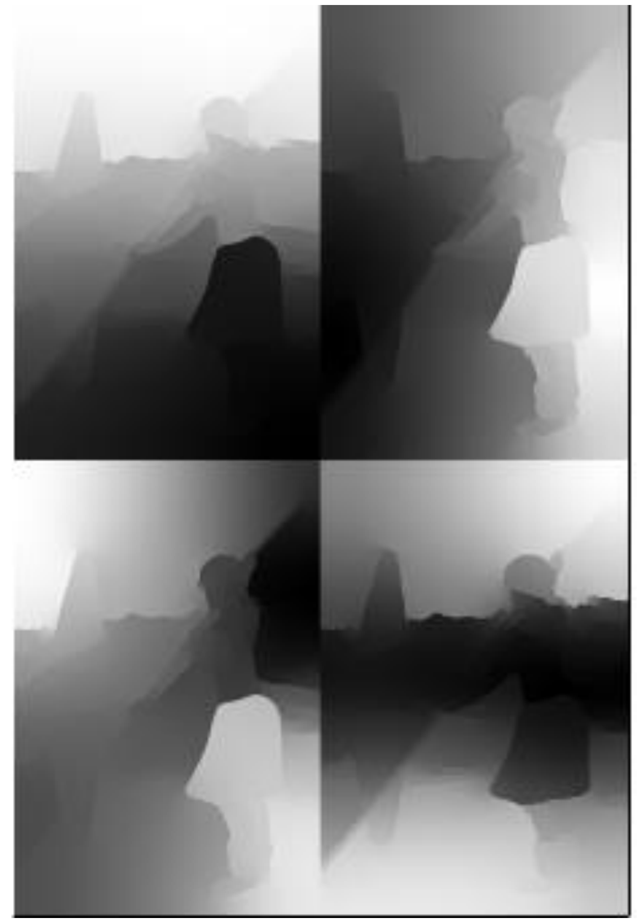
gPb (global *Pb*)



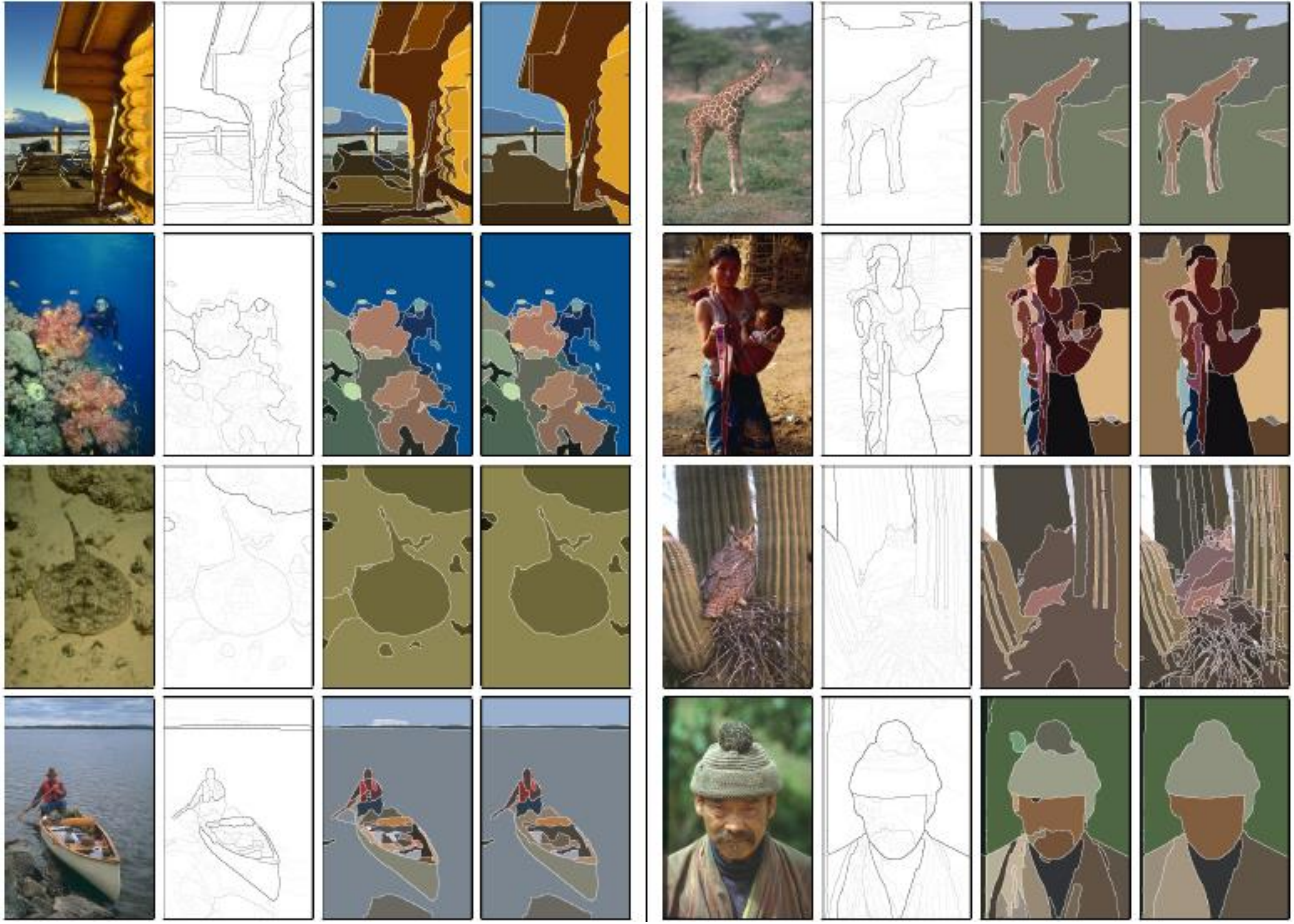
- Idea: We could use the *Pb* contours to generate an affinity matrix and then use Ncuts
- j and i have lower affinity because they cross higher *Pb* values

gPb (global *Pb*)

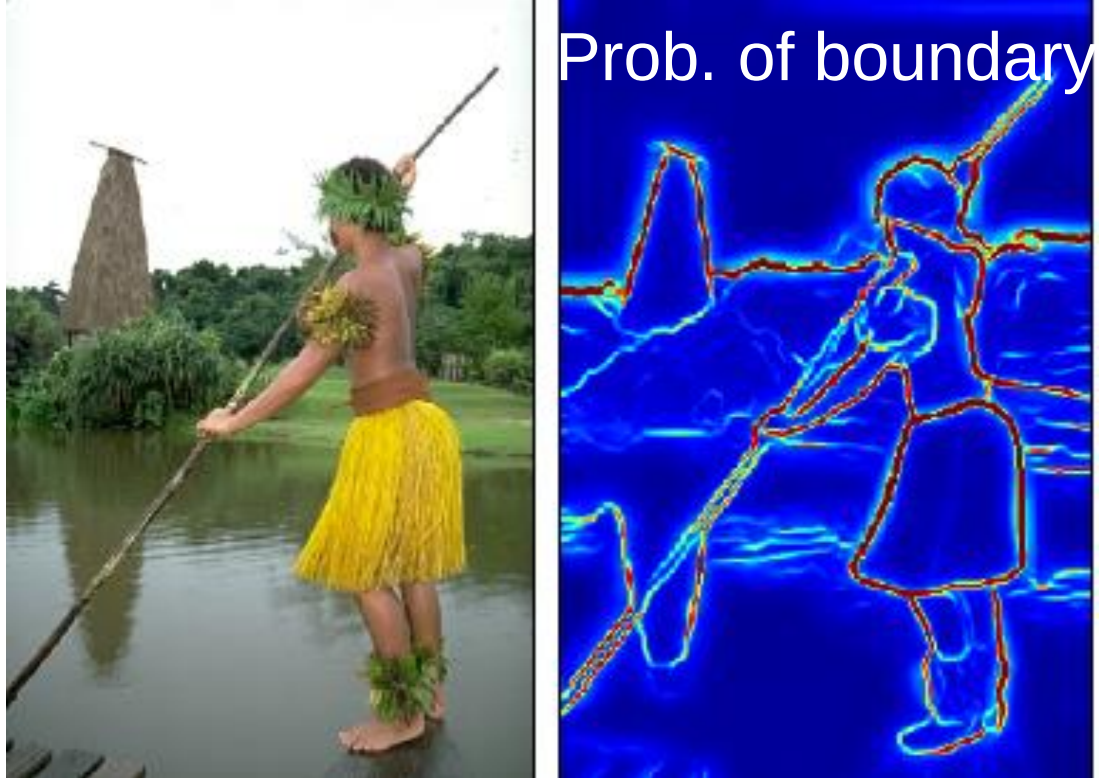
Not good: generate segmentation from the eigenvectors



Good: Combine the gradients of the eigenvectors!!



- Final step: Convert closed boundary (UCM = Ultrametric Contour Map)
- Different thresholds on contours yield segmentations at different levels of granularity
- Guaranteed to produce a hierarchical segmentation



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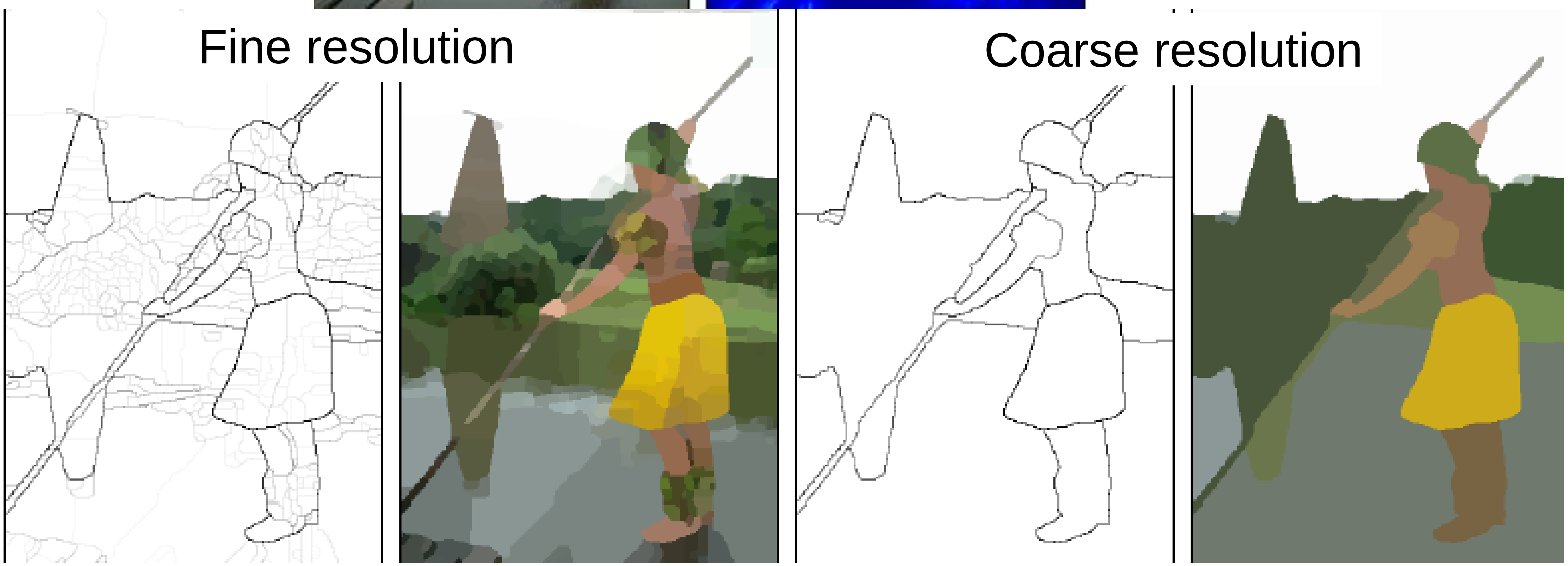


- Complete package: http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/resources.html

Clustering: group together similar points and represent them with a single token

Key Challenges:

1) What makes two points/images/patches similar?

2) How do we compute an overall grouping from pairwise similarities?

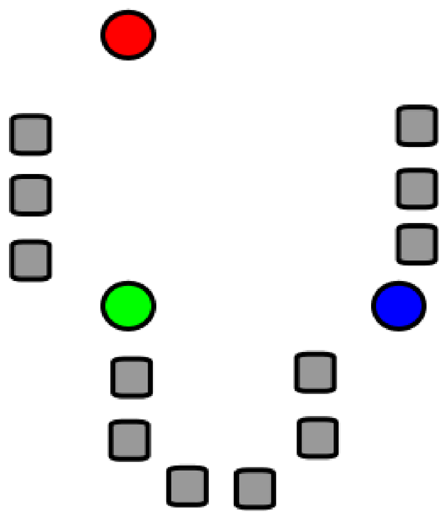
Why do we cluster?

- **Summarizing data**
  - Look at large amounts of data
  - Patch-based compression or denoising
  - Represent a large continuous vector with the cluster number
- **Counting**
  - Histograms of texture, color, SIFT vectors
- **Segmentation**
  - Separate the image into different regions
- **Prediction**
  - Images in the same cluster may have the same labels

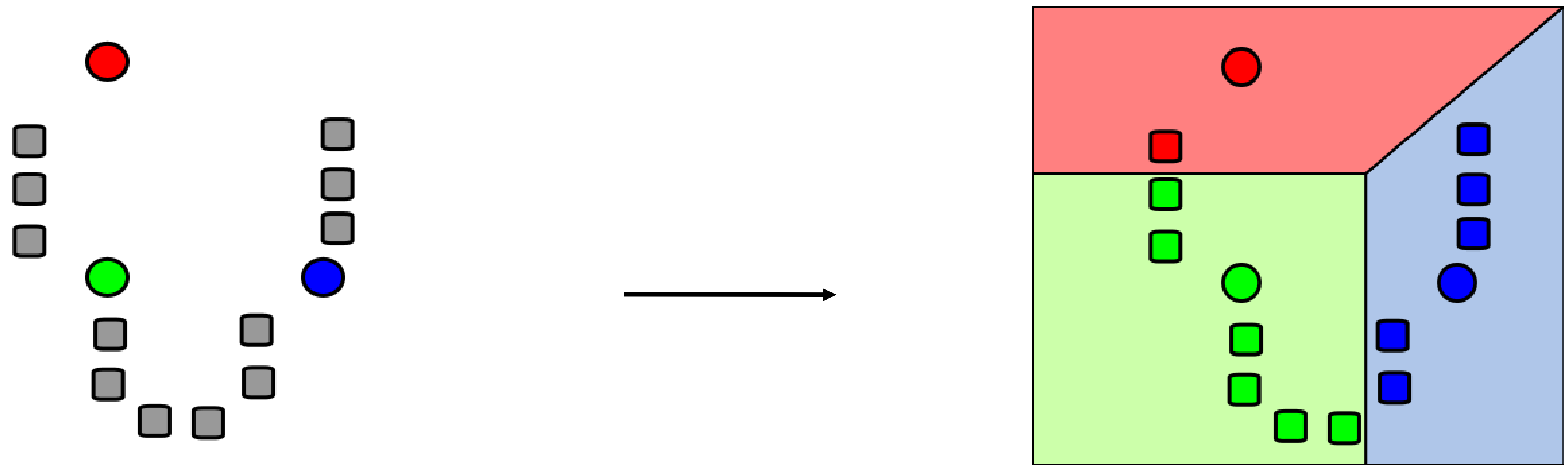
How do we cluster?

- K-means
  - Iteratively re-assign points to the nearest cluster center
- Agglomerative clustering
  - Start with each point as its own cluster and iteratively merge the closest clusters
- Mean-shift clustering
  - Estimate modes of pdf

K-means clustering



1. Select initial centroids at random



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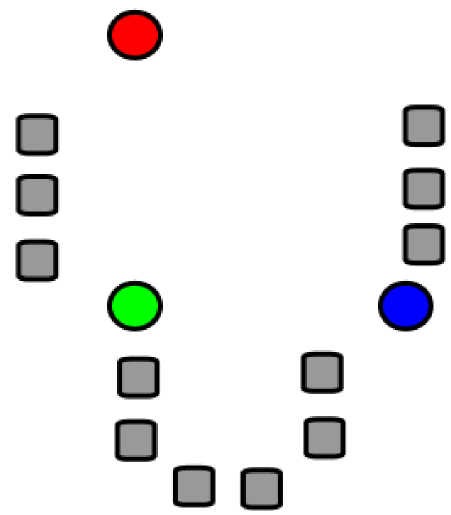
2. Assign each object to the cluster with the nearest centroid.



