Photometric stereo and shape from shading

16-385 Computer Vision
Spring 2018, Lecture 14

http://www.cs.cmu.edu/~16385/

Course announcements

- Homework 3 has been posted and is due on March 9th.
  - Any questions about the homework?
  - How many of you have looked at/started/finished homework 3?
- Office hours for Yannis' this week: Wednesday 3-5 pm.
- Gaurav's office hours now happen on Smith 200.
- Many more talks this week:

1. Manolis Savva, "Human-centric Understanding of 3D Environments," Wednesday March 7, 2:00 PM, NSH 3305.

2. David Fouhey, "Recovering a Functional and Three Dimensional Understanding of Images," Thursday March 8, 4:00 PM, NSH 3305.

Overview of today’s lecture

- Light sources.
- Shape from shading.
- Photometric stereo.

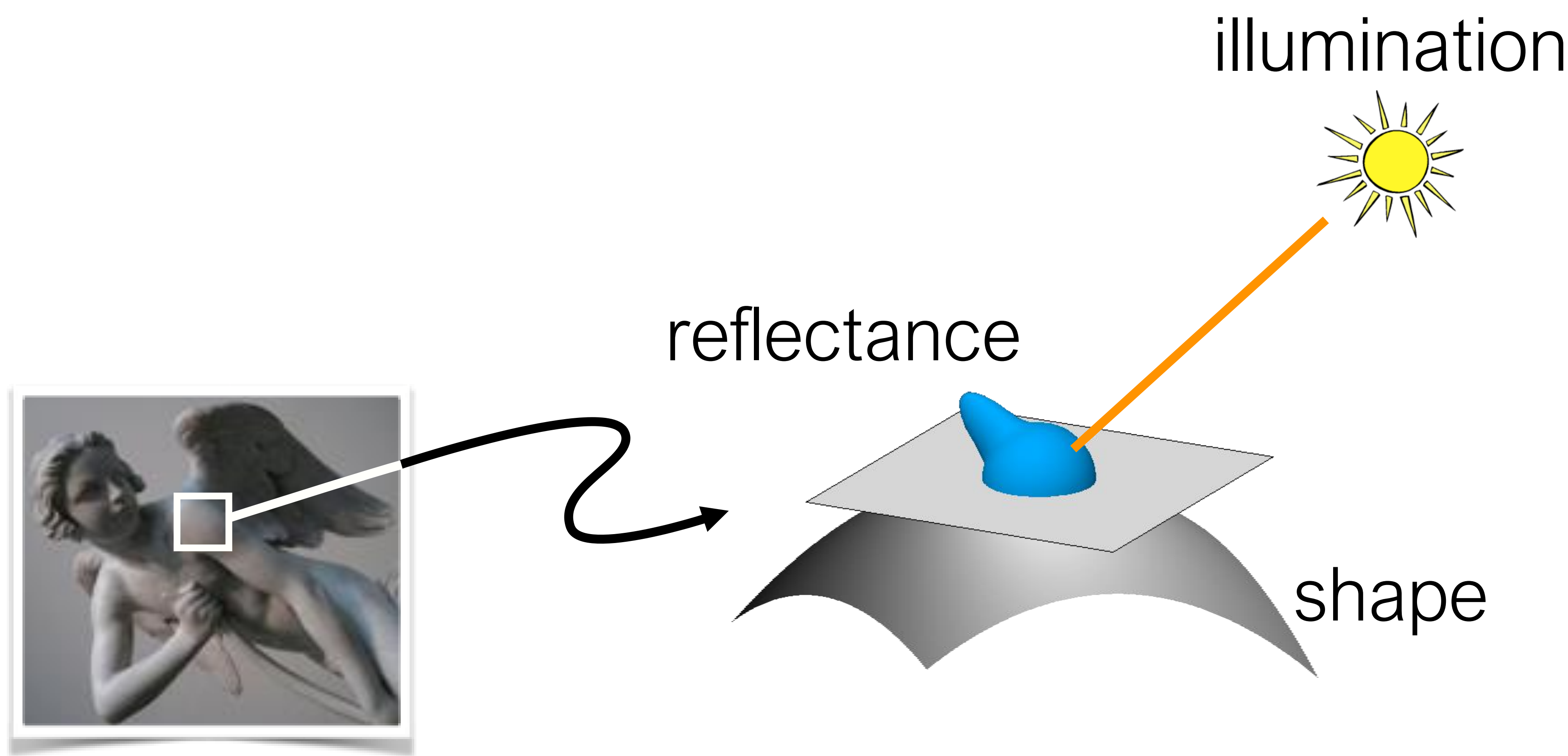
Slide credits

Most of these slides were adapted from:

- Srinivasa Narasimhan (16-385, Spring 2014).
- Todd Zickler (Harvard University).
- Steven Gortler (Harvard University).

Light sources

“Physics-based” computer vision
(a.k.a “inverse optics”)



$\mathbf{I} \Rightarrow$ shape, illumination, reflectance

Lighting models: Plenoptic function

- Radiance as a function of position and direction
- Radiance as a function of position, direction, and time
- Spectral radiance as a function of position, direction, time and wavelength

$$L(x, \omega, t, \lambda)$$

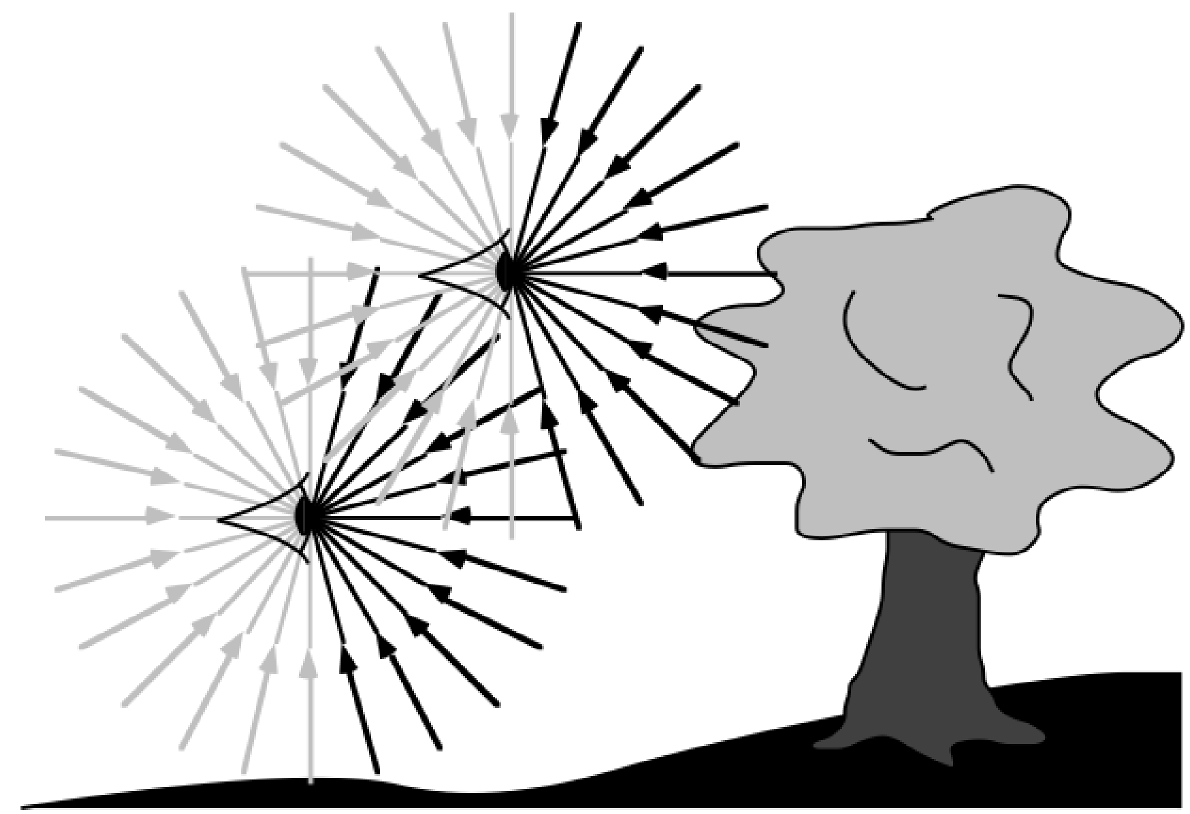


Fig.1.3
The plenoptic function describes the information available to an observer at any point in space and time. Shown here are two schematic eyes-which one should consider to have punctate pupils-gathering pencils of light rays. A real observer cannot see the light rays coming from behind, but the plenoptic function does include these rays.