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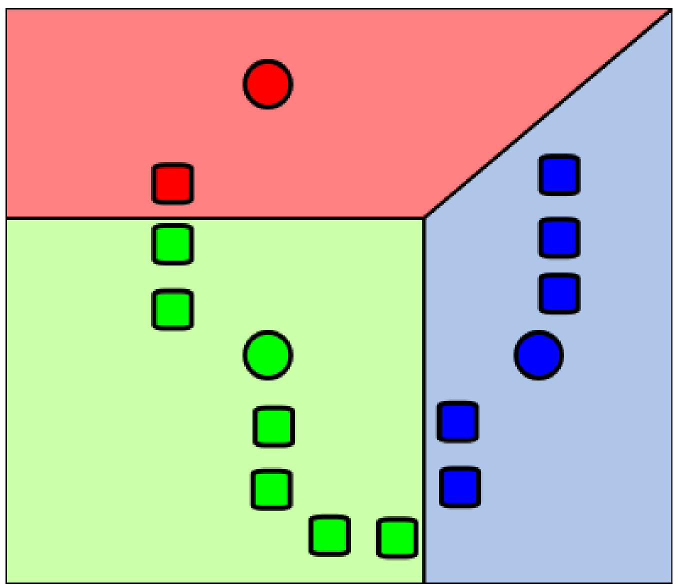

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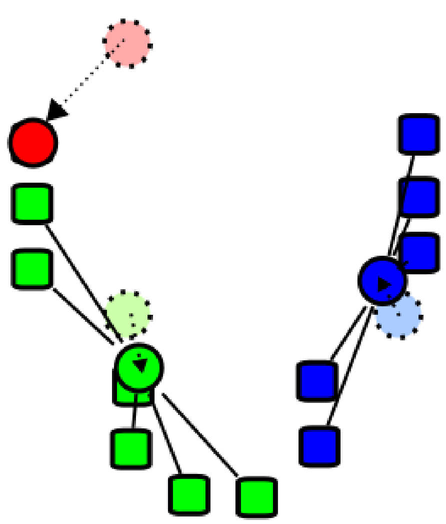
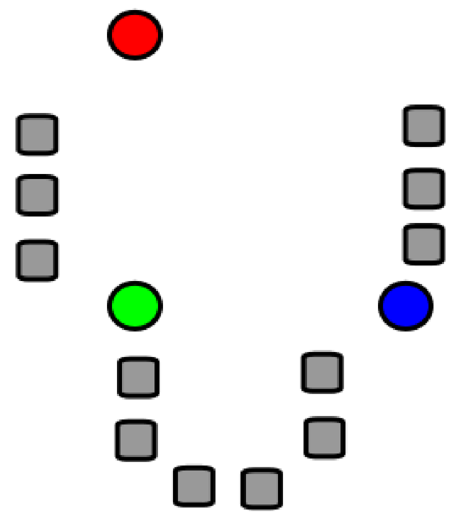


3. Compute each centroid as the mean of the objects assigned to it (go to 2)



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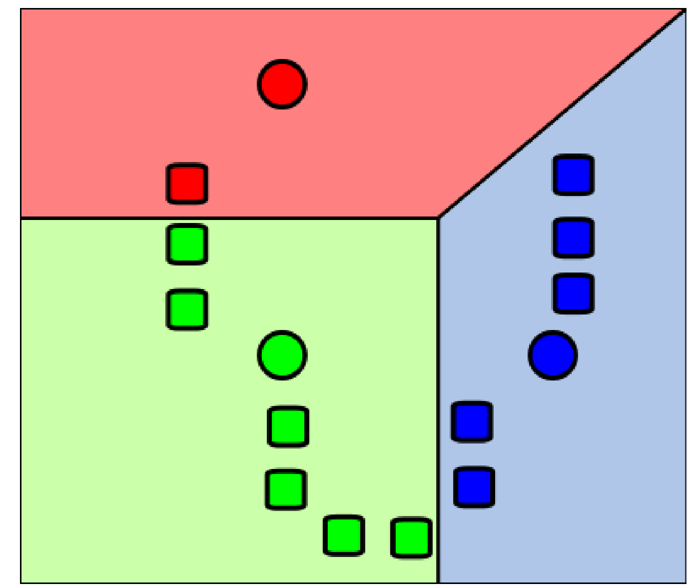

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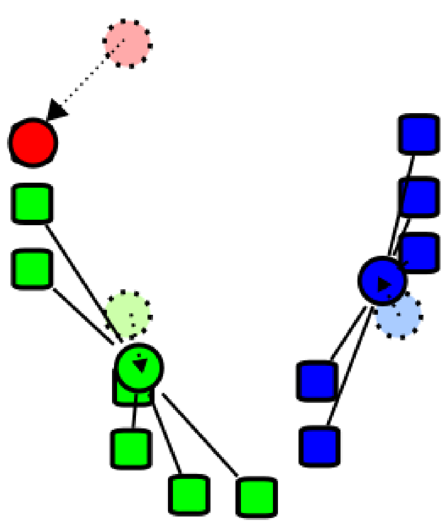

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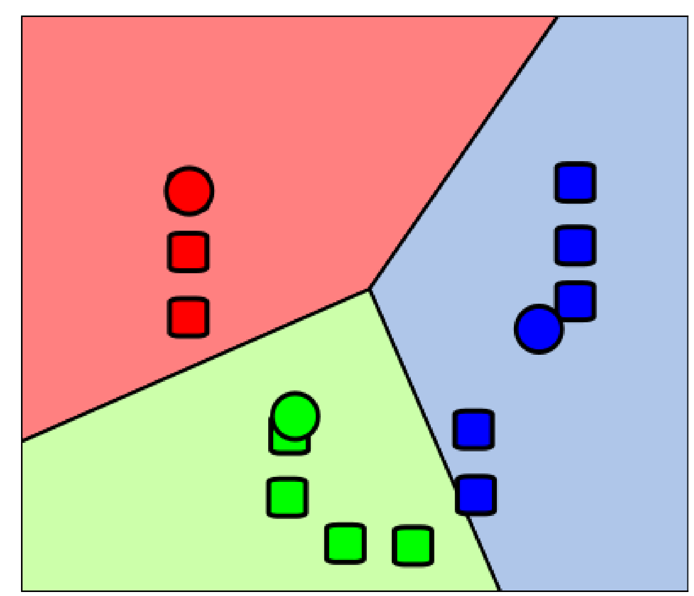

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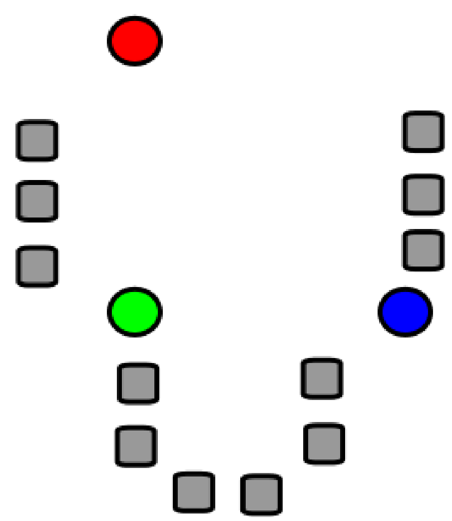

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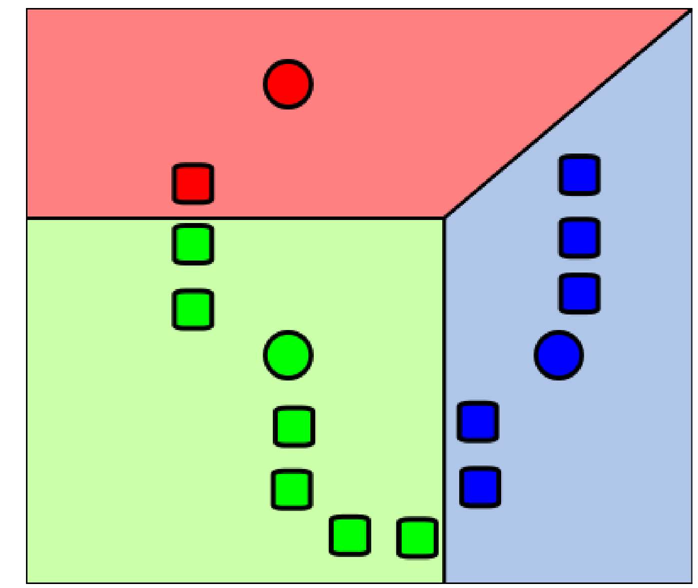

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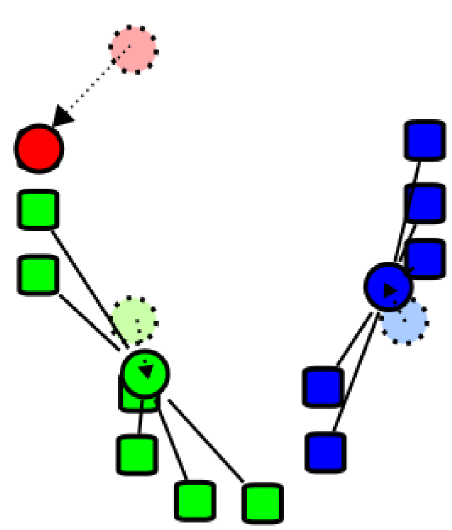

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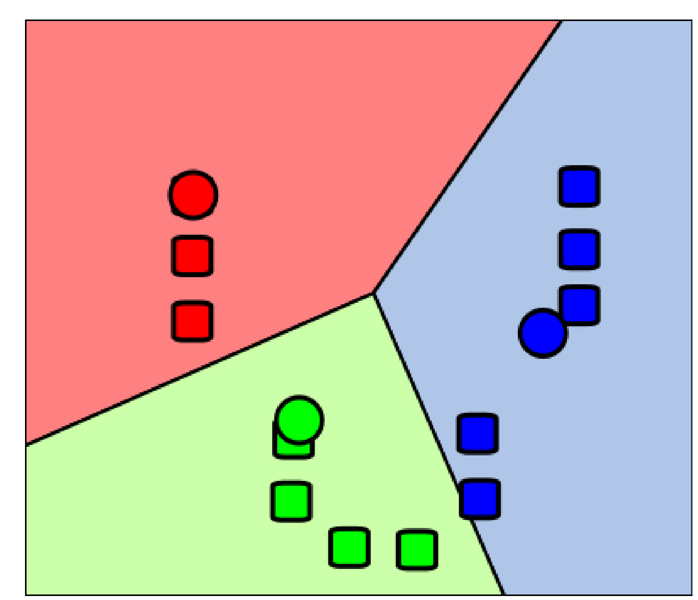


2. Assign each object to the cluster with the nearest centroid.

Repeat previous 2 steps until no change

K-means Clustering

Given k:

1. Select initial centroids at random.

2. Assign each object to the cluster with the nearest centroid.

3. Compute each centroid as the mean of the objects assigned to it.

4. Repeat previous 2 steps until no change.

K-means: design choices

- Initialization
  - Randomly select K points as initial cluster center
  - Or greedily choose K points to minimize residual
- Distance measures
  - Traditionally Euclidean, could be others
- Optimization
  - Will converge to a *local minimum*
  - May want to perform multiple restarts

K-means clustering using intensity or color

Image

Clusters on intensity

Clusters on color

How to choose the number of clusters?

- Minimum Description Length (MDL) principal for model comparison

- Minimize Schwarz Criterion
  - also called Bayes Information Criteria (BIC)

$$\text{Distortion} + \lambda\,(\#\text{parameters}) \log R$$

$$= \text{Distortion} + \lambda m k \log R$$

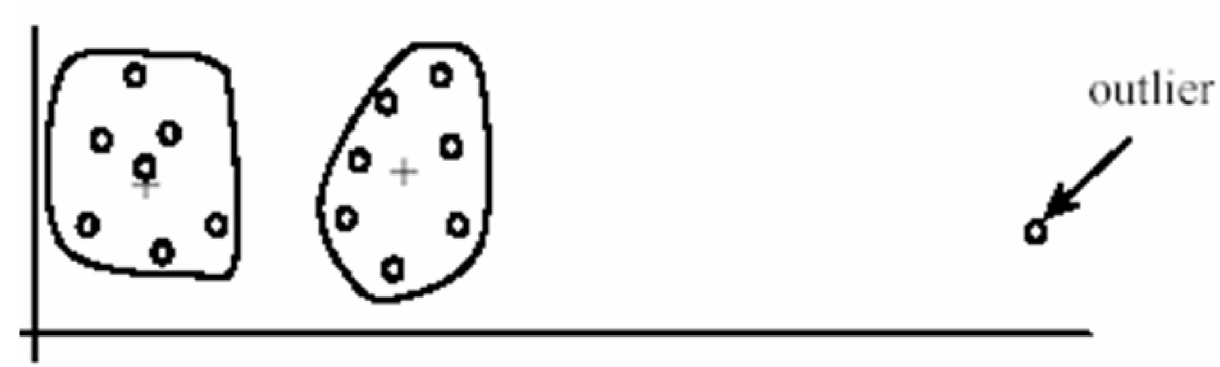
m=#dimensions

k=#Centers

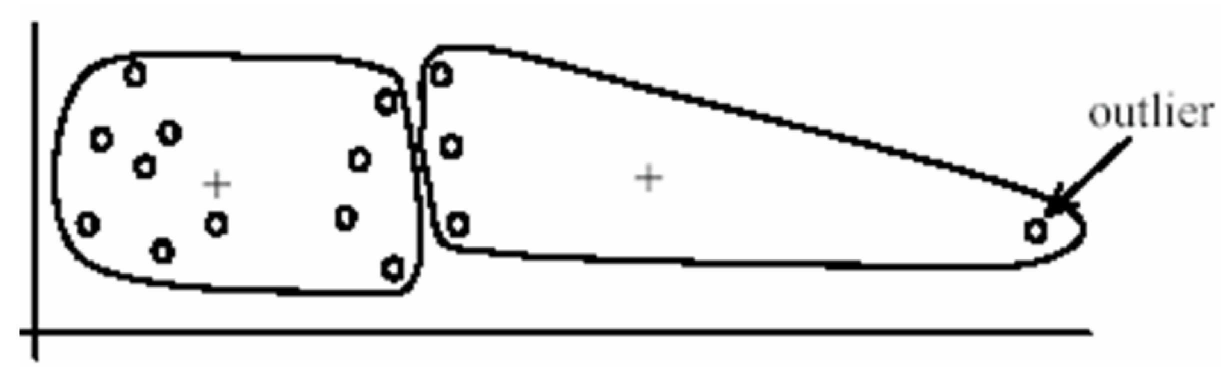
R=#Records

K-Means pros and cons

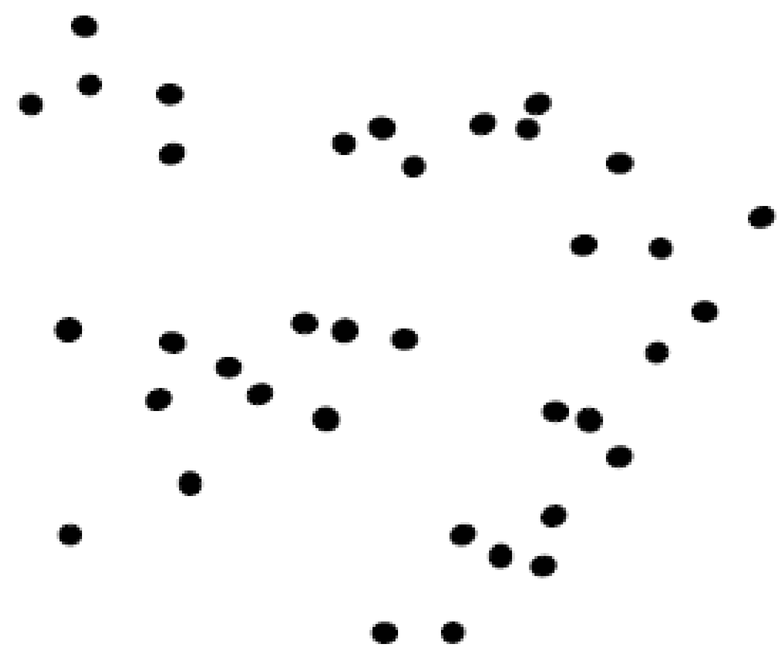
- Pros
  - Finds cluster centers that minimize conditional variance (good representation of data)
  - Simple and fast*
  - Easy to implement
- Cons
  - Need to choose K
  - Sensitive to outliers
  - Prone to local minima
  - All clusters have the same parameters (e.g., distance measure is non-adaptive)
  - *Can be slow: each iteration is O(KNd) for N d-dimensional points
- Usage
  - Rarely used for pixel segmentation



(B): Ideal clusters

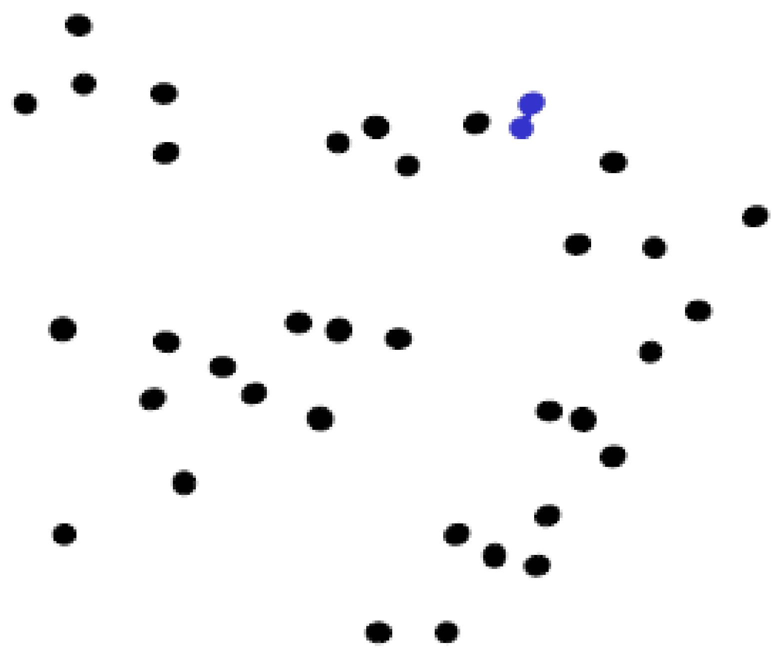


Agglomerative clustering



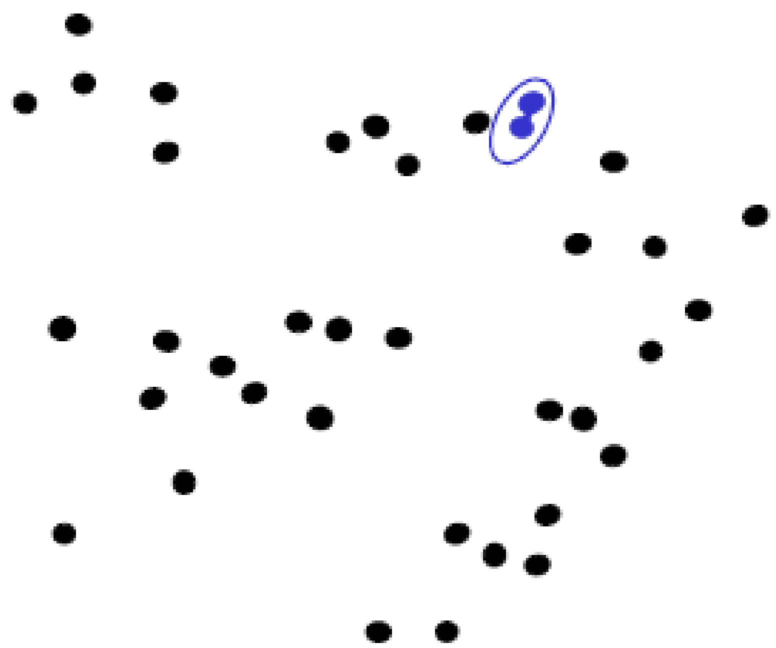
1. Say "Every point is its own cluster"