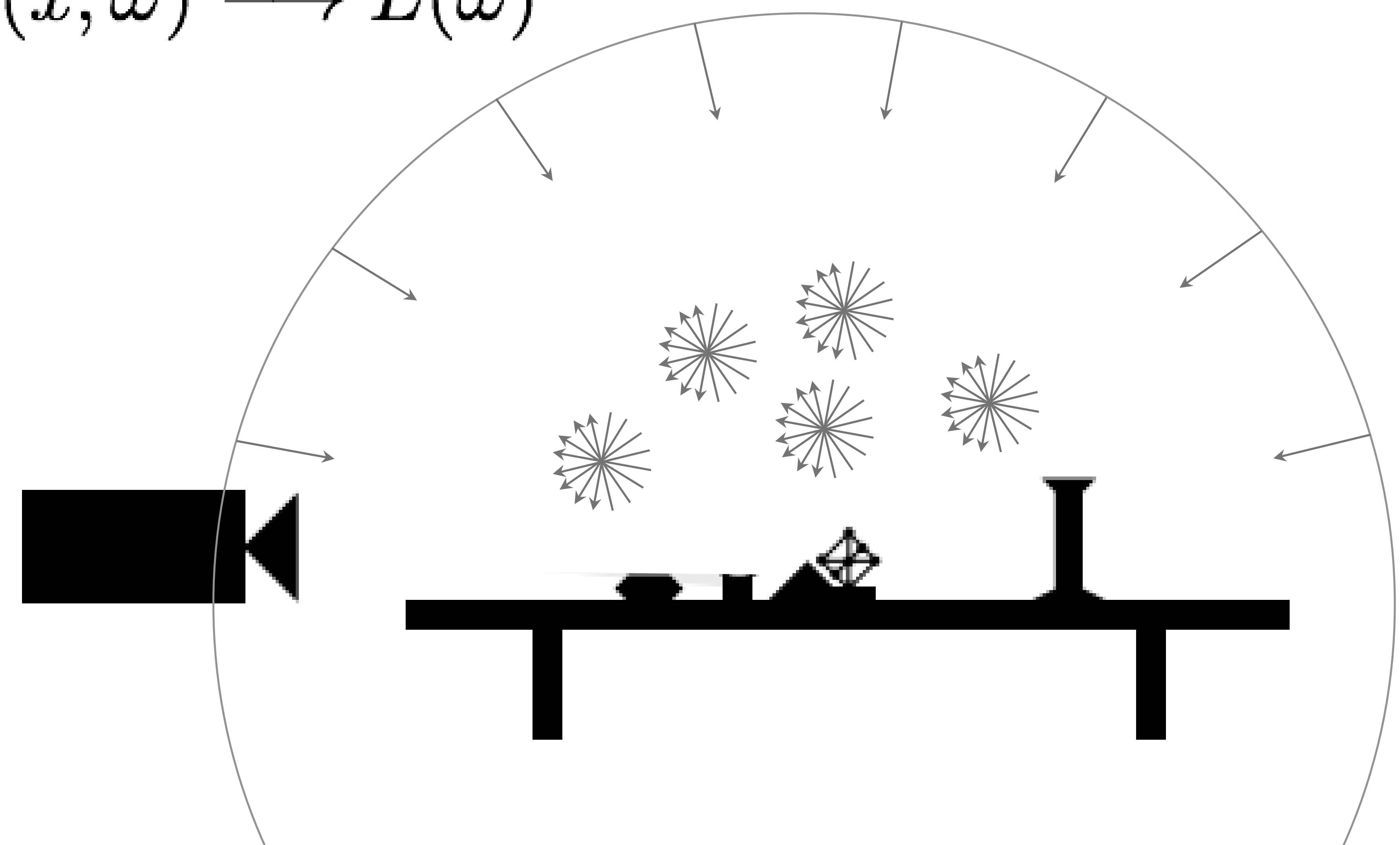
[Adelson and Bergen, 1991]

Lighting models: far-field approximation

- Assume that, over the observed region of interest, all source of incoming flux are relatively far away

$$L(x, \omega, t, \lambda) \longrightarrow L(\omega, t, \lambda)$$

$$L(x, \omega) \longrightarrow L(\omega)$$



[Debevec, 1998]

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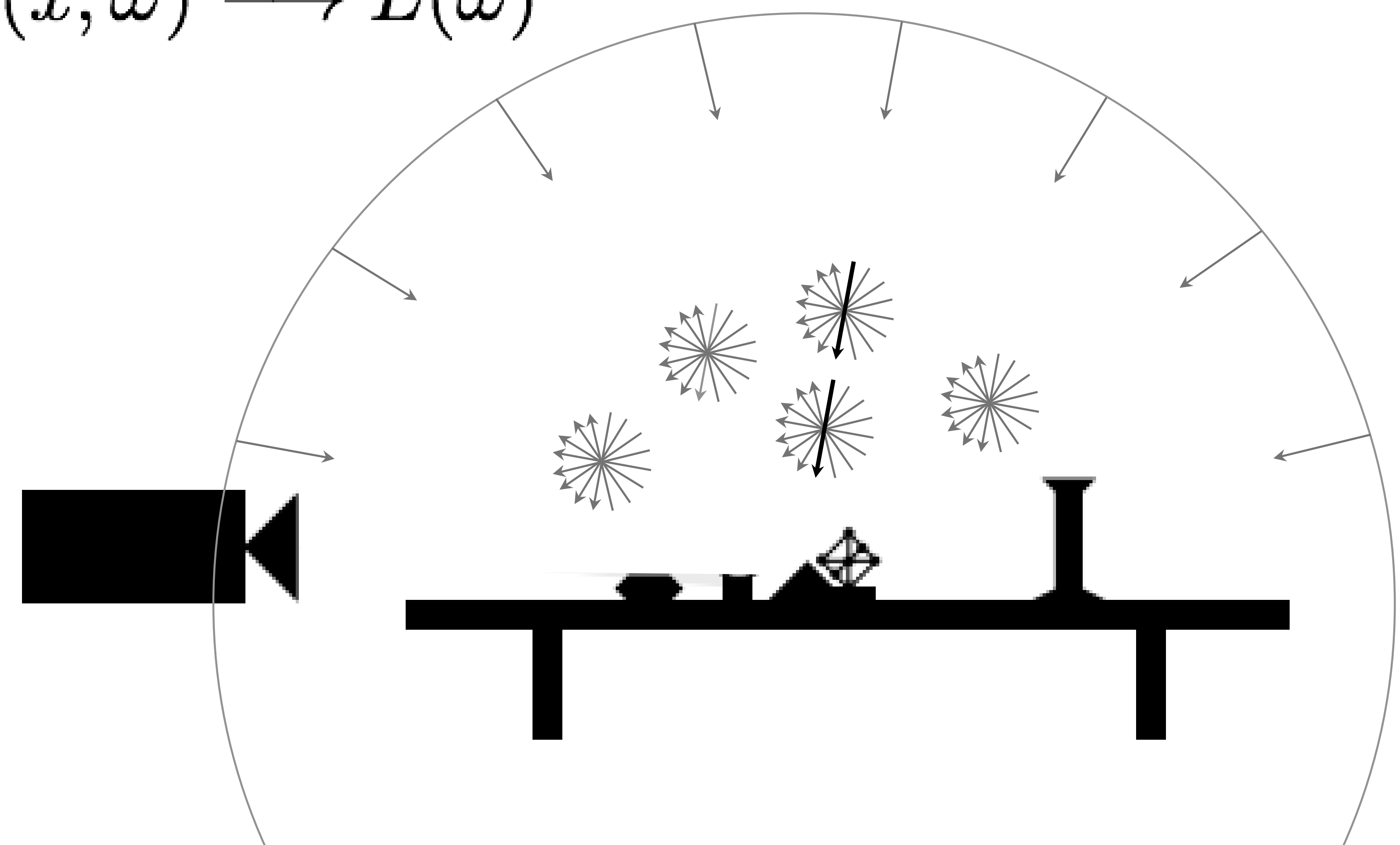

[Debevec, 1998]

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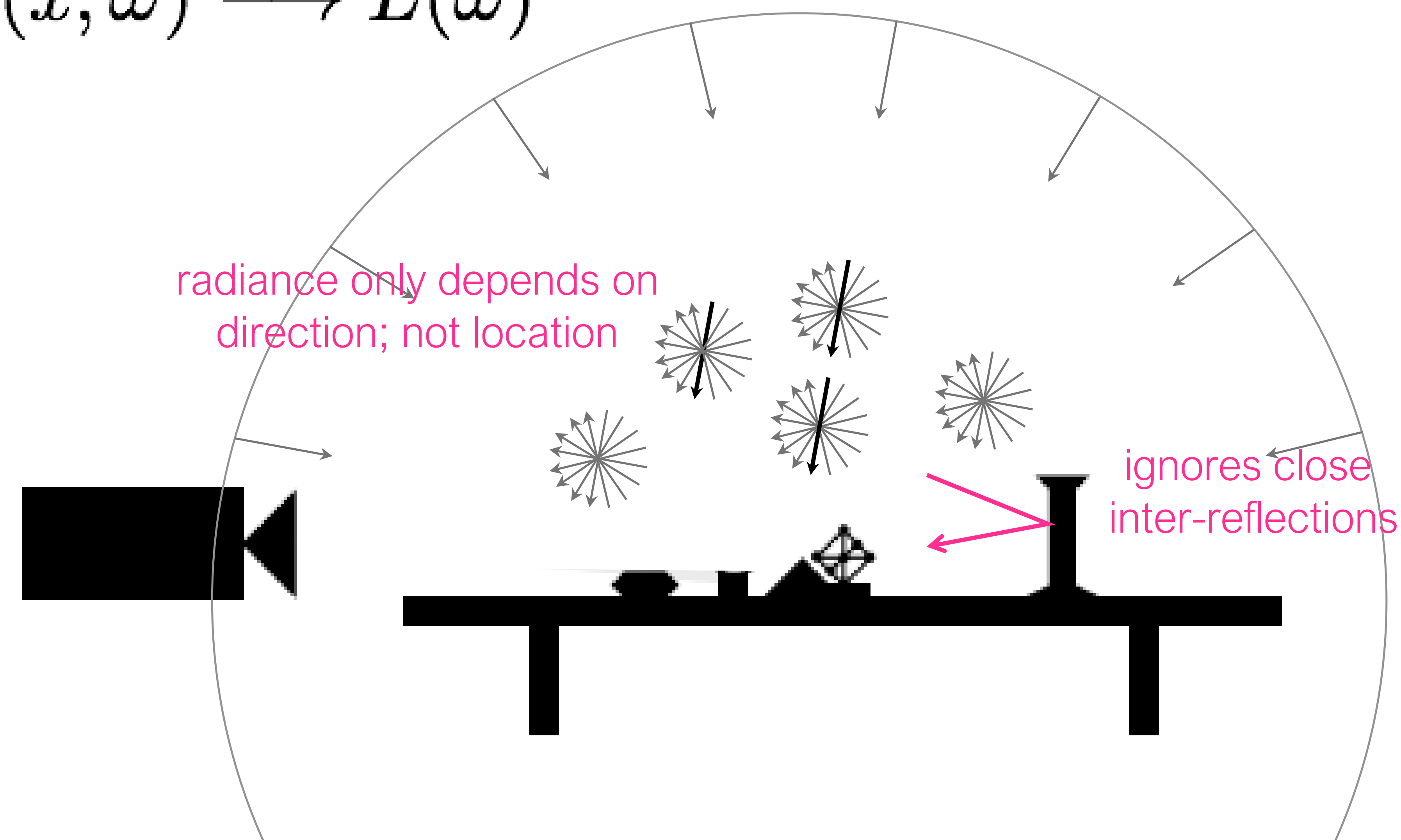
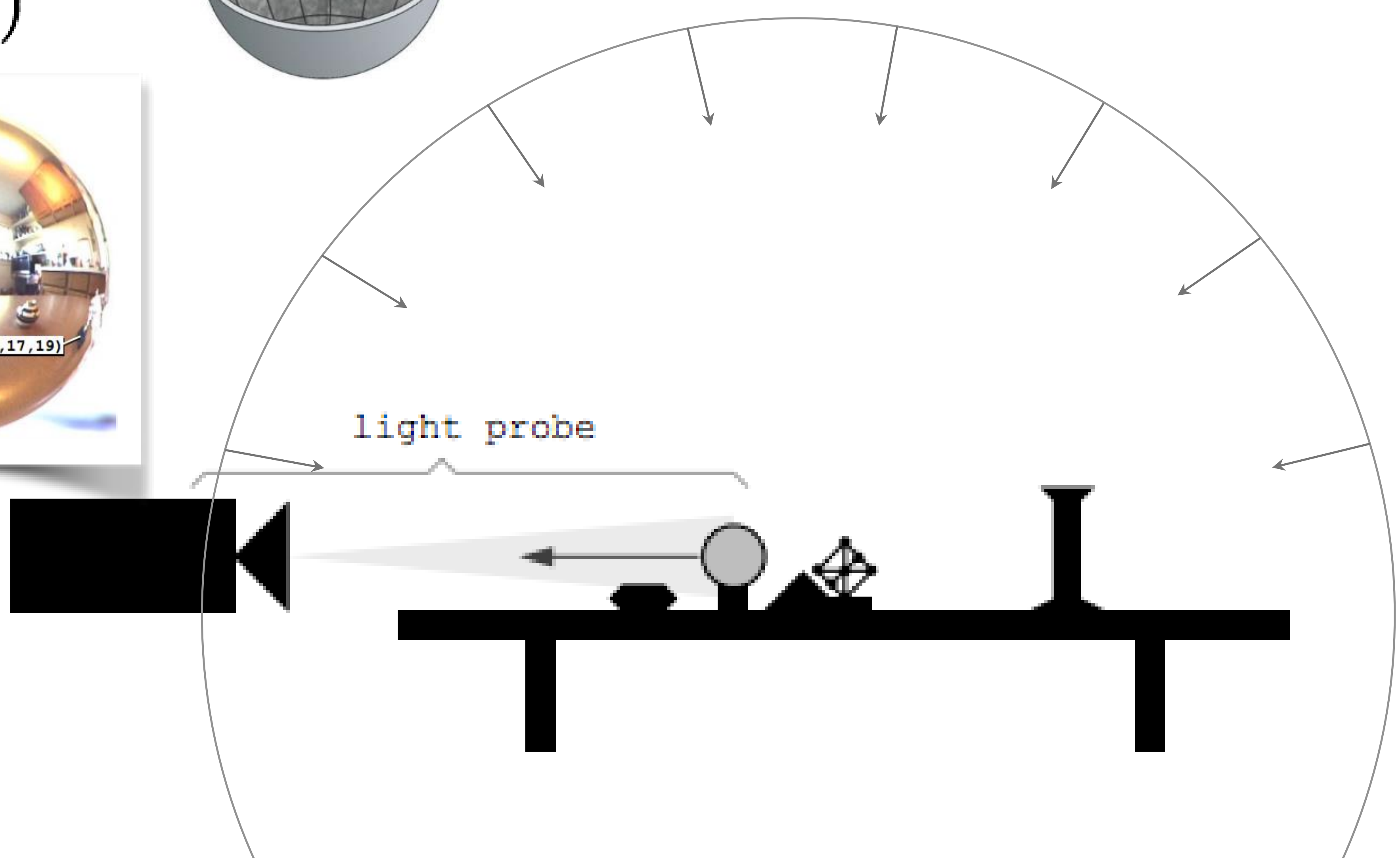