Agglomerative clustering



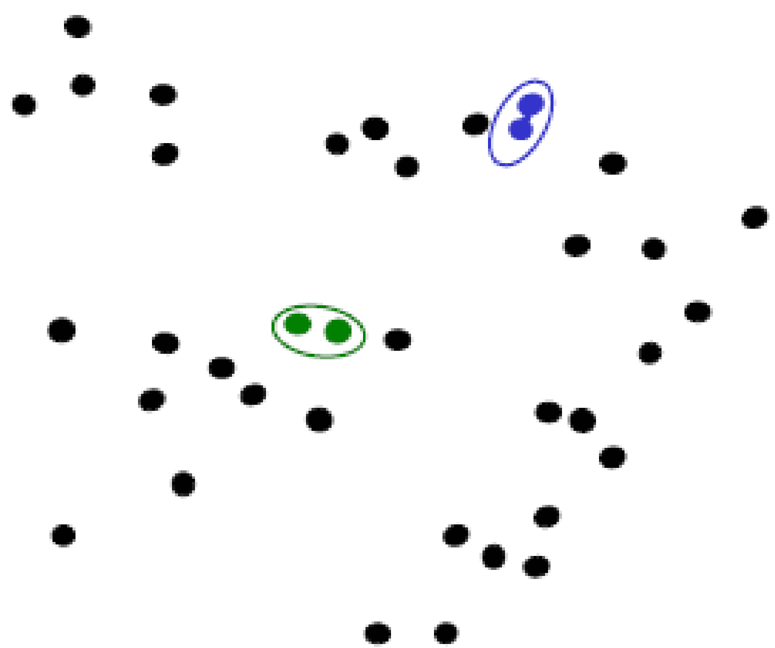
1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters

Agglomerative clustering

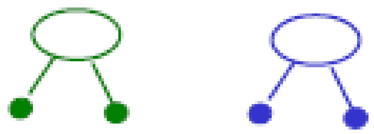


1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters
3. Merge it into a parent cluster

Agglomerative clustering



1. Say "Every point is its own cluster"
2. Find "most similar" pair of clusters
3. Merge it into a parent cluster
4. Repeat



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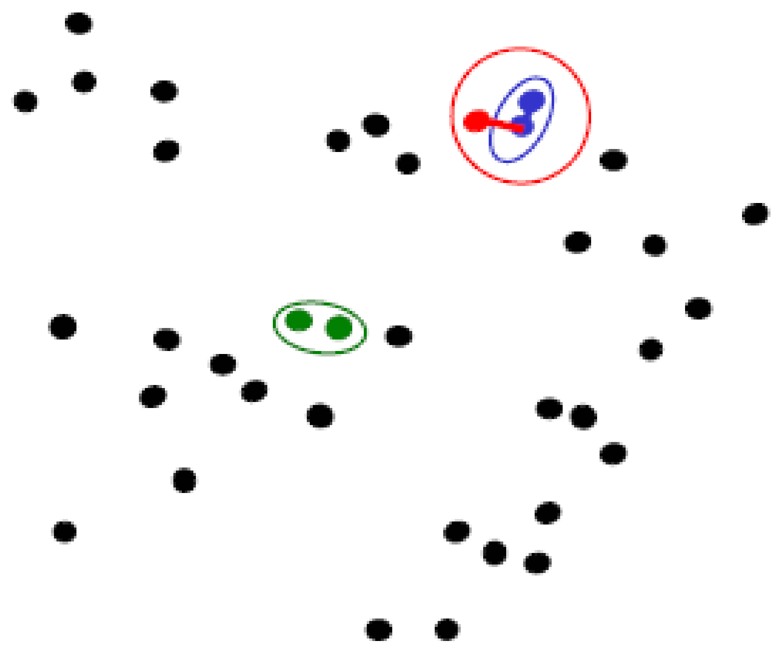

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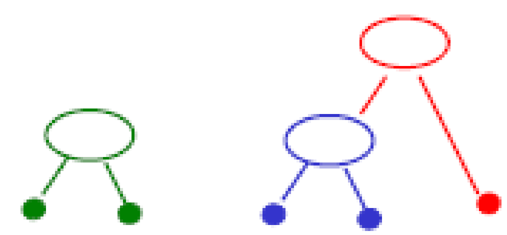
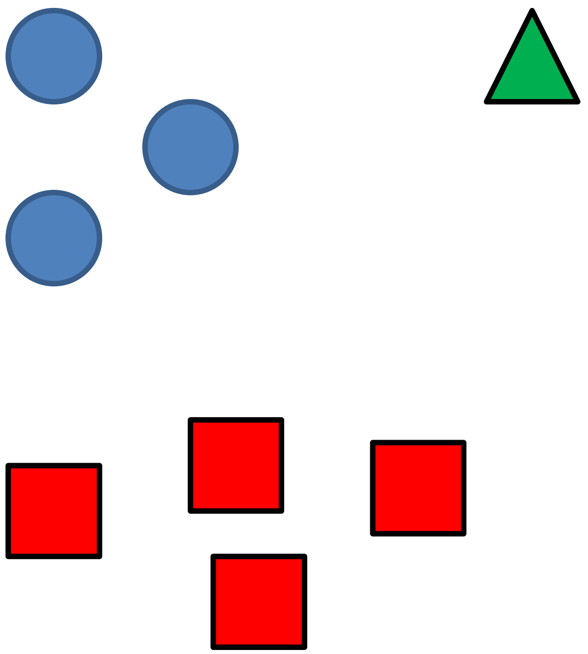
2. Find "most similar" pair of clusters
3. Merge it into a parent cluster
4. Repeat



Agglomerative clustering

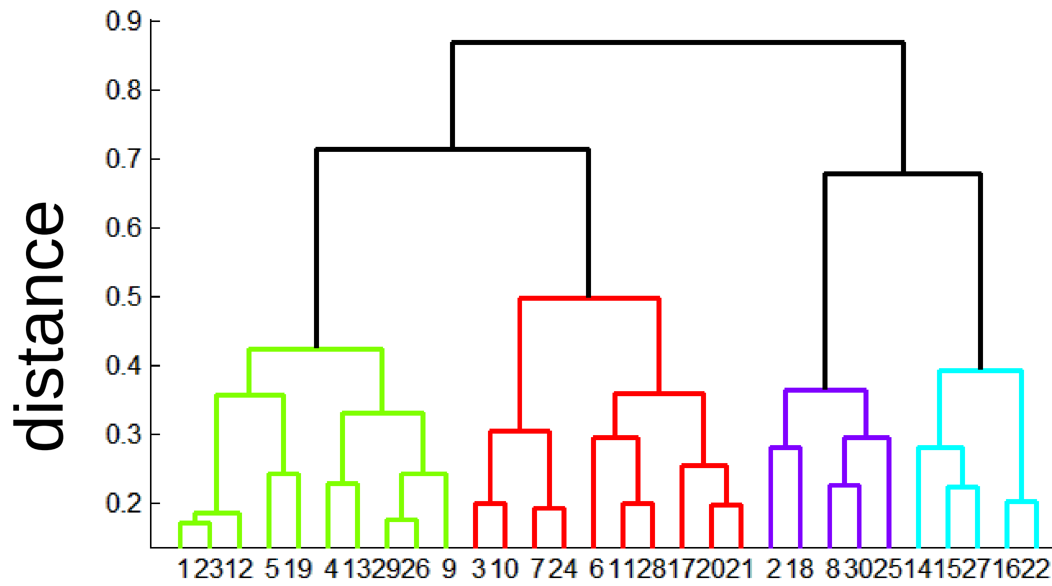
How to define cluster similarity?

- Average distance between points, maximum distance, minimum distance
- Distance between means or medoids



How many clusters?

- Clustering creates a dendrogram (a tree)
- Threshold based on max number of clusters or based on distance between merges



Conclusions: Agglomerative Clustering

Good

- Simple to implement, widespread application
- Clusters have adaptive shapes
- Provides a hierarchy of clusters

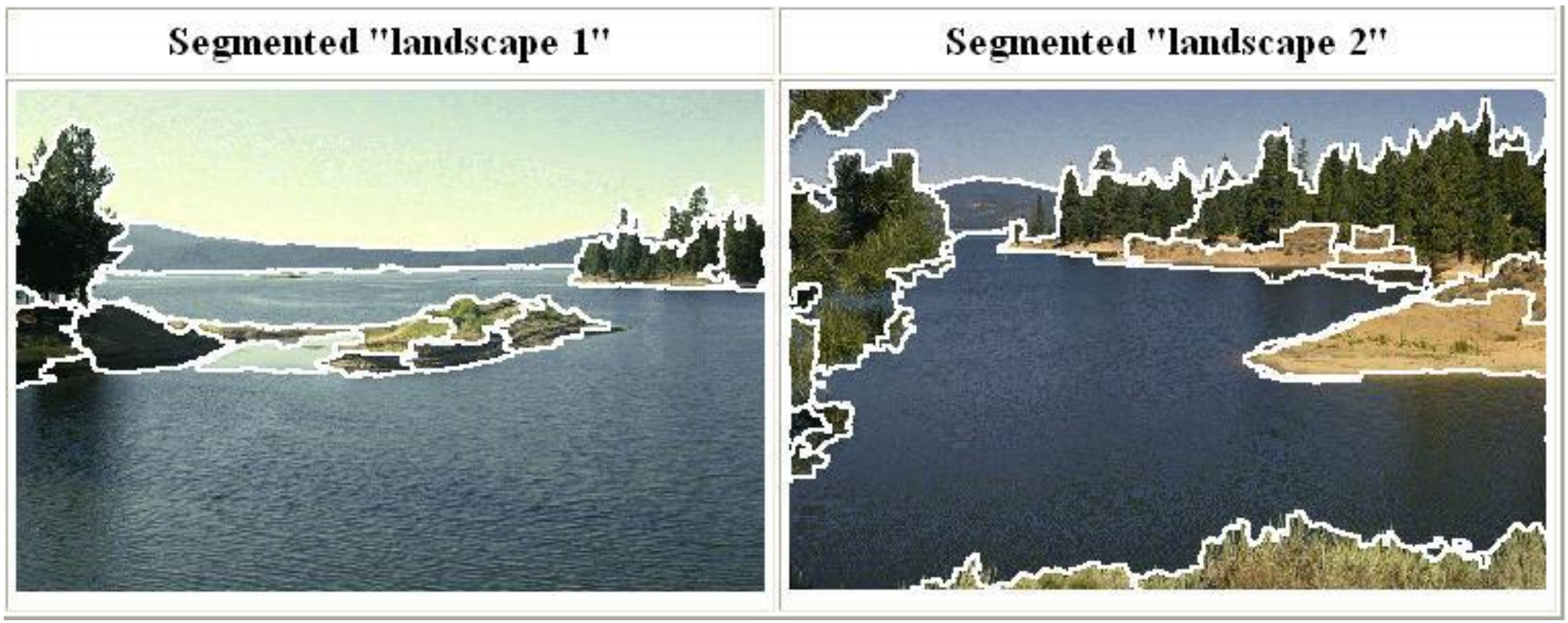
Bad

- May have imbalanced clusters
- Still have to choose number of clusters or threshold
- Need to use an “ultrametric” to get a meaningful hierarchy

Mean shift segmentation

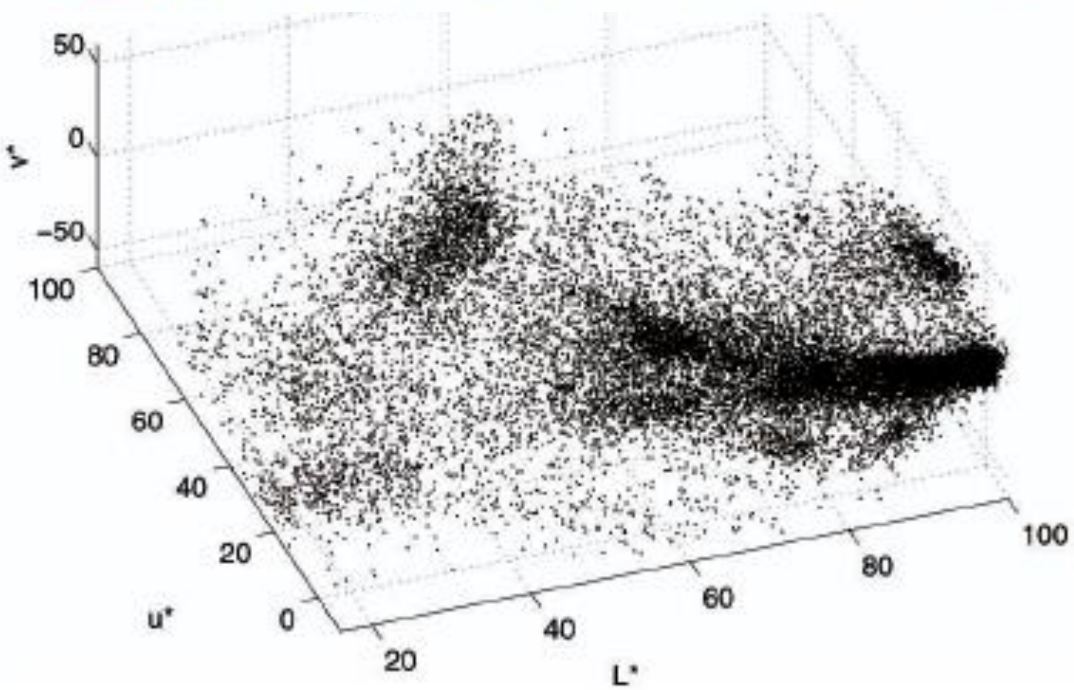
D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

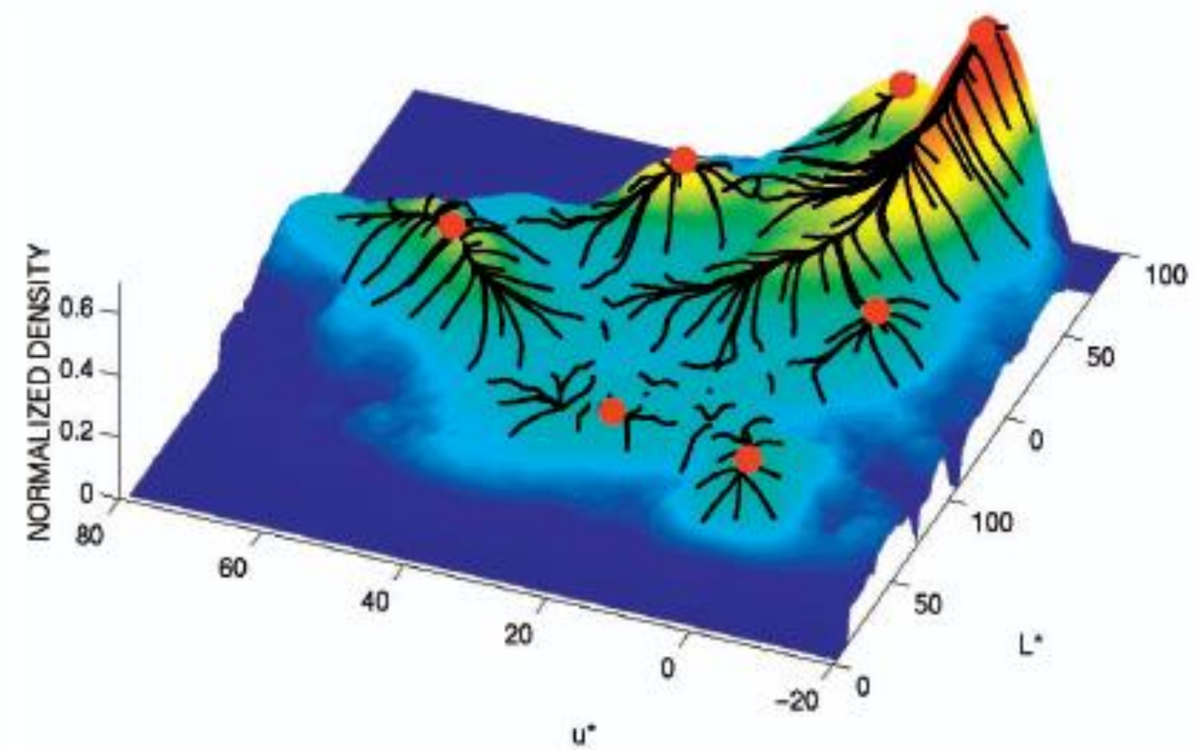
- Versatile technique for clustering-based segmentation

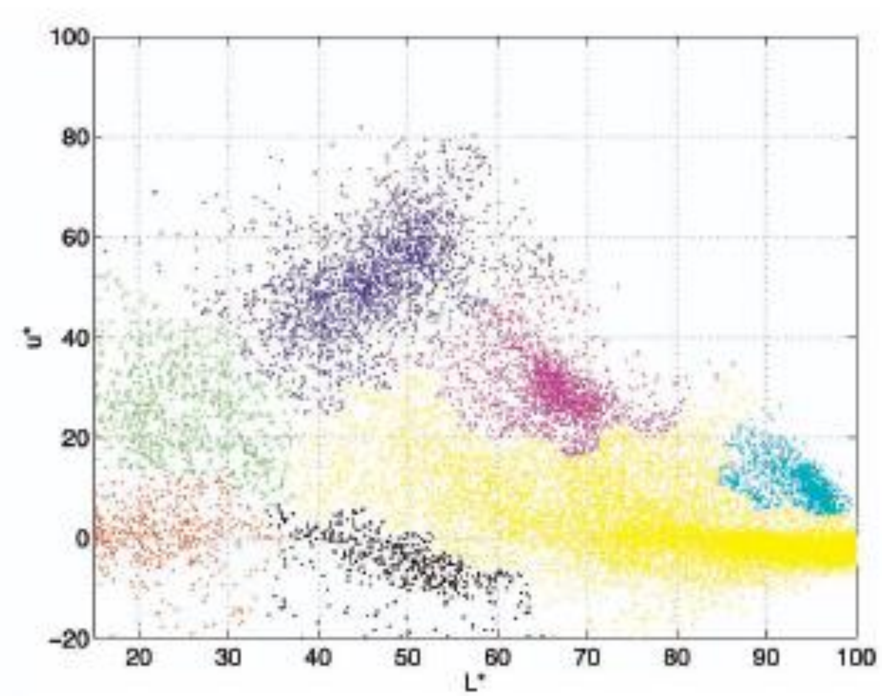


Mean shift algorithm

- Try to find *modes* of this non-parametric density







Mean Shift Algorithm

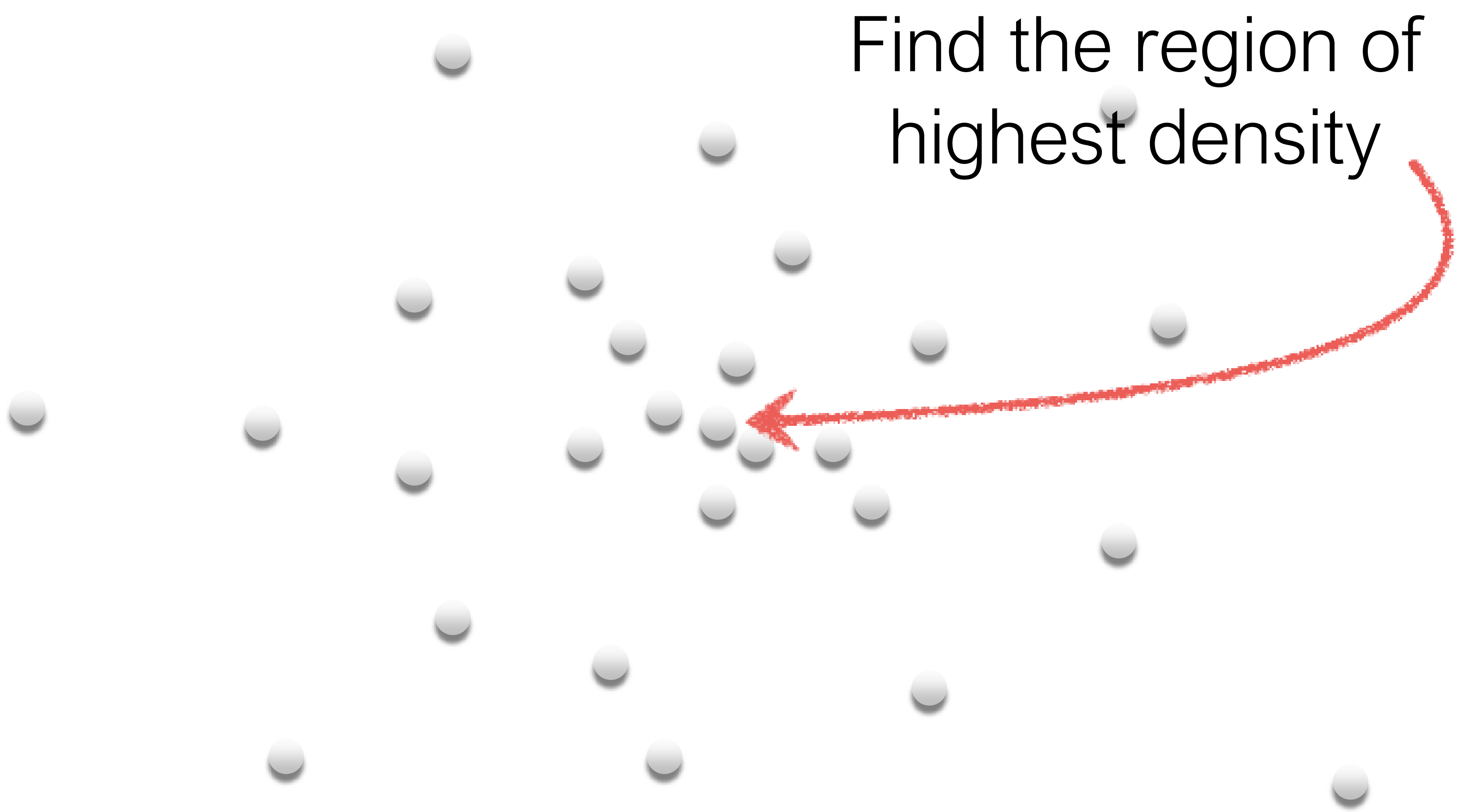
A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)

Mean Shift Algorithm

A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)



Mean Shift Algorithm

A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)

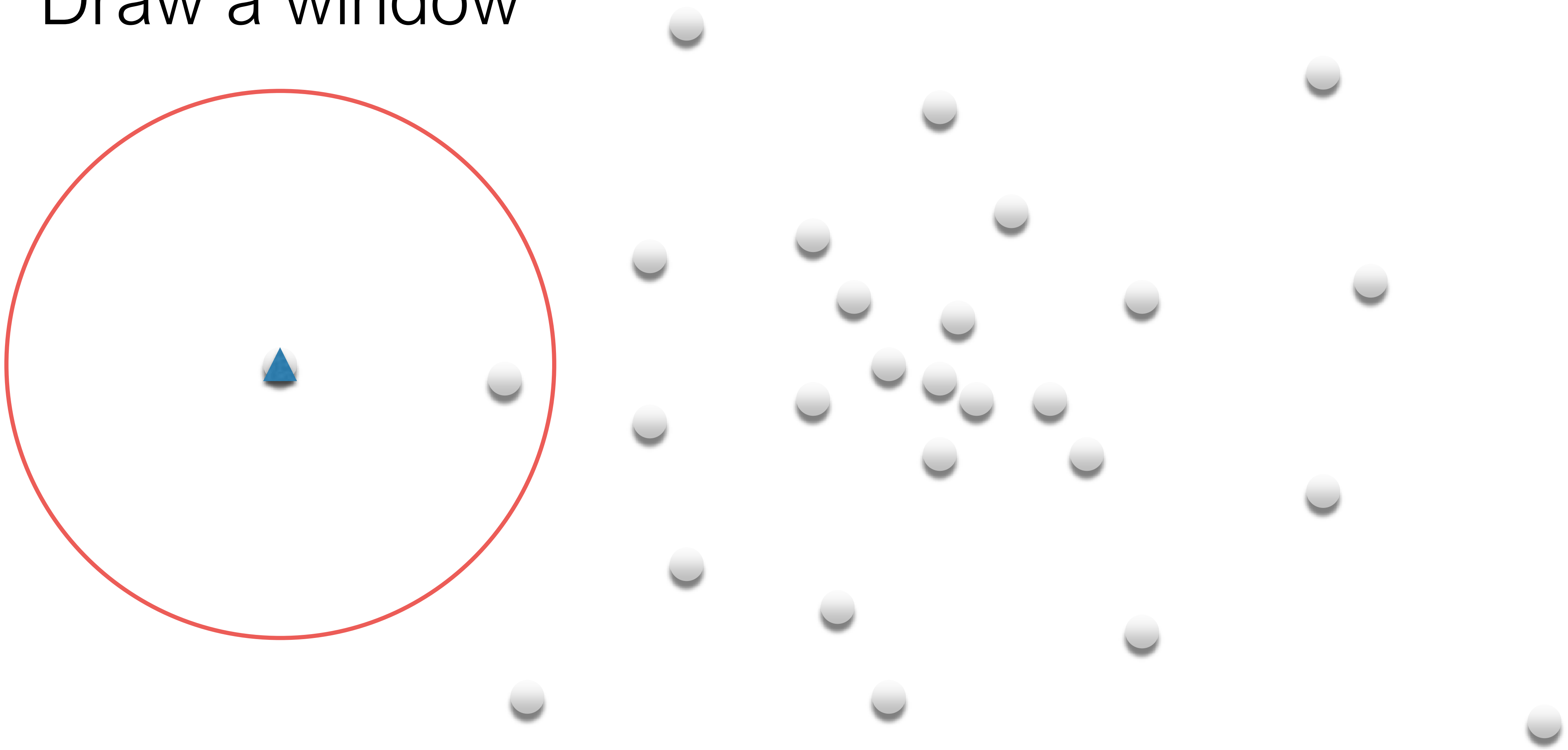
Pick a point

Mean Shift Algorithm

A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)

Draw a window

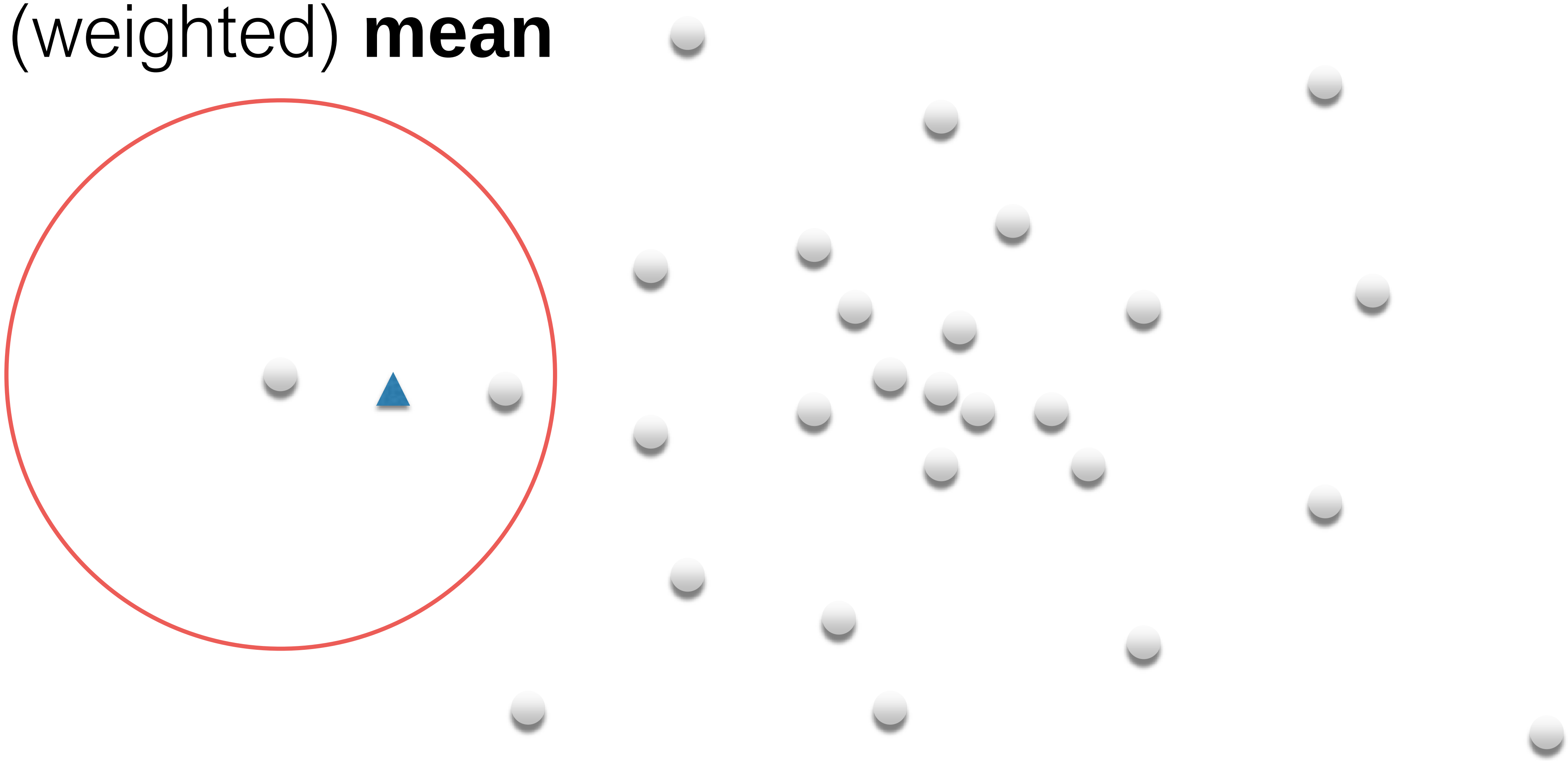


Mean Shift Algorithm

A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)

Compute the
(weighted) **mean**



Mean Shift Algorithm

A ‘mode seeking’ algorithm

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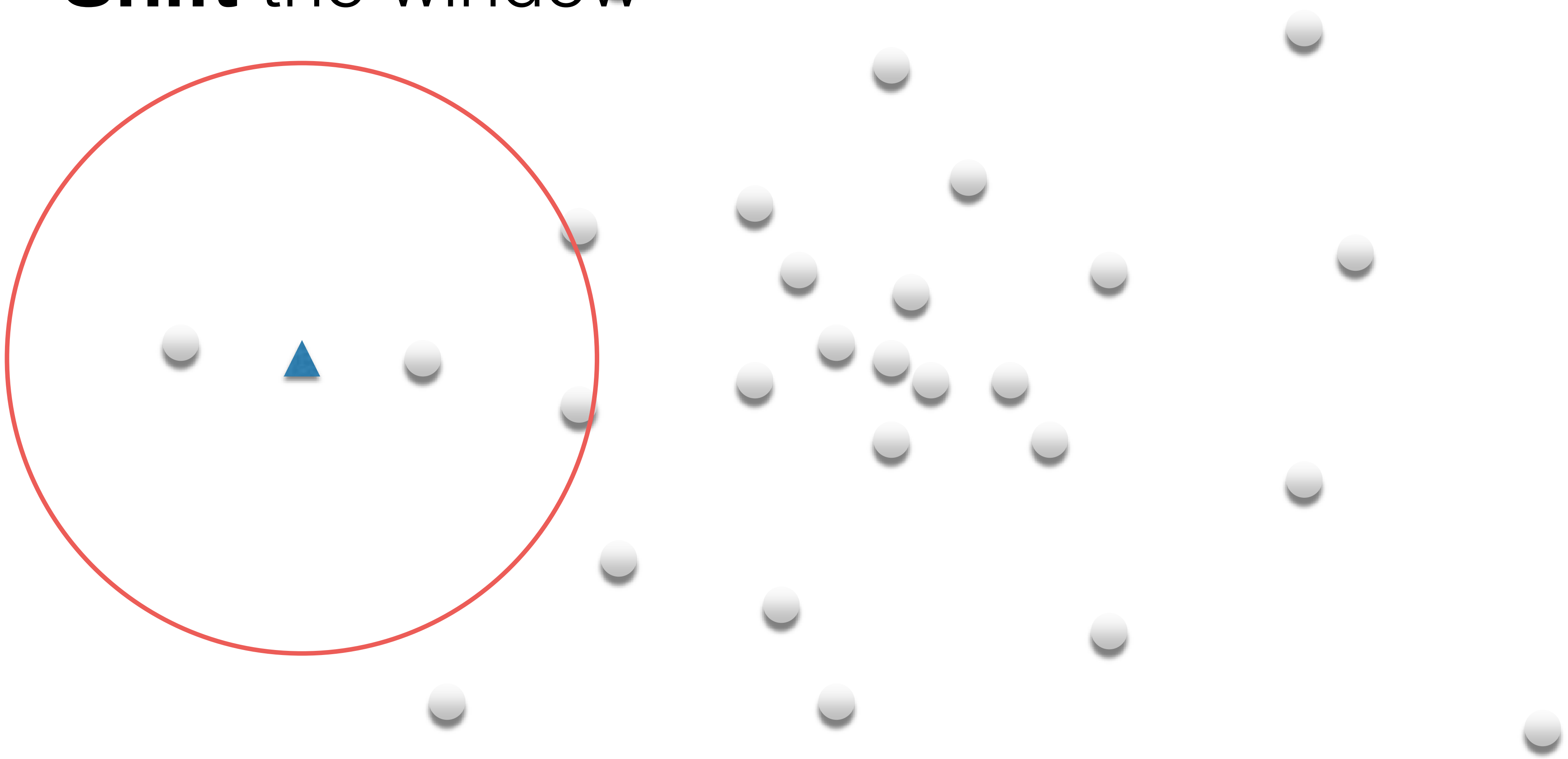
Shift the window