[Debevec, 1998]

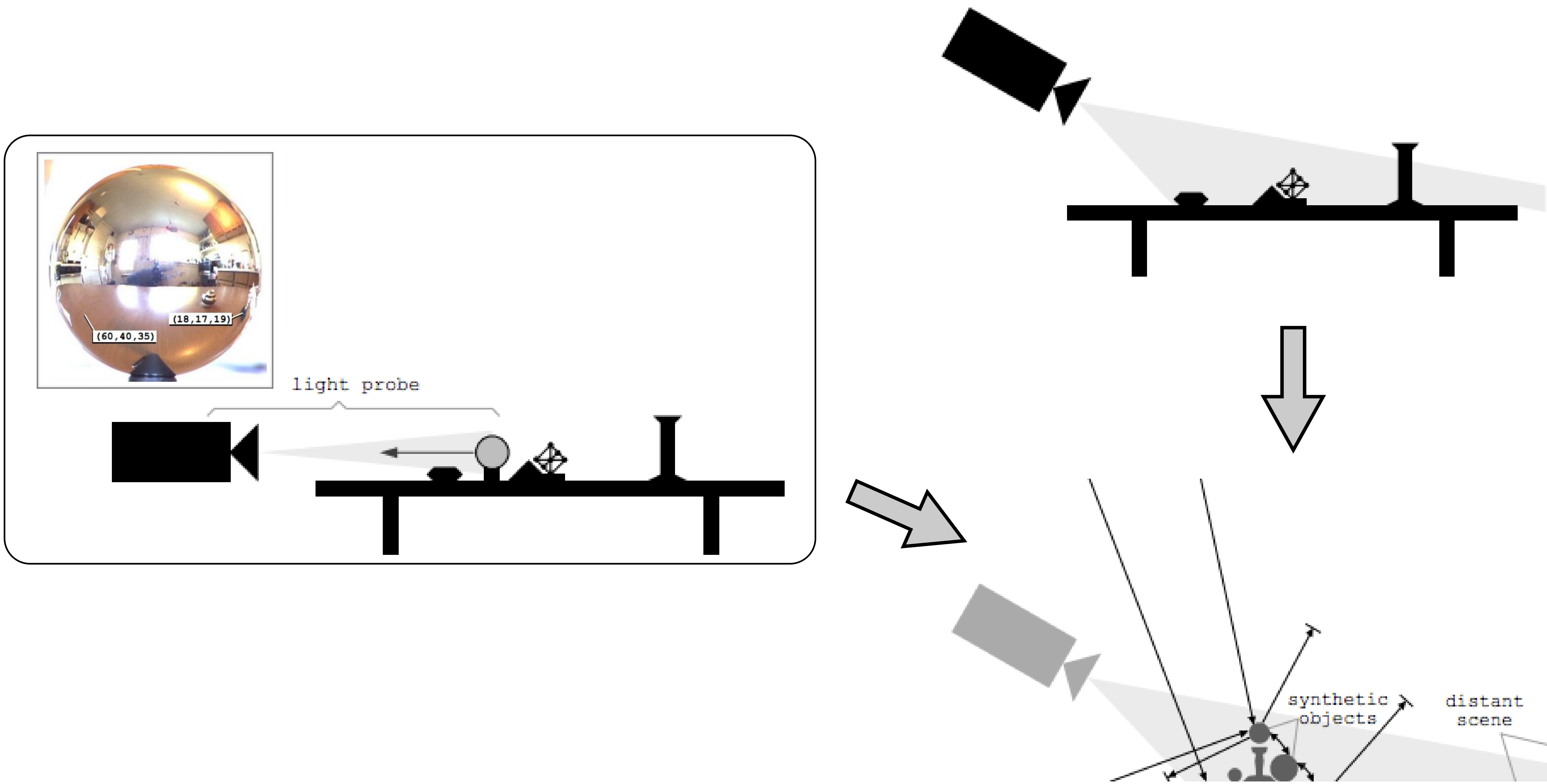
Application: augmented reality

$L(\omega)$



[Debevec, 1998]

Application: augmented reality



[Debevec, 1998]

Application: augmented reality

(a) Background photograph

(b) Camera calibration grid and light probe

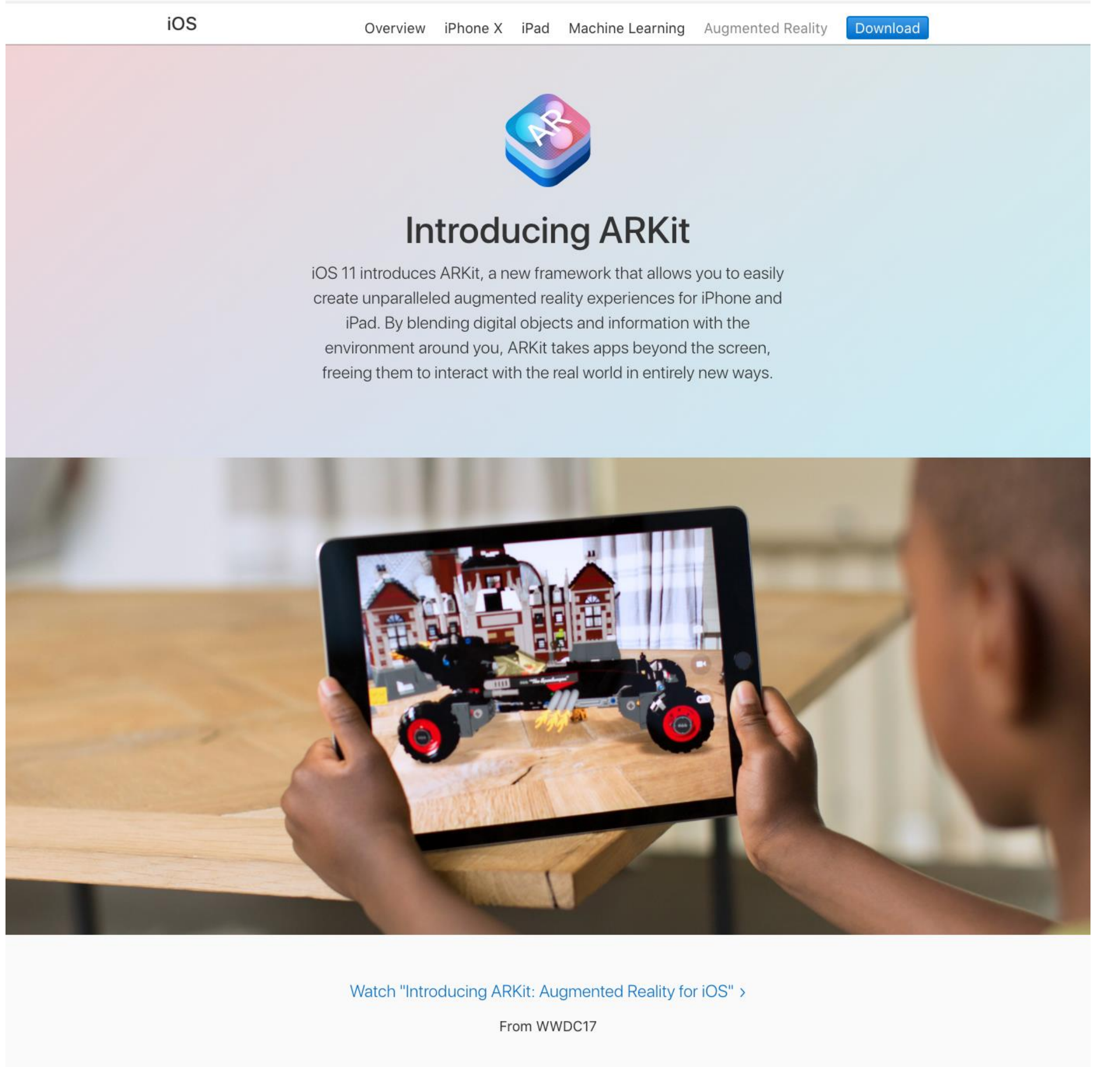
(g) Final result with differential rendering