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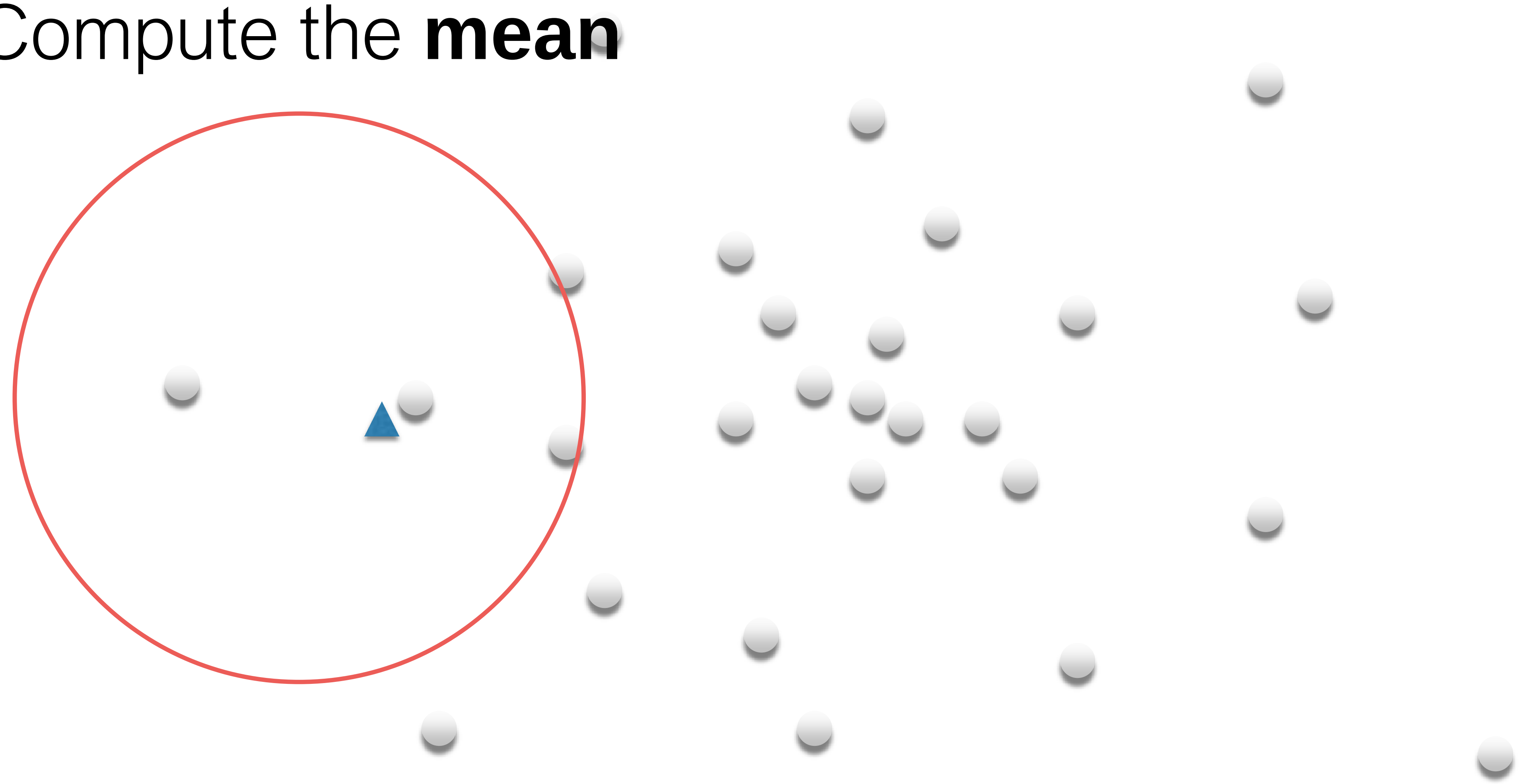
Compute the **mean**



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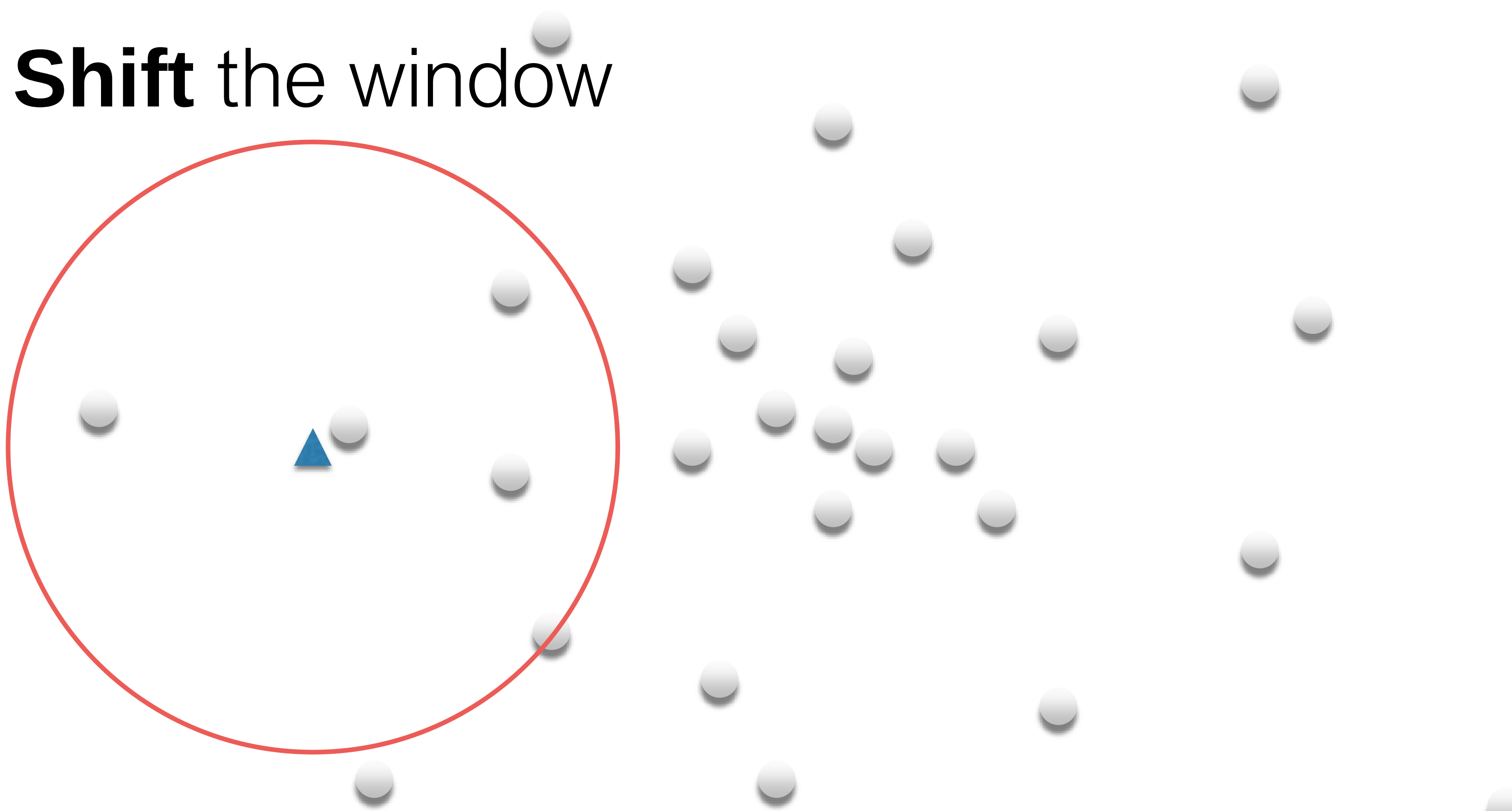
Shift the window



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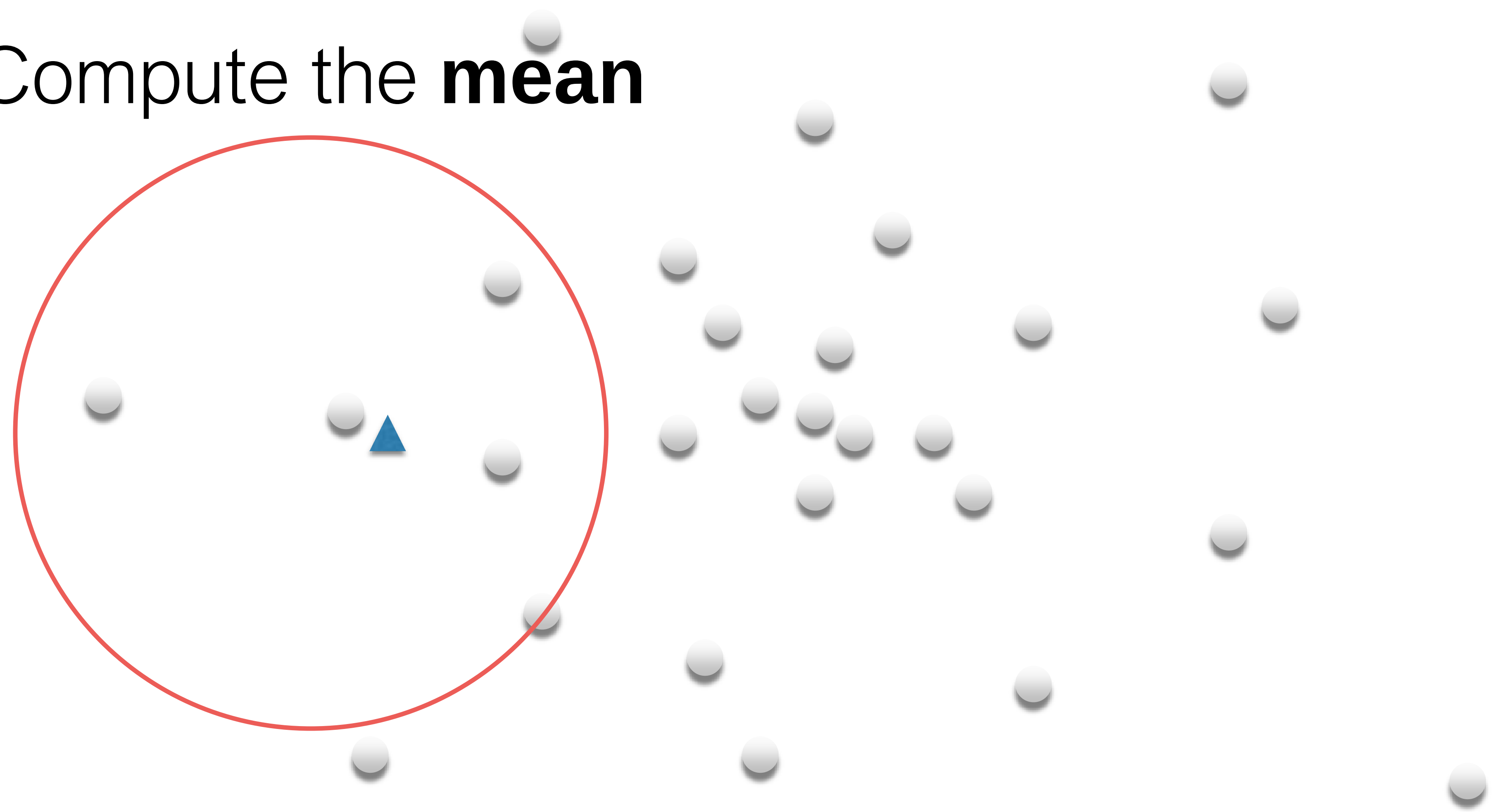

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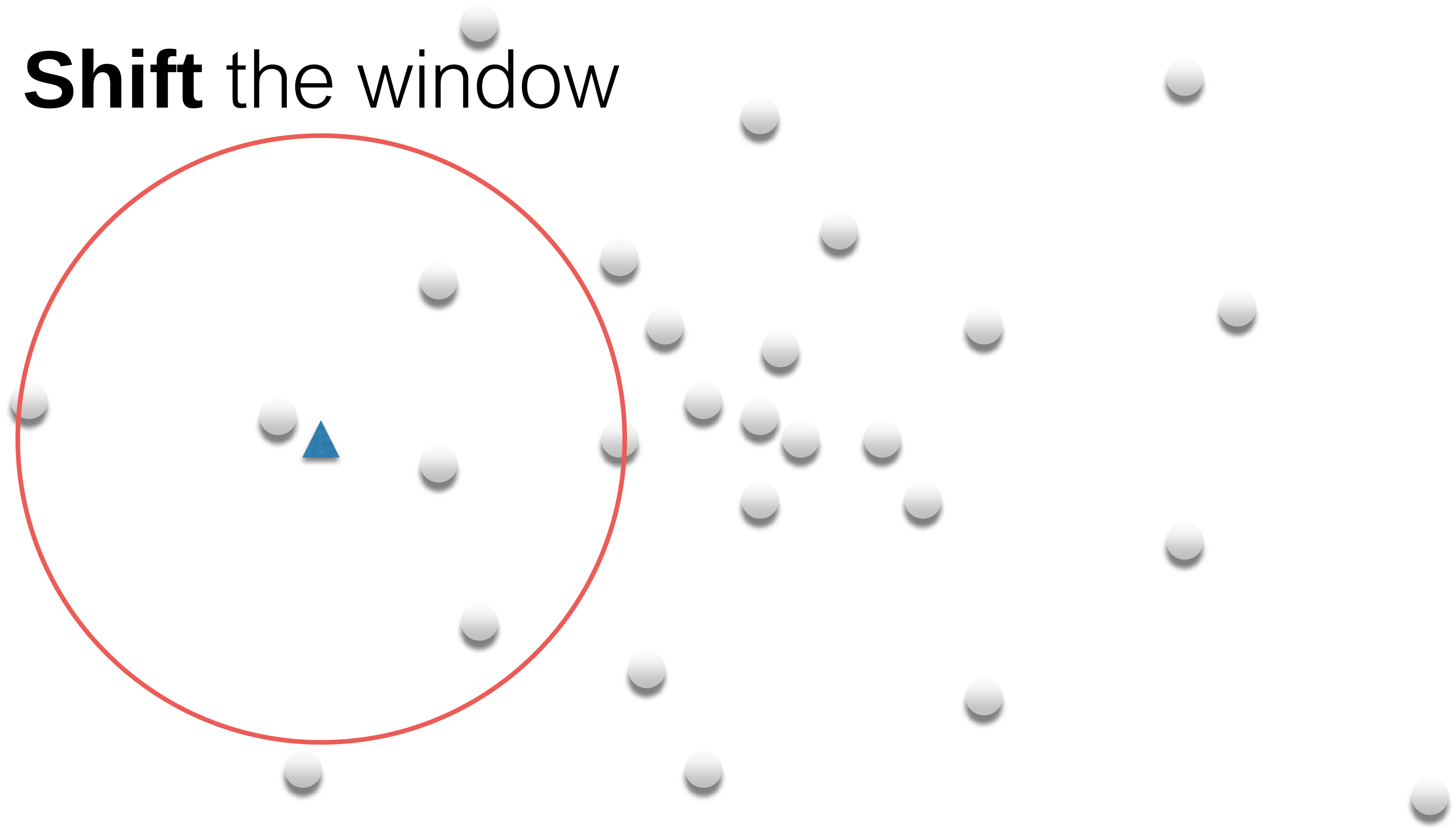

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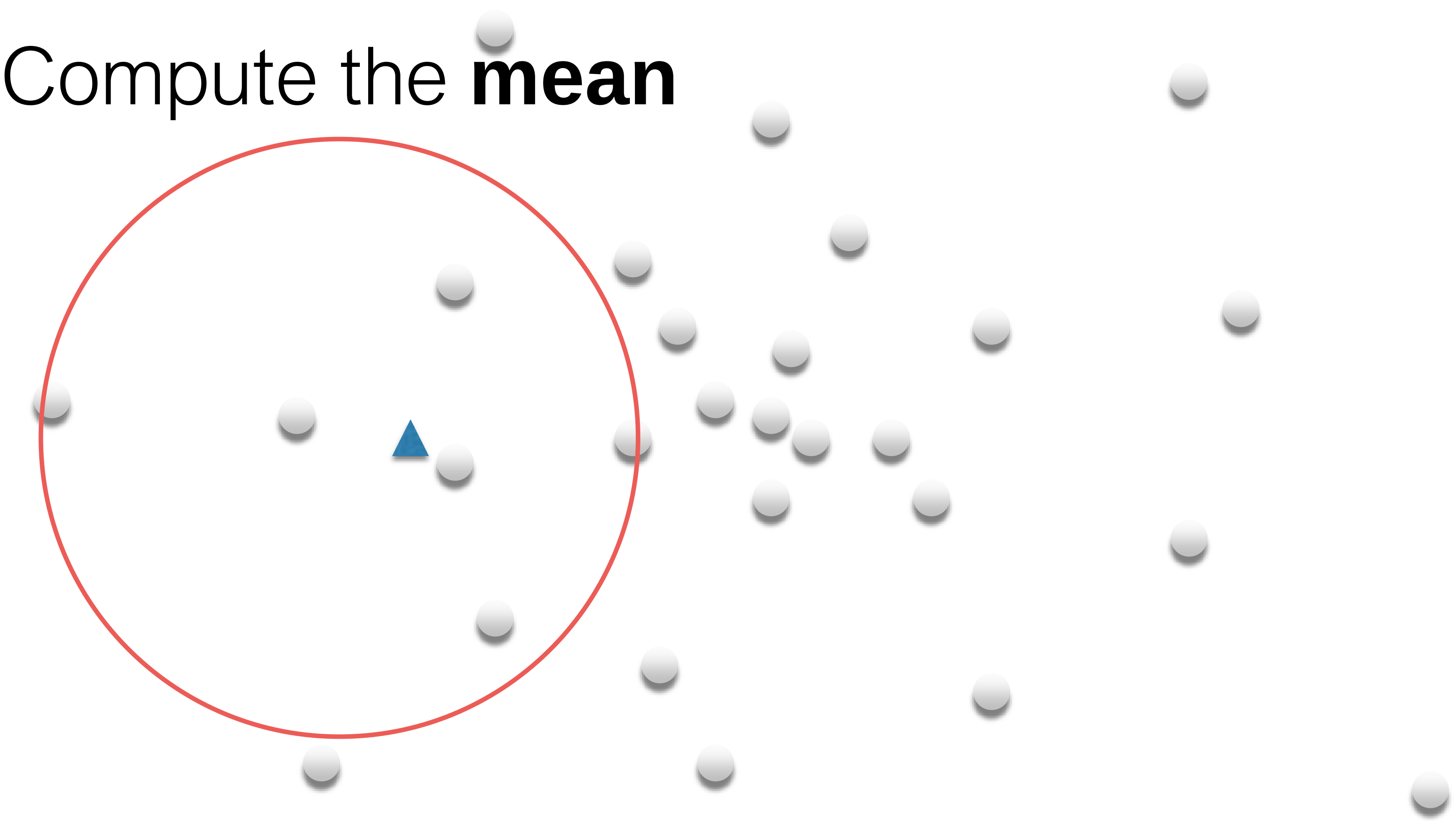