[Debevec, 1998]

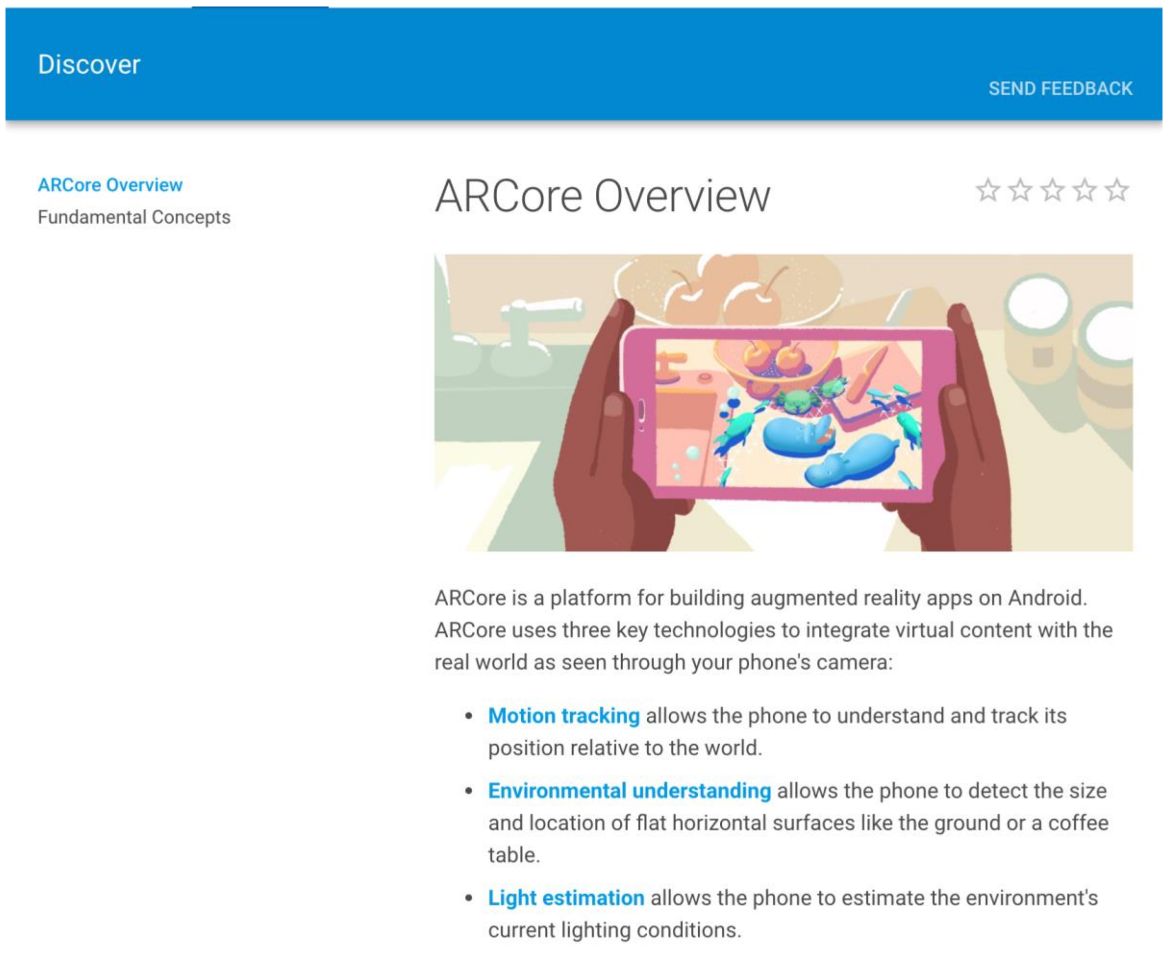
Application: augmented reality

http://gl.ict.usc.edu/LightStages/

Application: augmented reality



[https://developer.apple.com/arkit/]



[https://developers.google.com/ar/]

Lighting models: far-field approximation

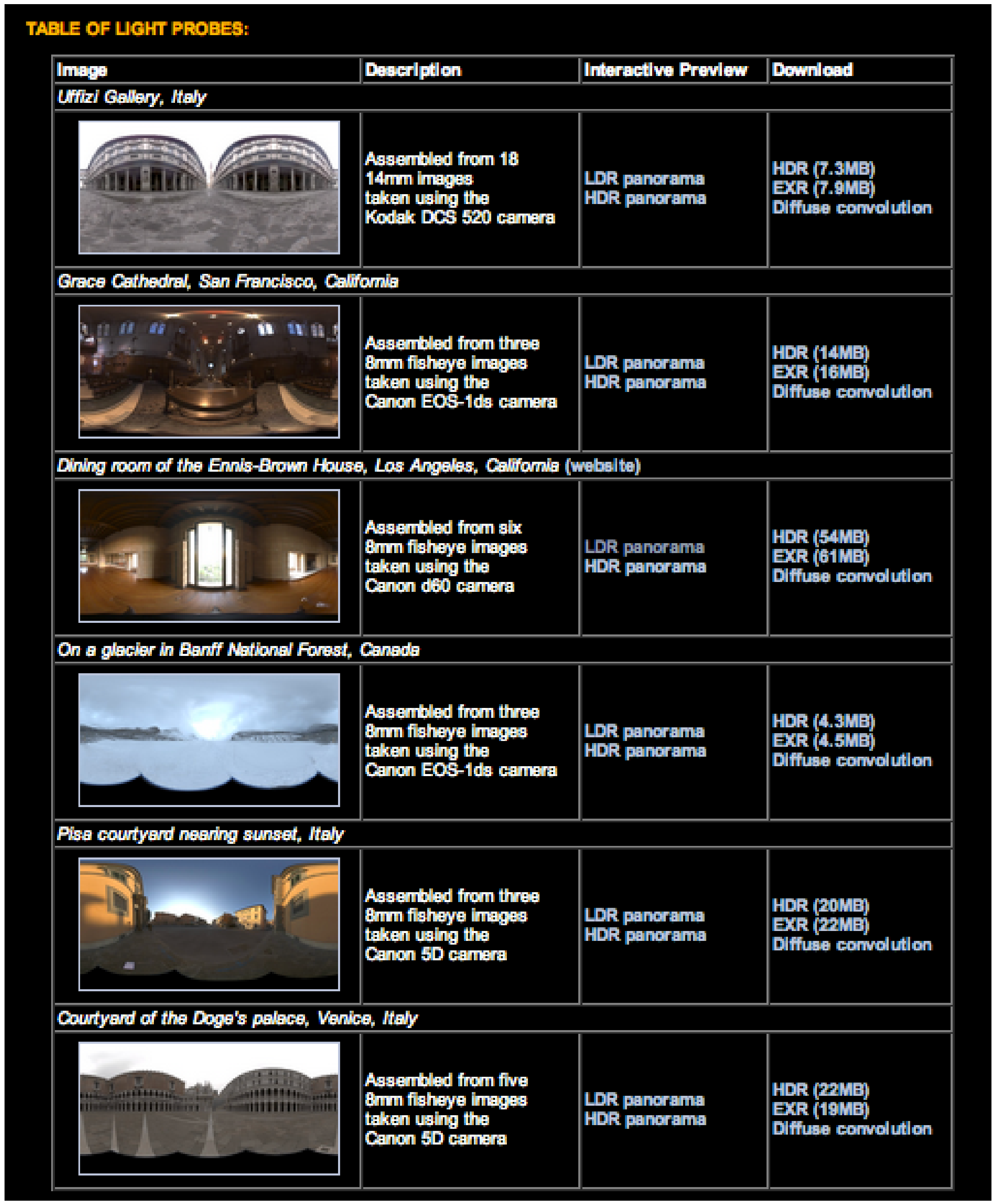


TABLE OF LIGHT PROBES:

Image	Description	Interactive Preview	Download
Uffizi Gallery, Italy			
	Assembled from 18 14mm images taken using the Kodak DCS 520 camera	LDR panorama HDR panorama	HDR (7.3MB) EXR (7.9MB) Diffuse convolution
Grace Cathedral, San Francisco, California			
	Assembled from three 8mm fisheye images taken using the Canon EOS-1ds camera	LDR panorama HDR panorama	HDR (14MB) EXR (16MB) Diffuse convolution
Dining room of the Ennis-Brown House, Los Angeles, California (website)			
	Assembled from six 8mm fisheye images taken using the Canon d60 camera	LDR panorama HDR panorama	HDR (54MB) EXR (61MB) Diffuse convolution
On a glacier in Banff National Forest, Canada			
	Assembled from three 8mm fisheye images taken using the Canon EOS-1ds camera	LDR panorama HDR panorama	HDR (4.3MB) EXR (4.5MB) Diffuse convolution
Pisa courtyard nearing sunset, Italy			
	Assembled from three 8mm fisheye images taken using the Canon 5D camera	LDR panorama HDR panorama	HDR (20MB) EXR (22MB) Diffuse convolution
Courtyard of the Doge's palace, Venice, Italy			
	Assembled from five 8mm fisheye images taken using the Canon 5D camera	LDR panorama HDR panorama	HDR (22MB) EXR (19MB) Diffuse convolution

- One can download far-field lighting environments that have been captured by others

  [http://gl.ict.usc.edu/Data/HighResProbes/]

- A number of apps and software exist to help you capture capture your own environments using a light probe

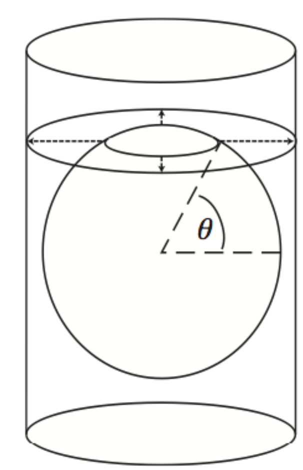


Figure 6. To produce the equal-area cylindrical projection of a spherical map, one projects each point on the surface of the sphere horizontally outward onto the cylinder, and then unwraps the cylinder to obtain a rectangular "panoramic" map.

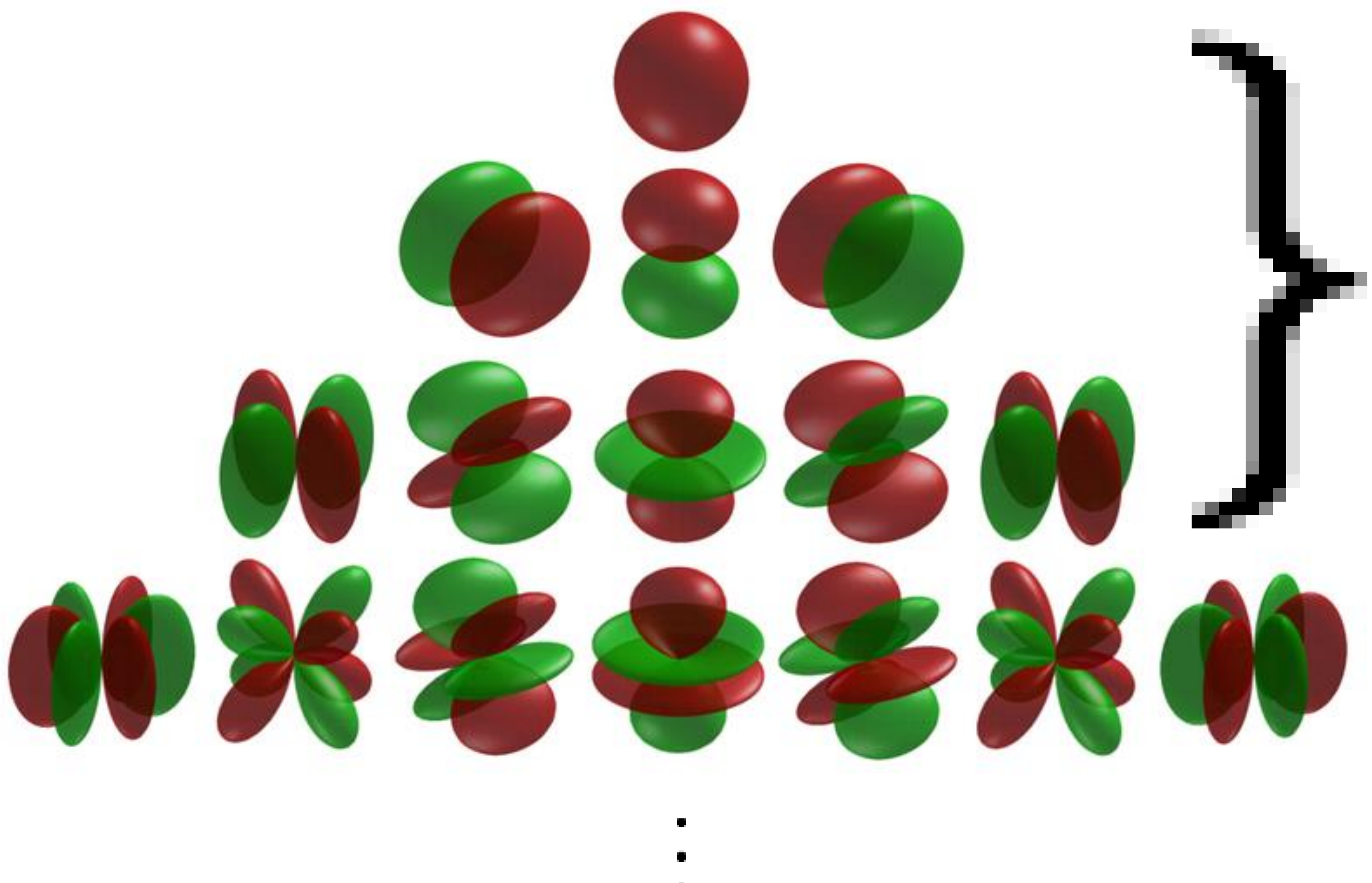
Application: inferring outdoor illumination

From a single image (left), we estimate the most likely sky appearance (middle) and insert a 3-D object (right). Illumination estimation was done entirely automatically.

[Lalonde et al., 2009]

A further simplification: Low-frequency illumination

$$L(\omega) = \sum_i a_i Y_i(\omega)$$



First nine basis functions are sufficient for re-creating Lambertian appearance

[Ramamoorthi and Hanrahan, 2001; Basri and Jacobs, 2003]

Low-frequency illumination

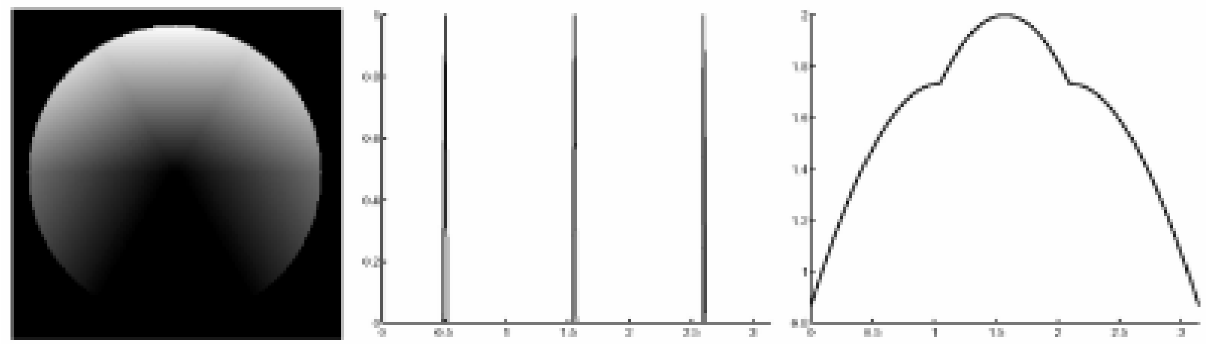


Fig. 2. On the left, a white sphere illuminated by three directional (distant point) sources of light. All the lights are parallel to the image plane, one source illuminates the sphere from above and the two others illuminate the sphere from diagonal directions. In the middle, a cross-section of the lighting function with three peaks corresponding to the three light sources. On the right, a cross-section indicating how the sphere reflects light. We will make precise the intuition that the material acts as a low-pass filtering, smoothing the light as it reflects it.

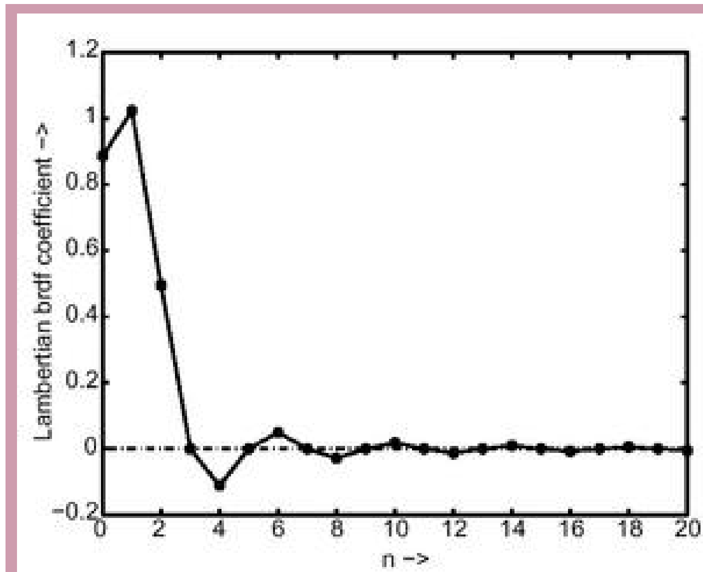


Figure 3. Plot of spherical harmonic terms in Lambertian BRDF filter.

[Ramamoorthi and Hanrahan, 2001; Basri and Jacobs, 2003]

Low-frequency illumination

$$L(\omega) = \sum_i a_i Y_i(\omega)$$

Truncate to first 9 terms

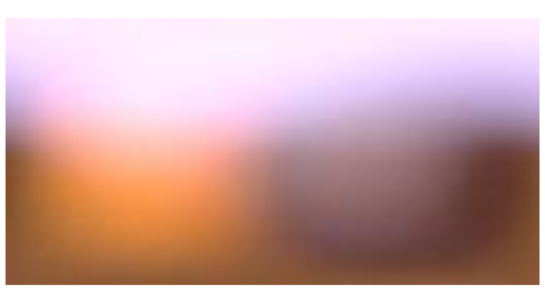
$$\vec{\ell} = (\ell_1, \ldots, \ell_9)$$

[Ramamoorthi and Hanrahan, 2001; Basri and Jacobs, 2003]

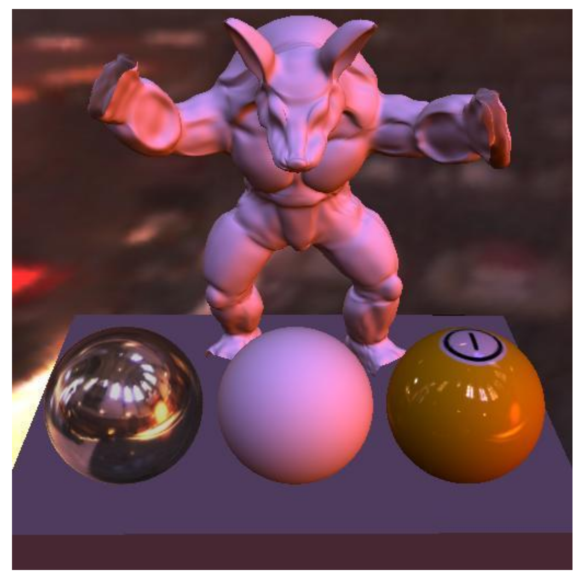
Application: Trivial rendering

Capture light probe

Low-pass filter (truncate to first nine SHs)



Rendering a (convex) diffuse object in this environment simply requires a lookup based on the surface normal at each pixel



White-out: Snow and Overcast Skies

CAN’T perceive the shape of the snow covered terrain!