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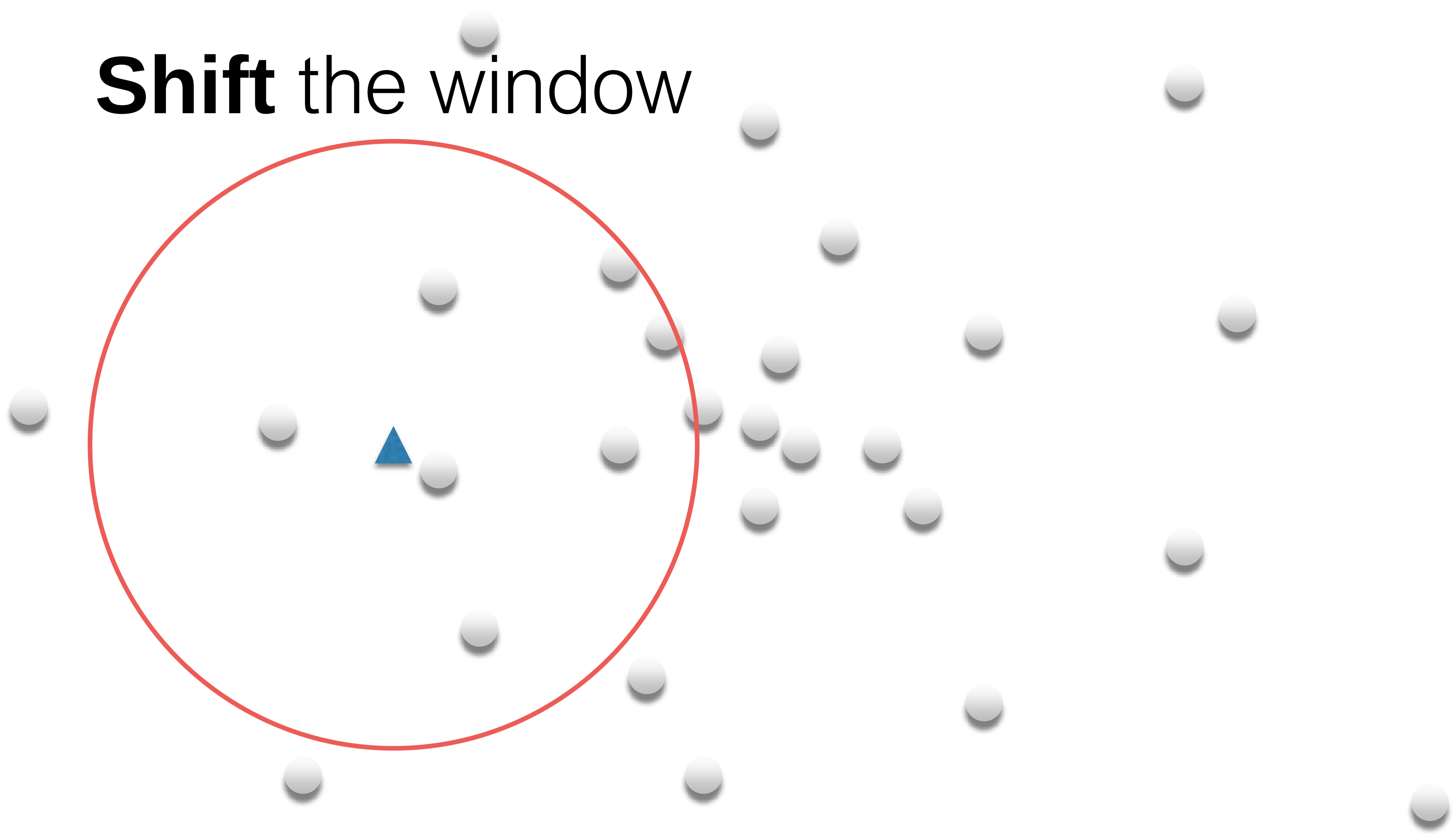

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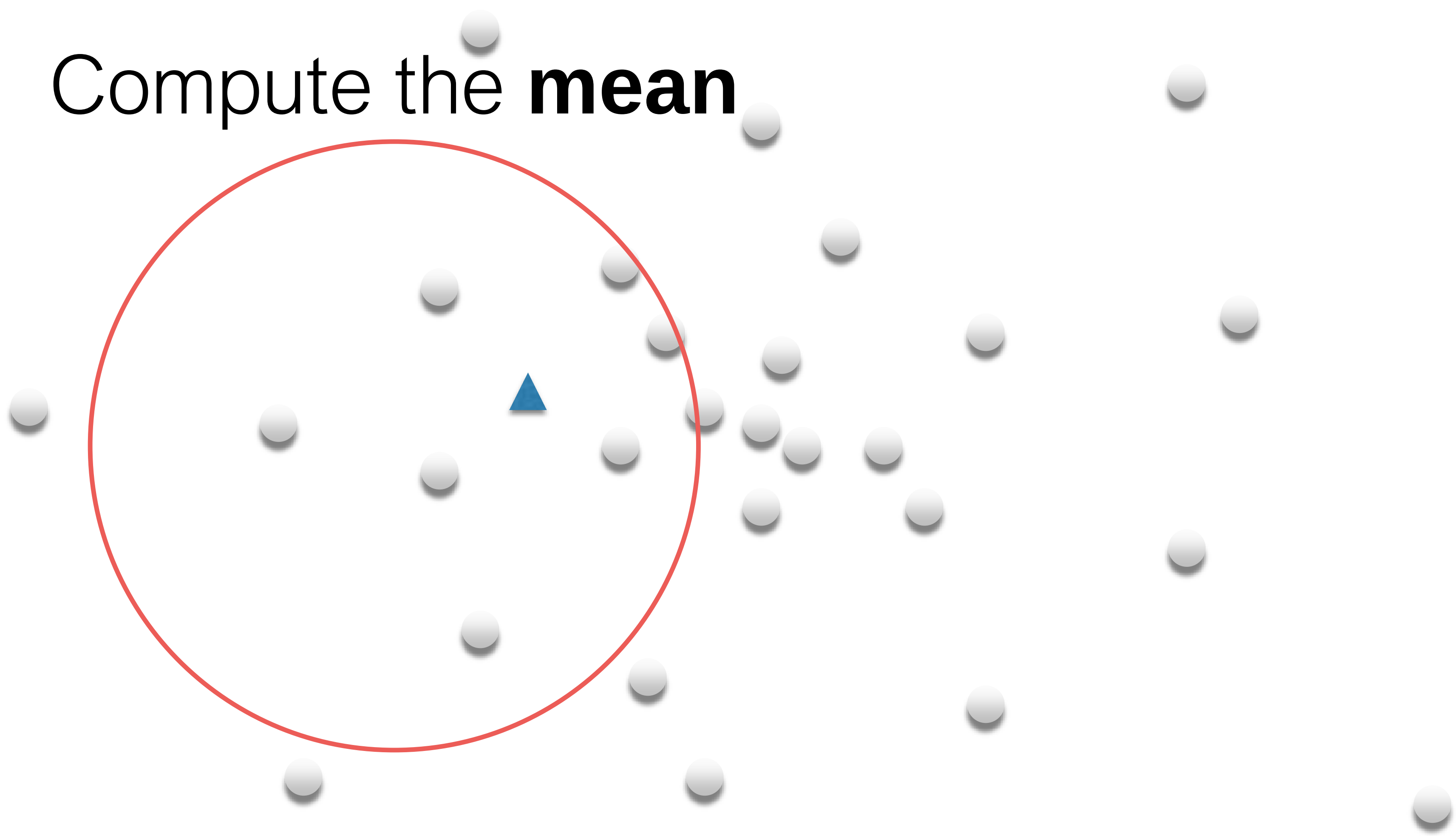

Mean Shift Algorithm

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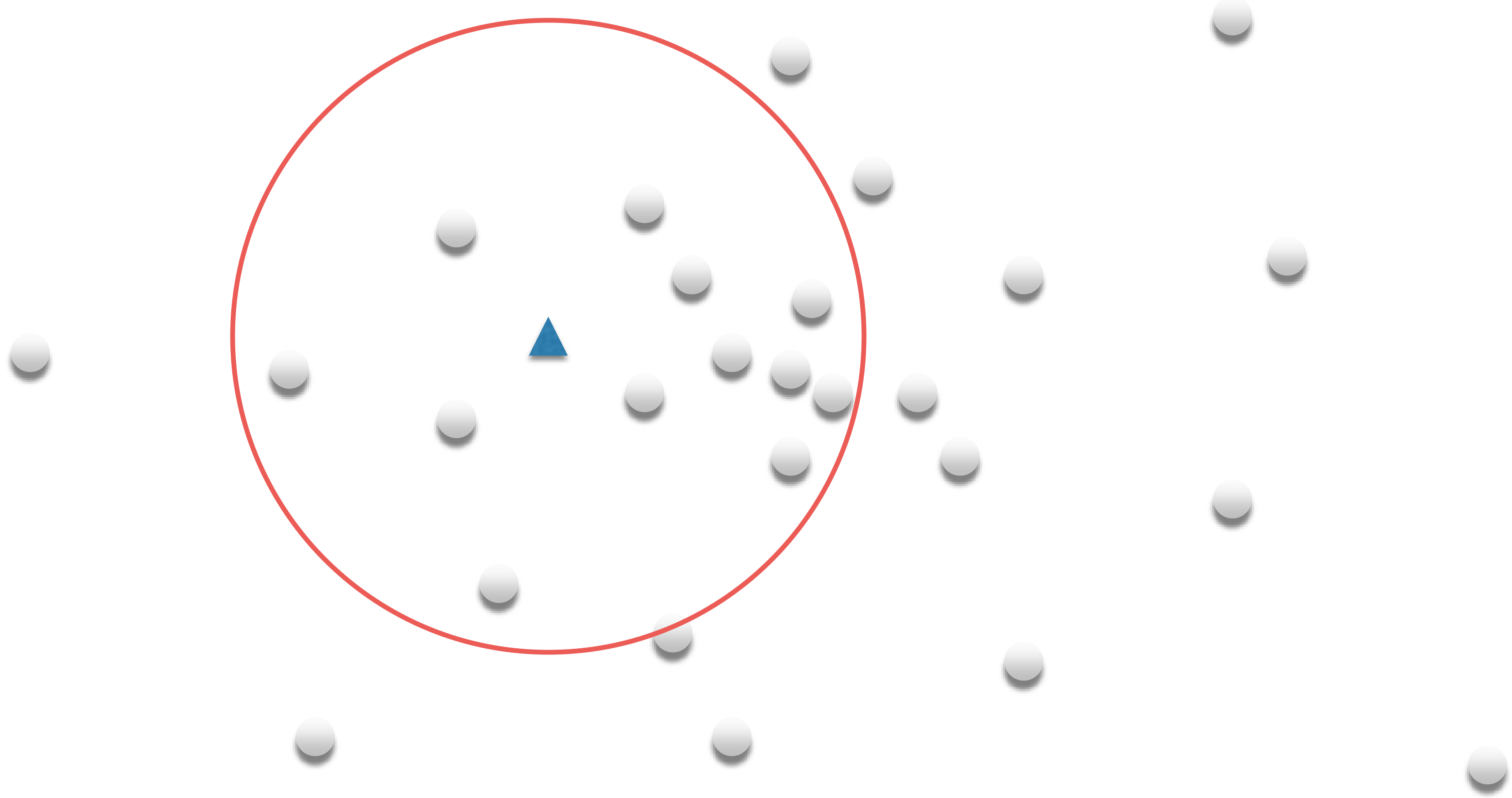
Fukunaga & Hostetler (1975)

Shift the window



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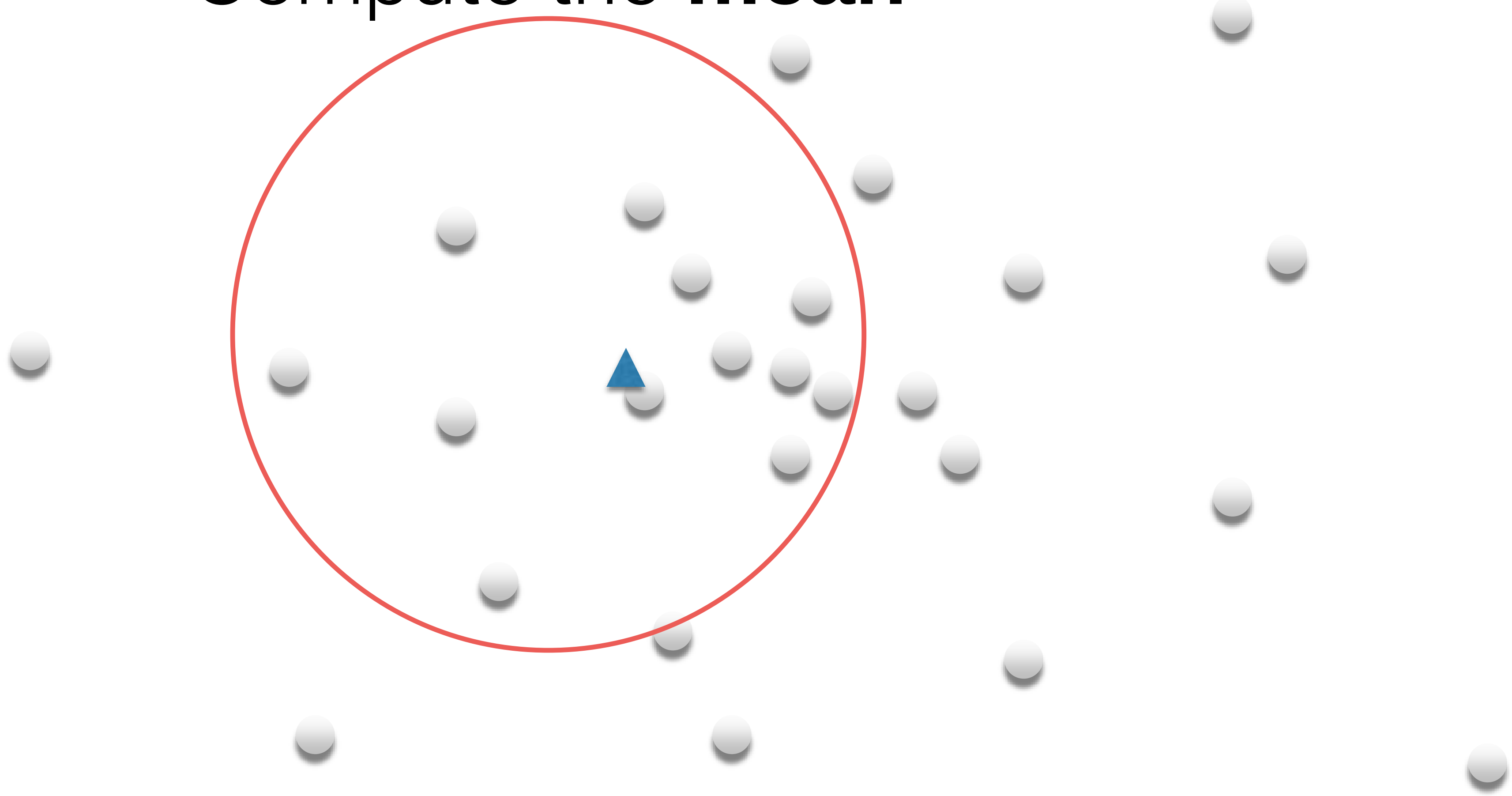

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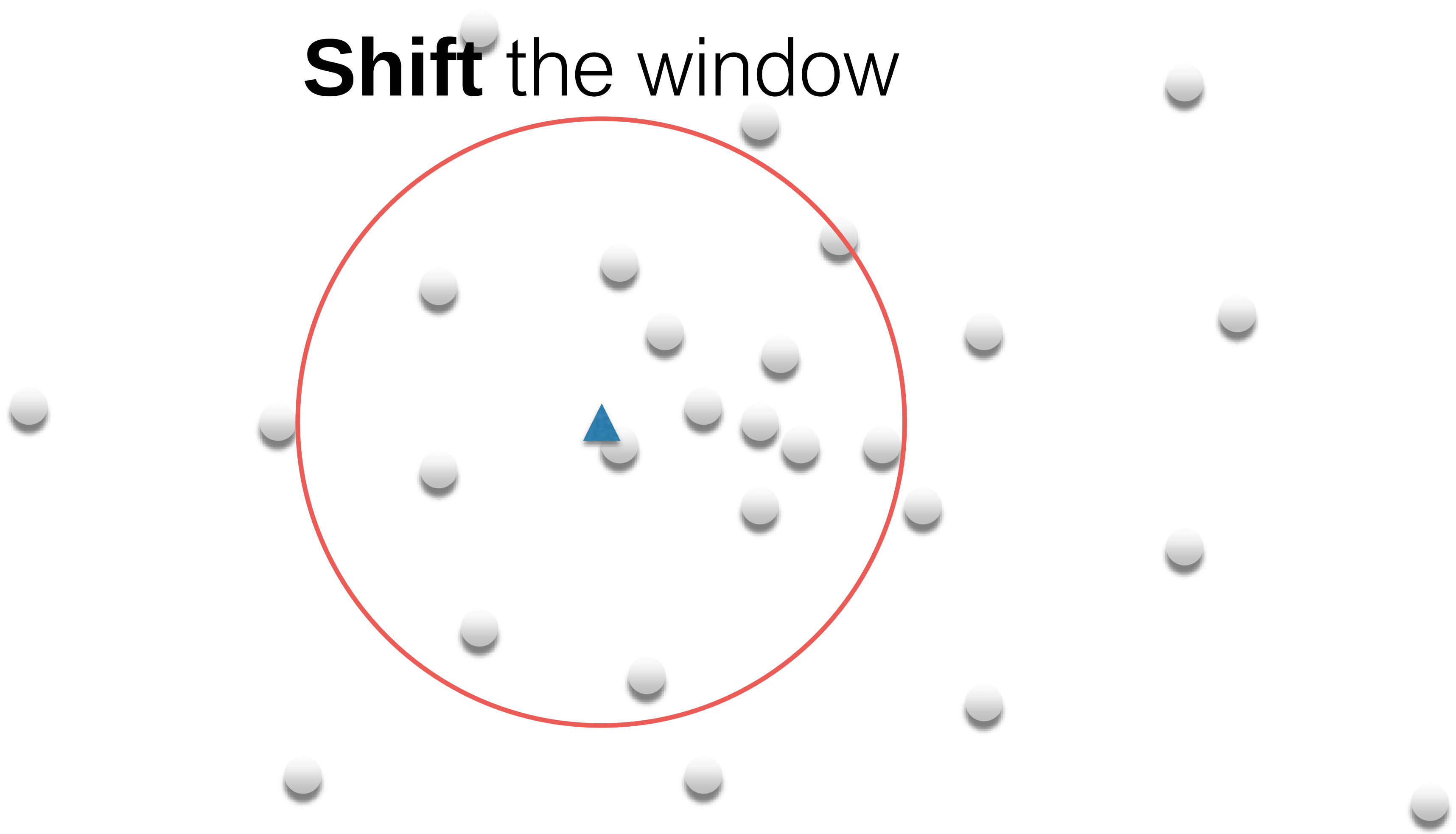