CAN perceive shape in regions lit by the street lamp!!

WHY?

Diffuse Reflection from Uniform Sky

$$L^{surface}(\theta_r, \phi_r) = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} L^{src}(\theta_i, \phi_i) \, f(\theta_i, \phi_i; \theta_r, \phi_r) \cos\theta_i \, \sin\theta_i \, d\theta_i \, d\phi_i$$

- Assume Lambertian Surface with Albedo = 1 (no absorption)

$$f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{1}{\pi}$$

- Assume Sky radiance is constant

$$L^{src}(\theta_i, \phi_i) = L^{sky}$$

- Substituting in above Equation:

$$\boxed{L^{surface}(\theta_r, \phi_r) = L^{sky}}$$

Radiance of any patch is the same as Sky radiance !! (white-out condition)

Even simpler:
Directional lighting

- Assume that, over the observed region of interest, all source of incoming flux is from one direction

$$L(x, \omega, t, \lambda) \longrightarrow L(x, t, \lambda) \longrightarrow s(t, \lambda)\delta(\omega = \omega_o(t))$$

$$L(x, \omega) \longrightarrow L(\omega) \longrightarrow s\delta(\omega = \omega_o)$$

- Convenient representation

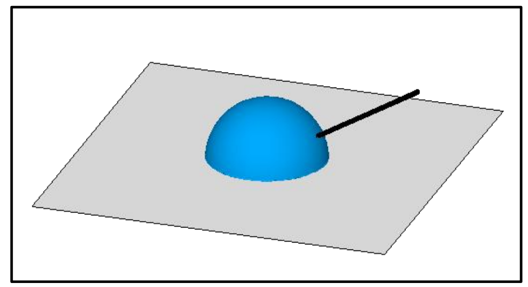
$$\vec{\boldsymbol{\ell}} = (\ell_x, \ell_y, \ell_z)$$

"light direction" $\hat{\boldsymbol{\ell}} = \dfrac{\vec{\boldsymbol{\ell}}}{||\vec{\boldsymbol{\ell}}||}$

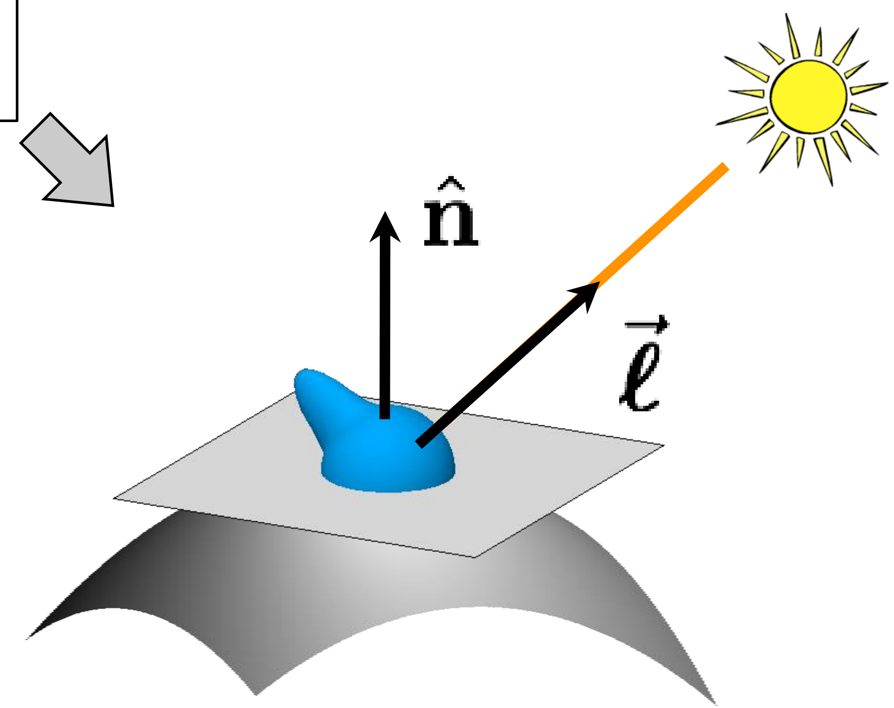
"light strength" $||\vec{\boldsymbol{\ell}}||$

Simple shading

ASSUMPTION 1:
LAMBERTIAN

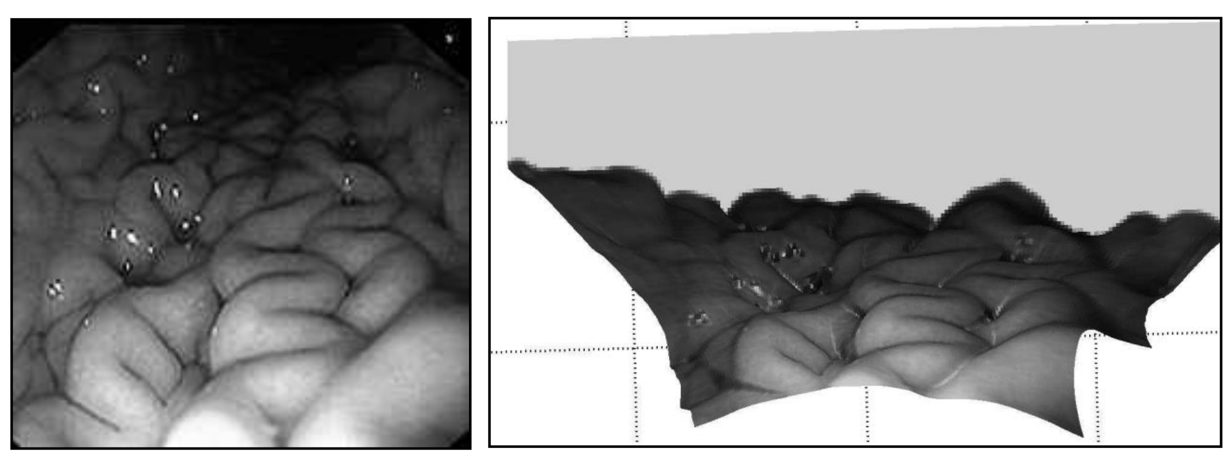


ASSUMPTION 2:
DIRECTIONAL LIGHTING



$$L^{\text{out}}(\hat{\boldsymbol{\omega}}) = \int_{\Omega_{\text{in}}} f(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) L^{\text{in}}(\hat{\boldsymbol{\omega}}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\boldsymbol{\omega}}_{\text{in}}$$

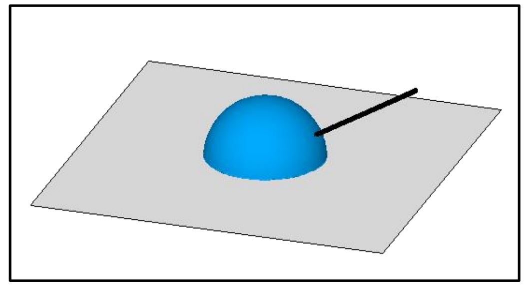
$$I = a\hat{\mathbf{n}}^\top \vec{\boldsymbol{\ell}}$$



[Prados, 2004]

“N-dot-l” shading

ASSUMPTION 1:
LAMBERTIAN



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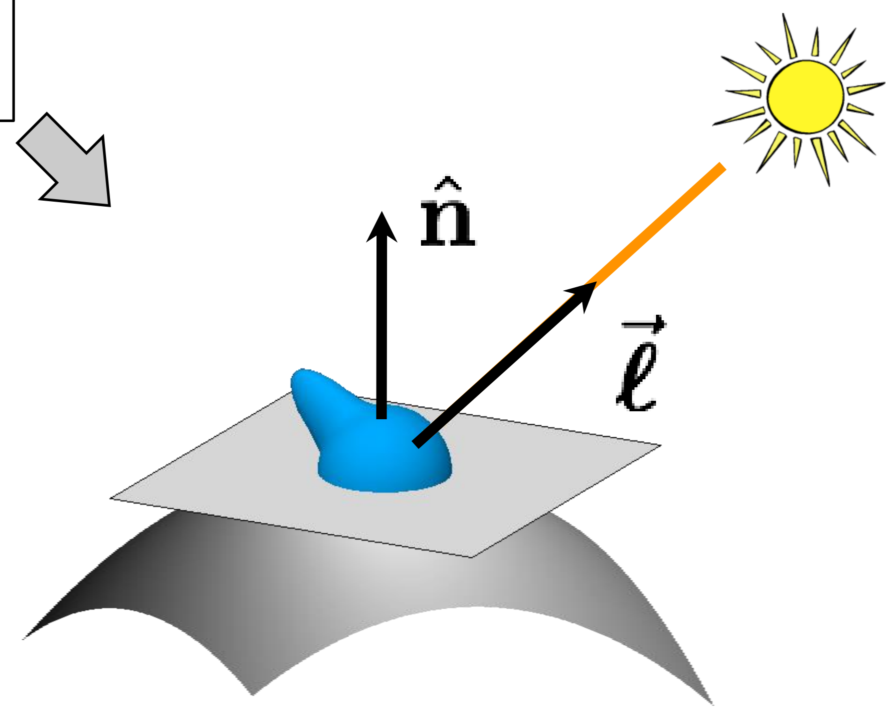

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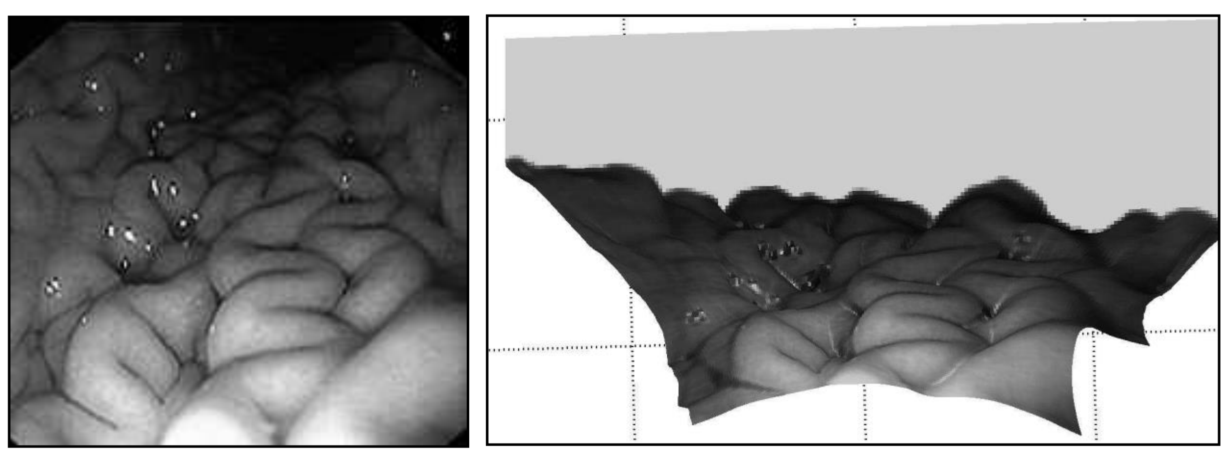
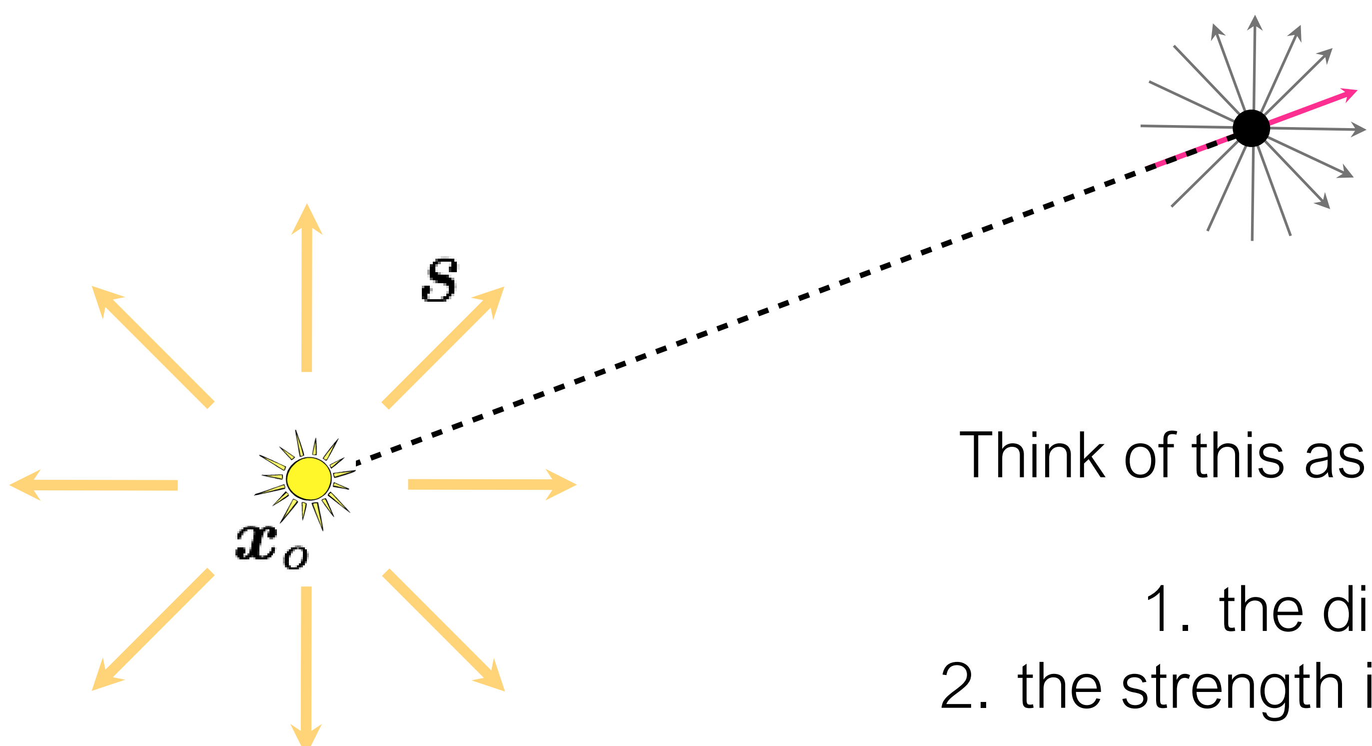


[Prados, 2004]

An ideal point light source

$$L(\boldsymbol{x}, \boldsymbol{\omega}) = \frac{s}{||\boldsymbol{x} - \boldsymbol{x}_o||^2} \ \delta\left(\boldsymbol{\omega} = \frac{\boldsymbol{x} - \boldsymbol{x}_o}{||\boldsymbol{x} - \boldsymbol{x}_o||}\right)$$



Think of this as a spatially-varying directional source where

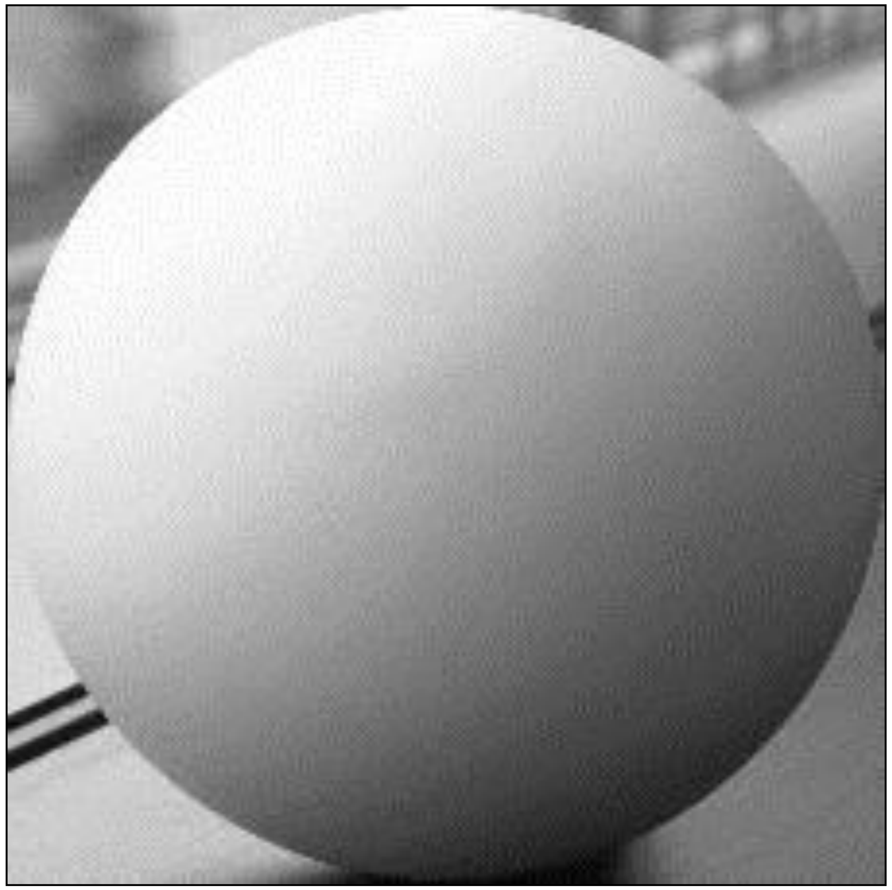
1. the direction is away from x_o
2. the strength is proportional to 1/(distance)^2

Summary of some useful lighting models

- plenoptic function (function on 5D domain)
- far-field illumination (function on 2D domain)
- low-frequency far-field illumination (nine numbers)
- directional lighting (three numbers = direction and strength)
- point source (four numbers = location and strength)

Shape from shading

Image Intensity and 3D Geometry



- Shading as a cue for shape reconstruction
- What is the relation between intensity and shape?
  - Reflectance Map