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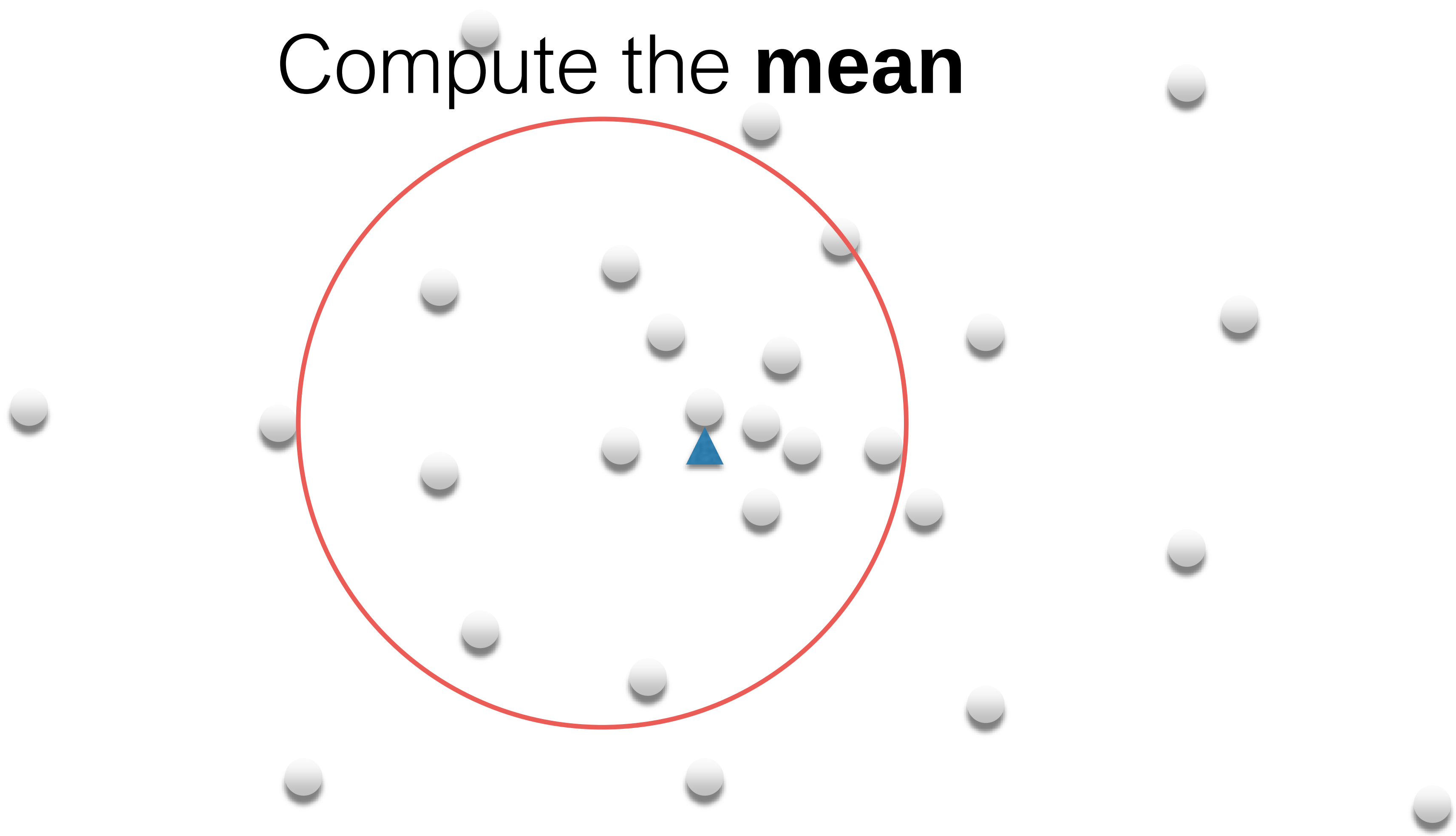

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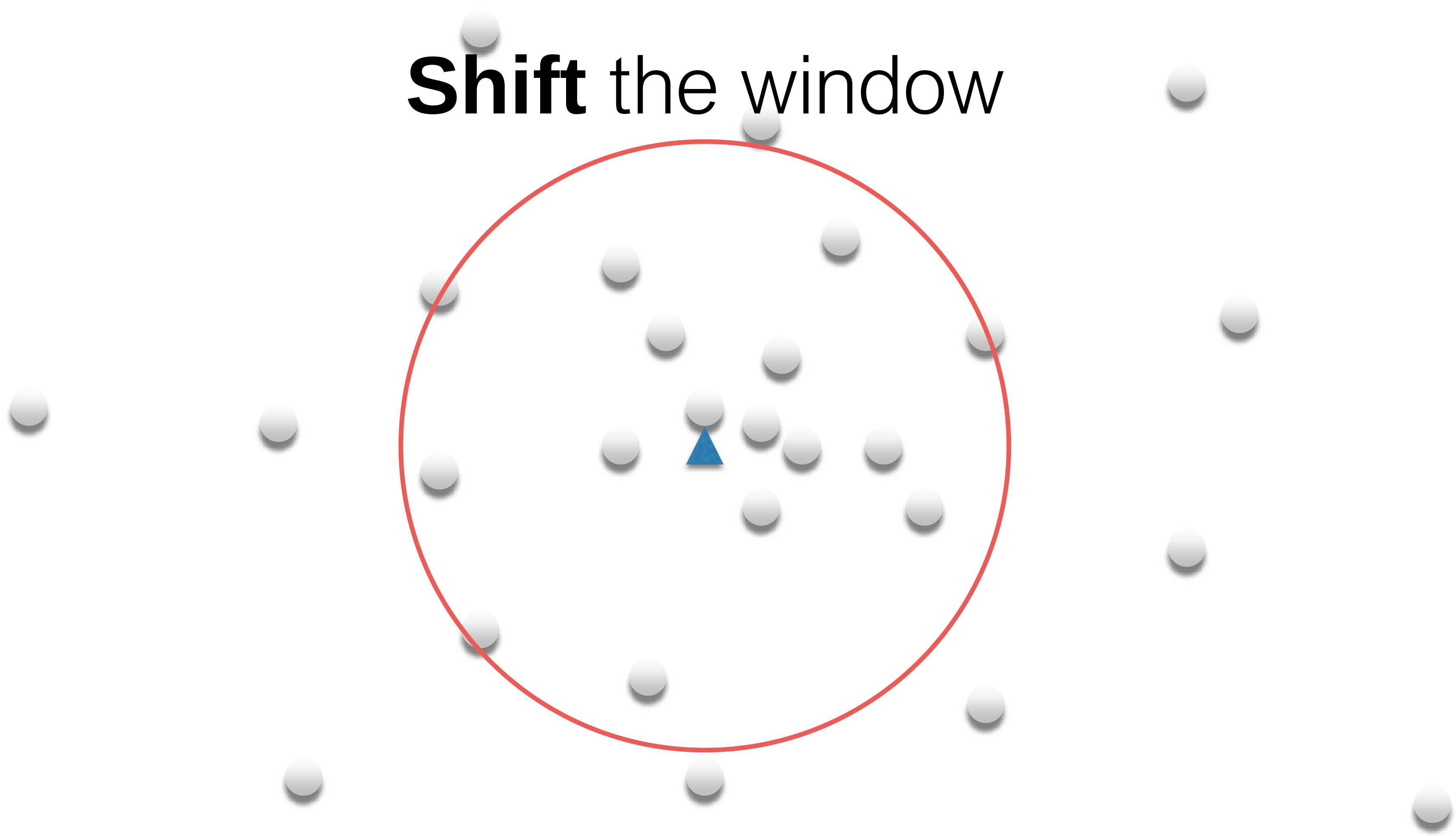

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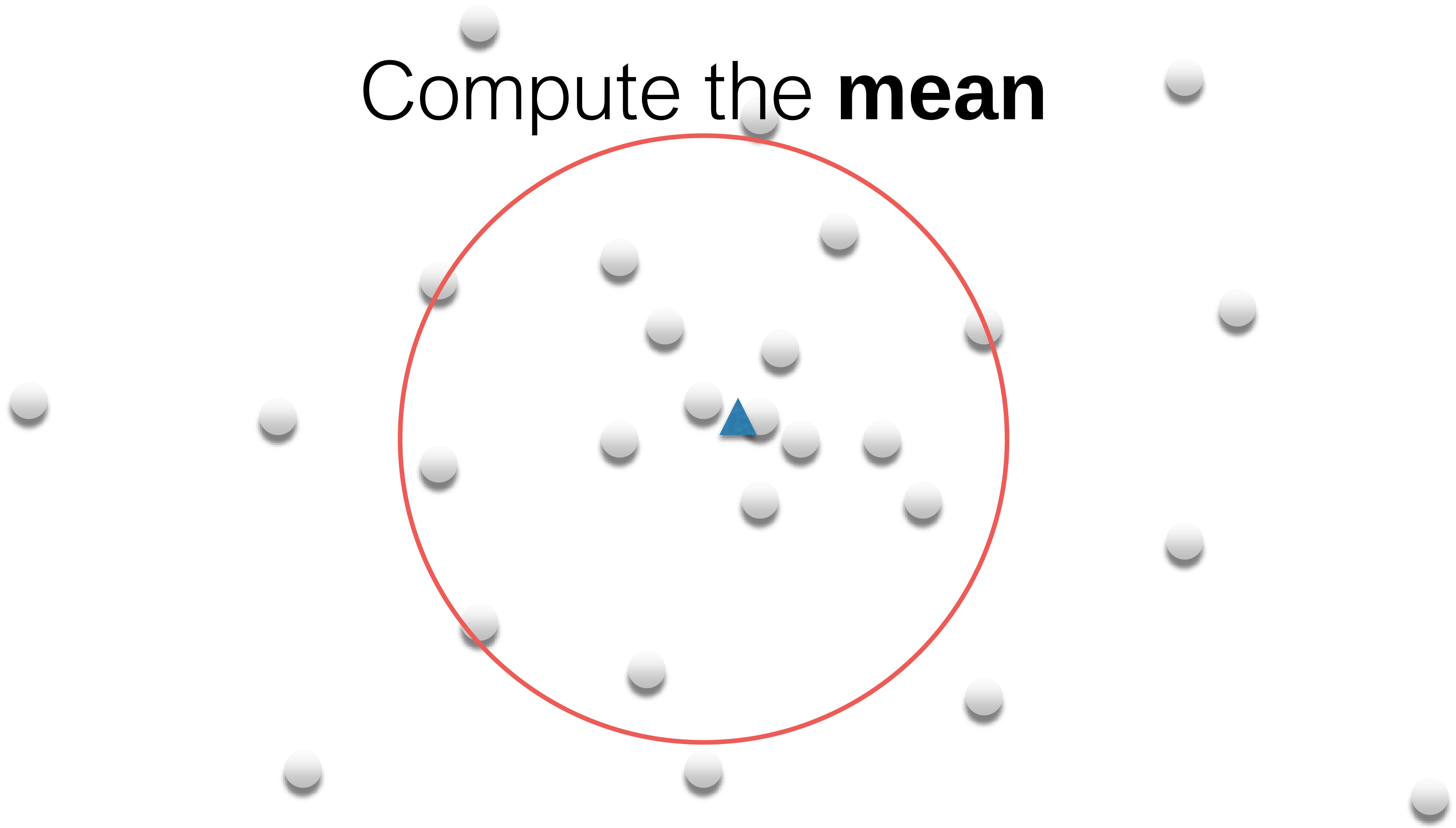

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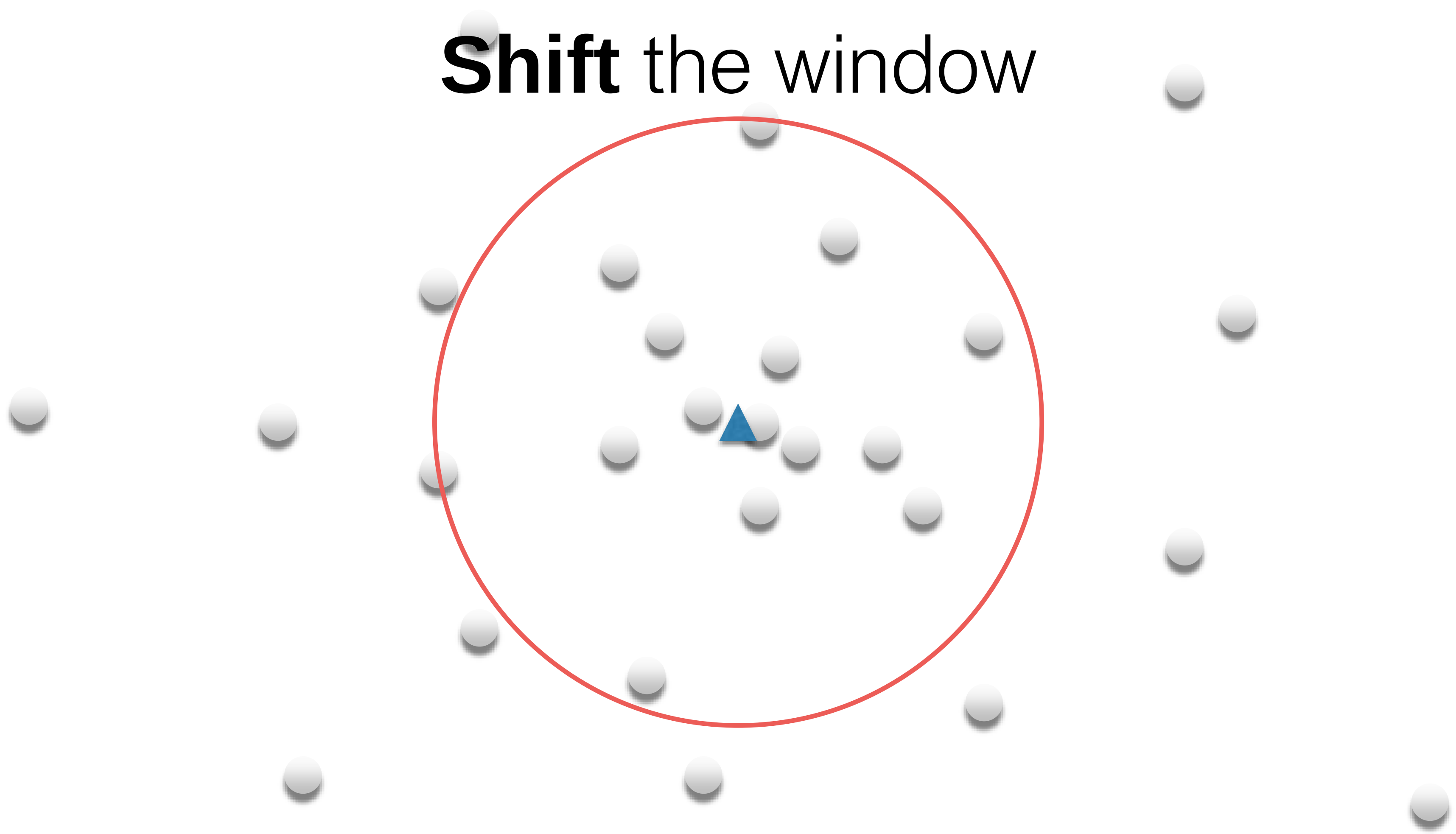
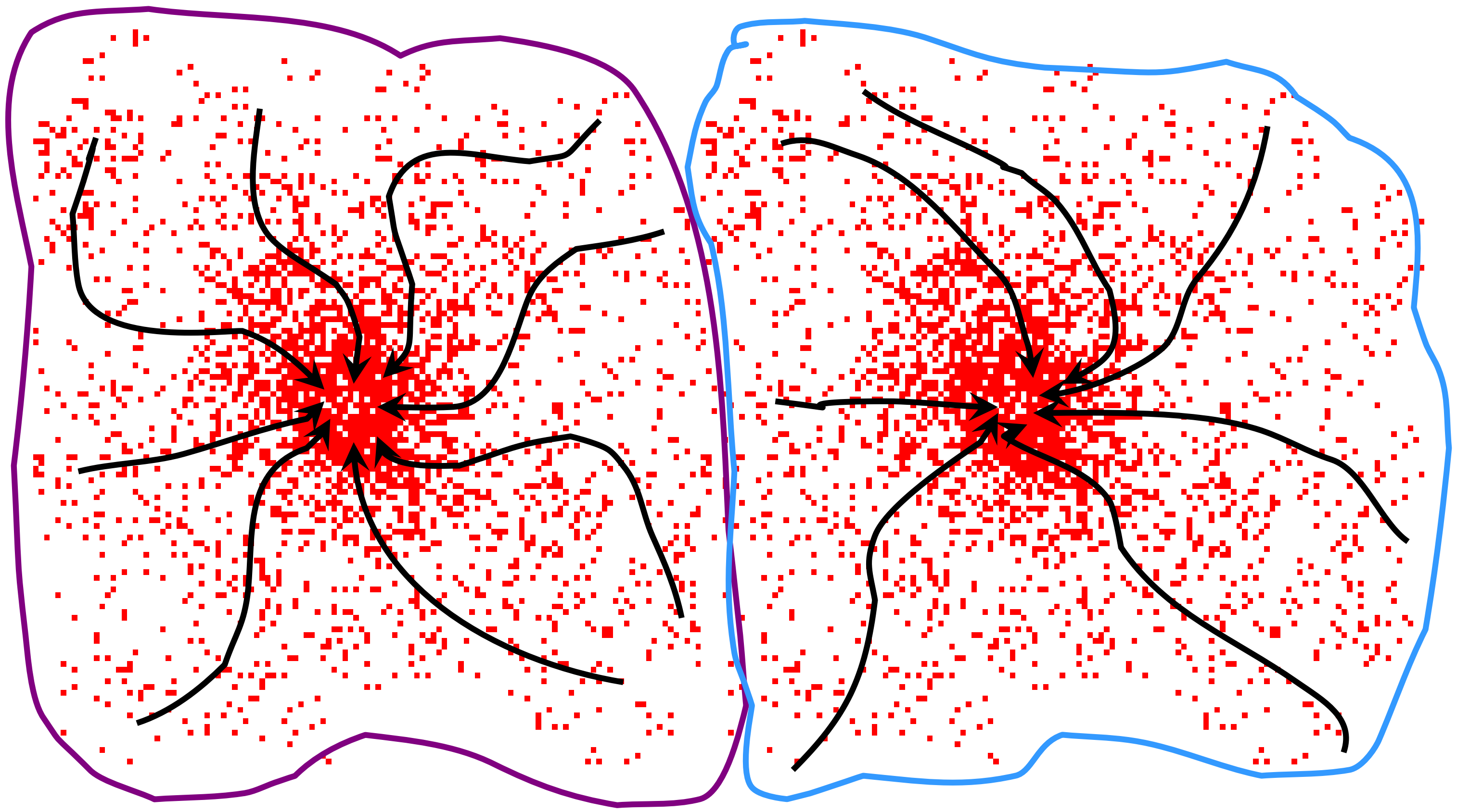


To understand the theory behind this we need to understand…

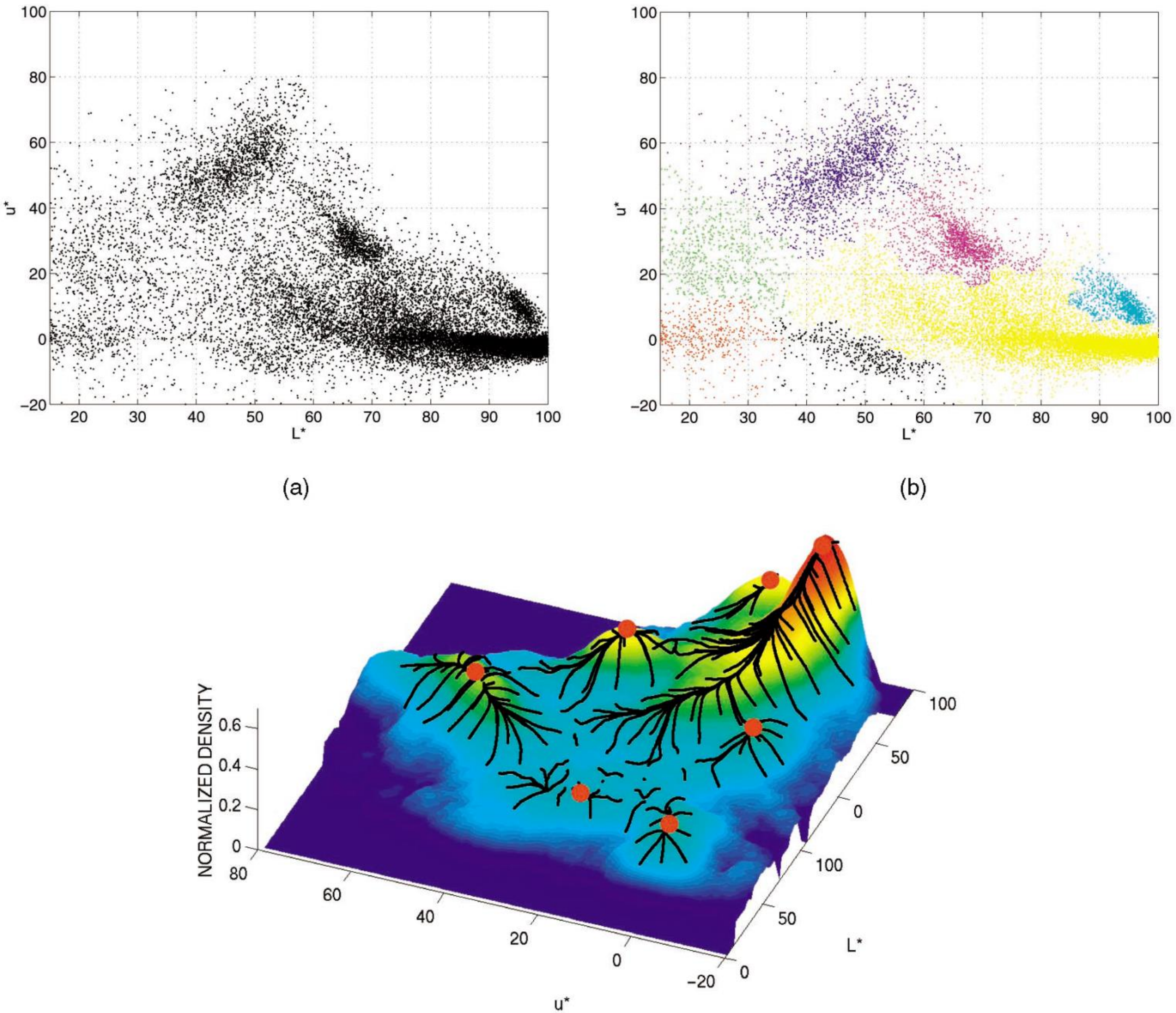
Attraction basin

- Attraction basin: the region for which all trajectories lead to the same mode
- Cluster: all data points in the attraction basin of a mode



Slide by Y. Ukrainitz & B. Sarel

Attraction basin

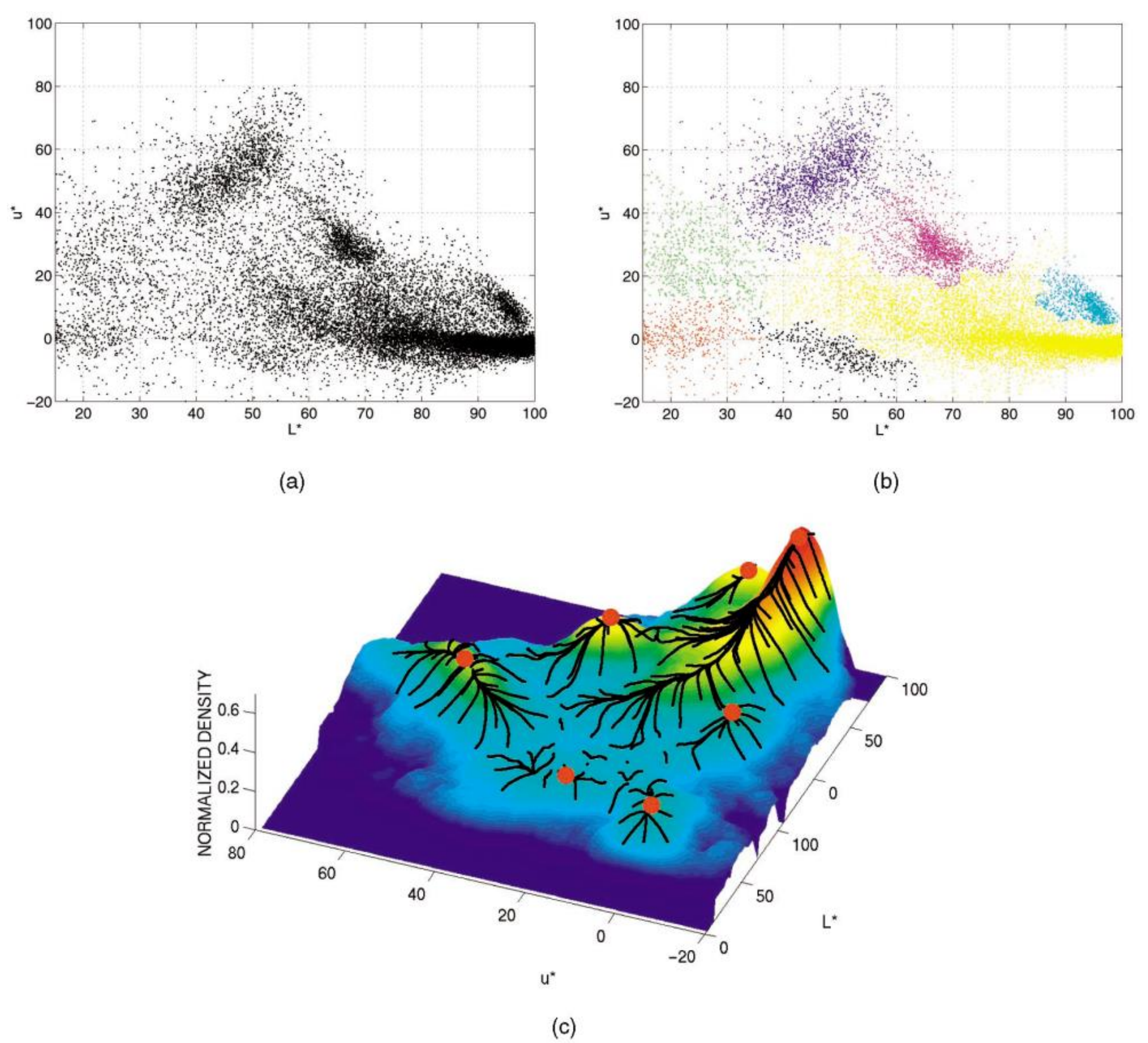


Mean shift clustering

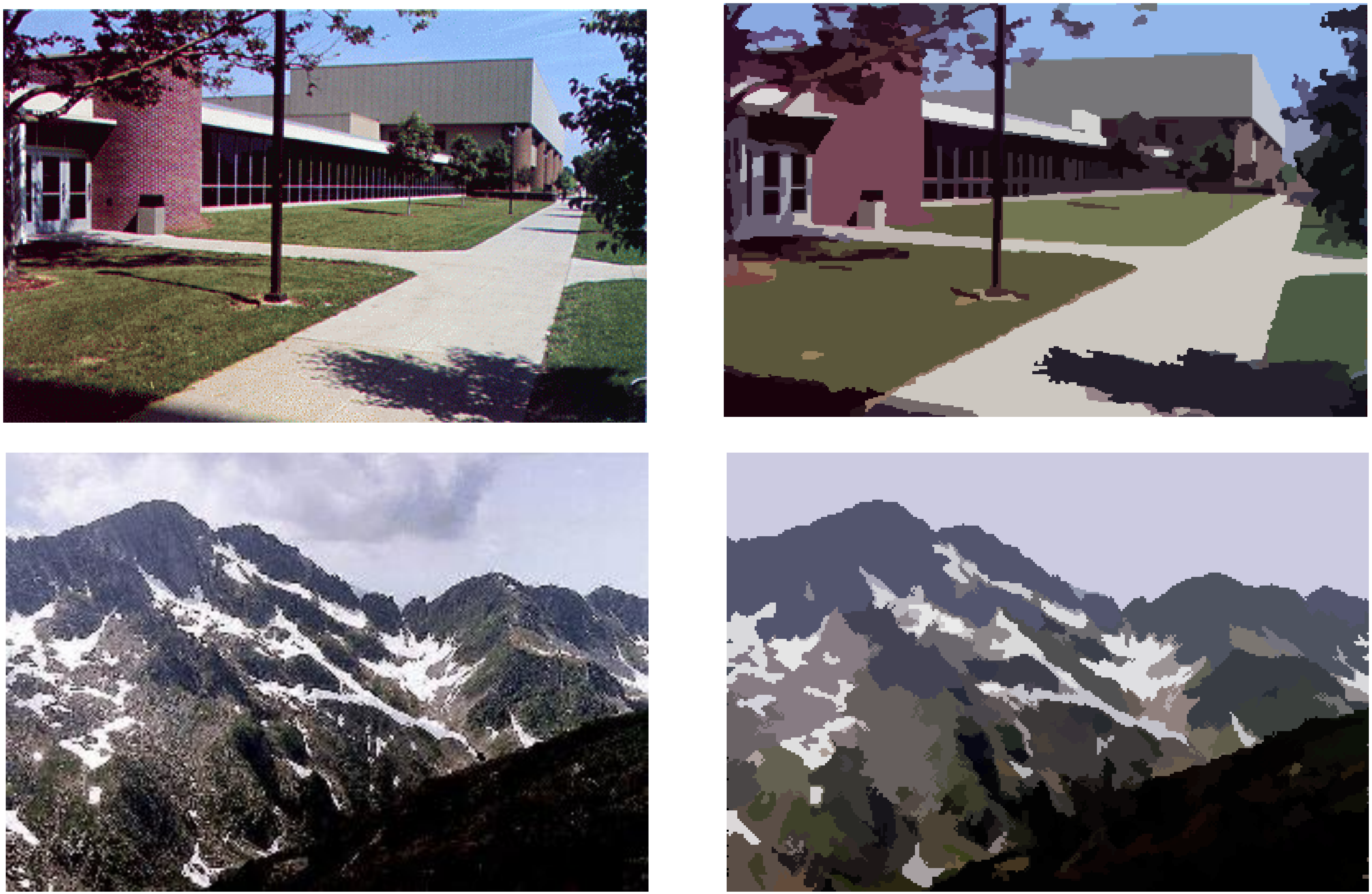
- The mean shift algorithm seeks *modes* of the given set of points
  1. Choose kernel and bandwidth
  2. For each point:
     a) Center a window on that point
     b) Compute the mean of the data in the search window
     c) Center the search window at the new mean location
     d) Repeat (b,c) until convergence
  3. Assign points that lead to nearby modes to the same cluster

Segmentation by Mean Shift

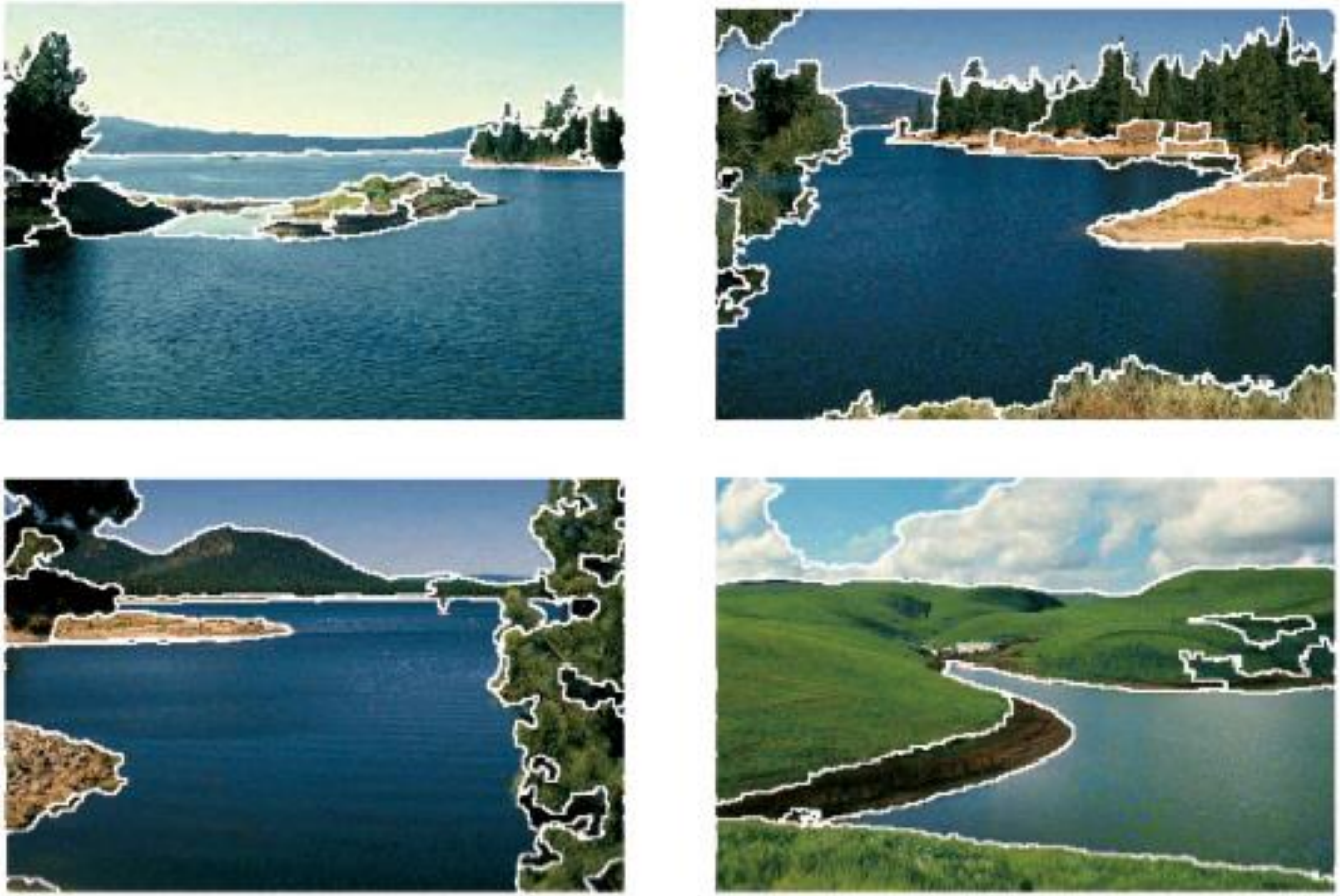
- Compute features for each pixel (color, gradients, texture, etc)
- Set kernel size for features K_f and position K_s
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that are within width of K_f and K_s



Mean shift segmentation results



http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html



http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

Mean-shift: other issues

- Speedups
  - Binned estimation
  - Fast search of neighbors
  - Update each window in each iteration (faster convergence)

- Other tricks
  - Use kNN to determine window sizes adaptively

- Lots of theoretical support

  D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

References

Basic reading:

- Szeliski, Sections 5.2, 5.3, 5.4, 5.5.