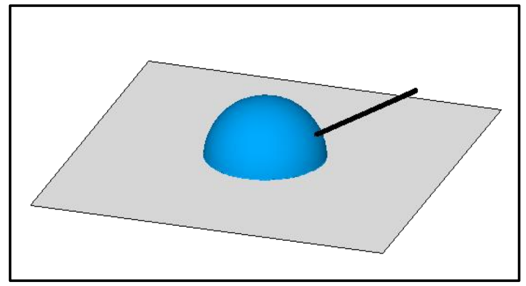
Application: Detecting composite photos

Real photo

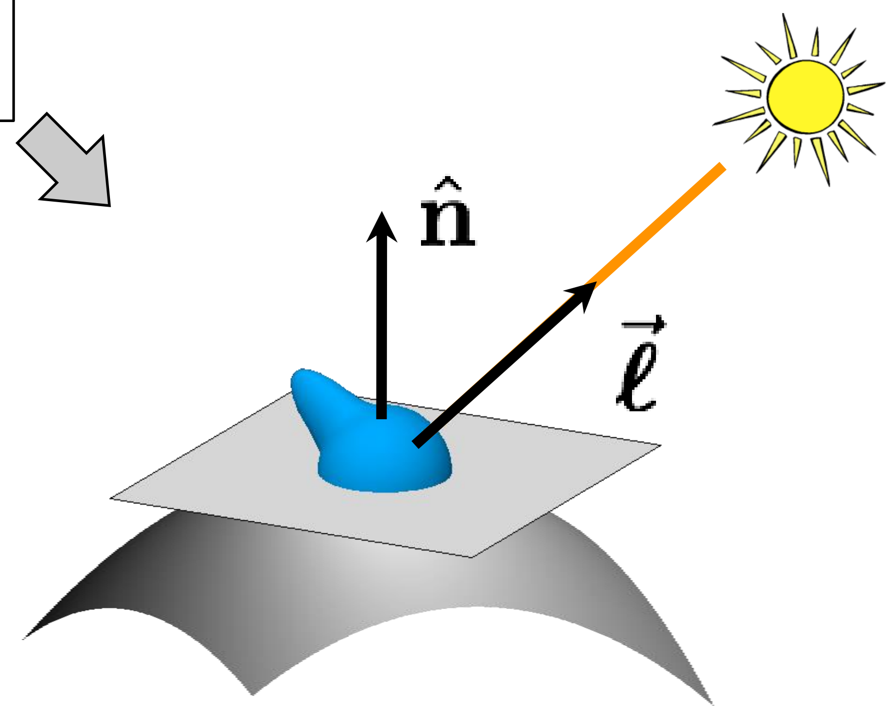
Fake photo

“N-dot-l” shading

ASSUMPTION 1:
LAMBERTIAN



ASSUMPTION 2:
DIRECTIONAL LIGHTING



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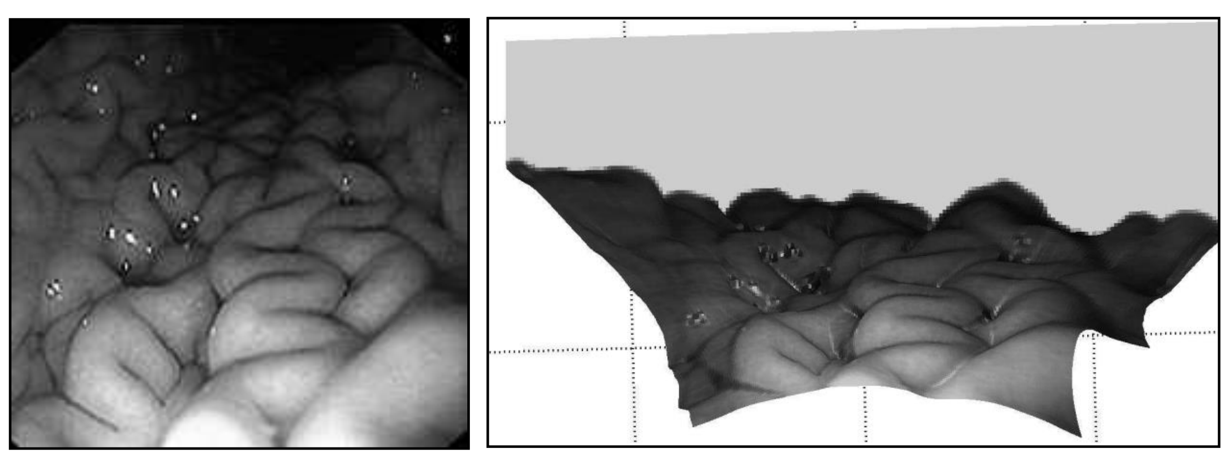
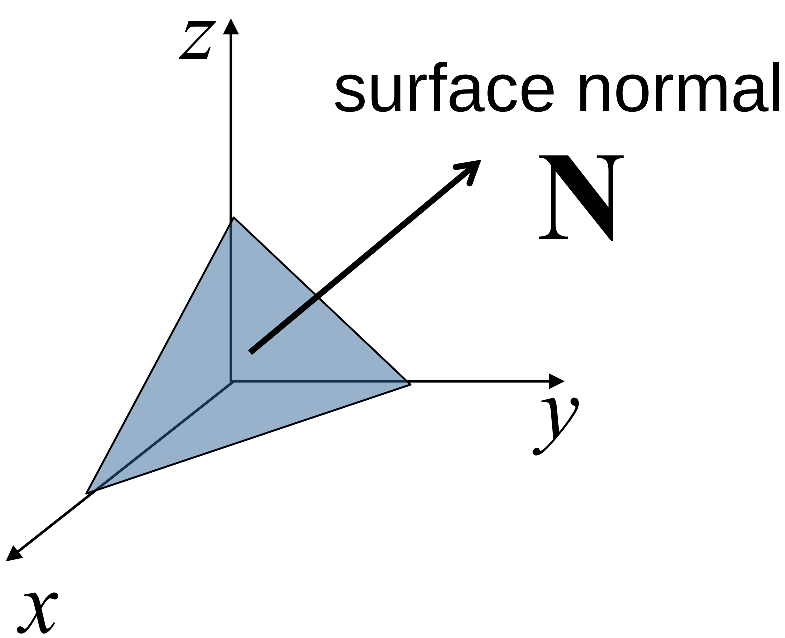


[Prados, 2004]

Surface Normal



Equation of plane $Ax + By + Cz + D = 0$

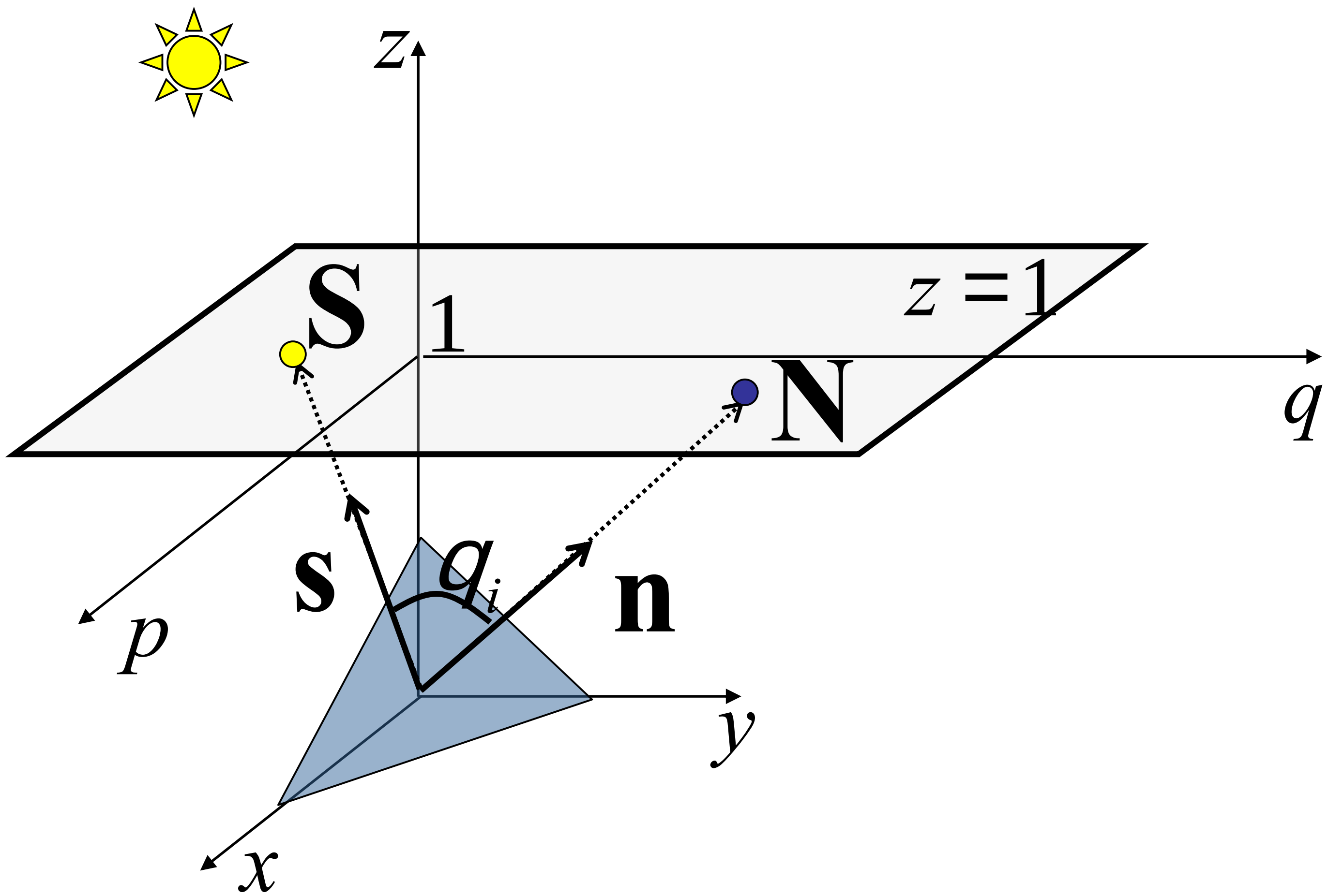
or $\frac{A}{C}x + \frac{B}{C}y + z + \frac{D}{C} = 0$

Let $-\frac{\partial z}{\partial x} = \frac{A}{C} = p \qquad -\frac{\partial z}{\partial y} = \frac{B}{C} = q$

Surface normal

$$\mathbf{N} = \left(\frac{A}{C}, \frac{B}{C}, 1\right) = (p, q, 1)$$

Gradient Space



Normal vector

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

Source vector

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_S, q_S, 1)}{\sqrt{p_S^2 + q_S^2 + 1}}$$

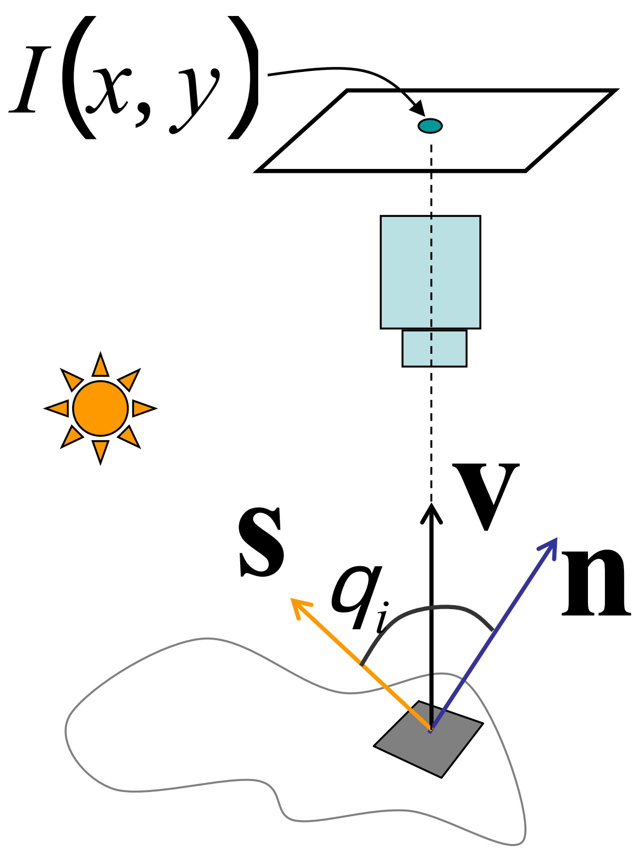
$$\cos q_i = \mathbf{n} \times \mathbf{s} = \frac{(pp_S + qq_S + 1)}{\sqrt{p^2 + q^2 + 1}\sqrt{p_S^2 + q_S^2 + 1}}$$

$z = 1$ plane is called the Gradient Space (*pq* plane)

- Every point on it corresponds to a particular surface orientation

Reflectance Map

- Relates image irradiance *I(x,y)* to surface orientation *(p,q)* for given source direction and surface reflectance
- Lambertian case:



k : source brightness

r : surface albedo (reflectance)

c : constant (optical system)

Image irradiance:

$$I = \frac{r}{p} kc \cos q_i = \frac{r}{p} kc\,\mathbf{n} \times \mathbf{s}$$

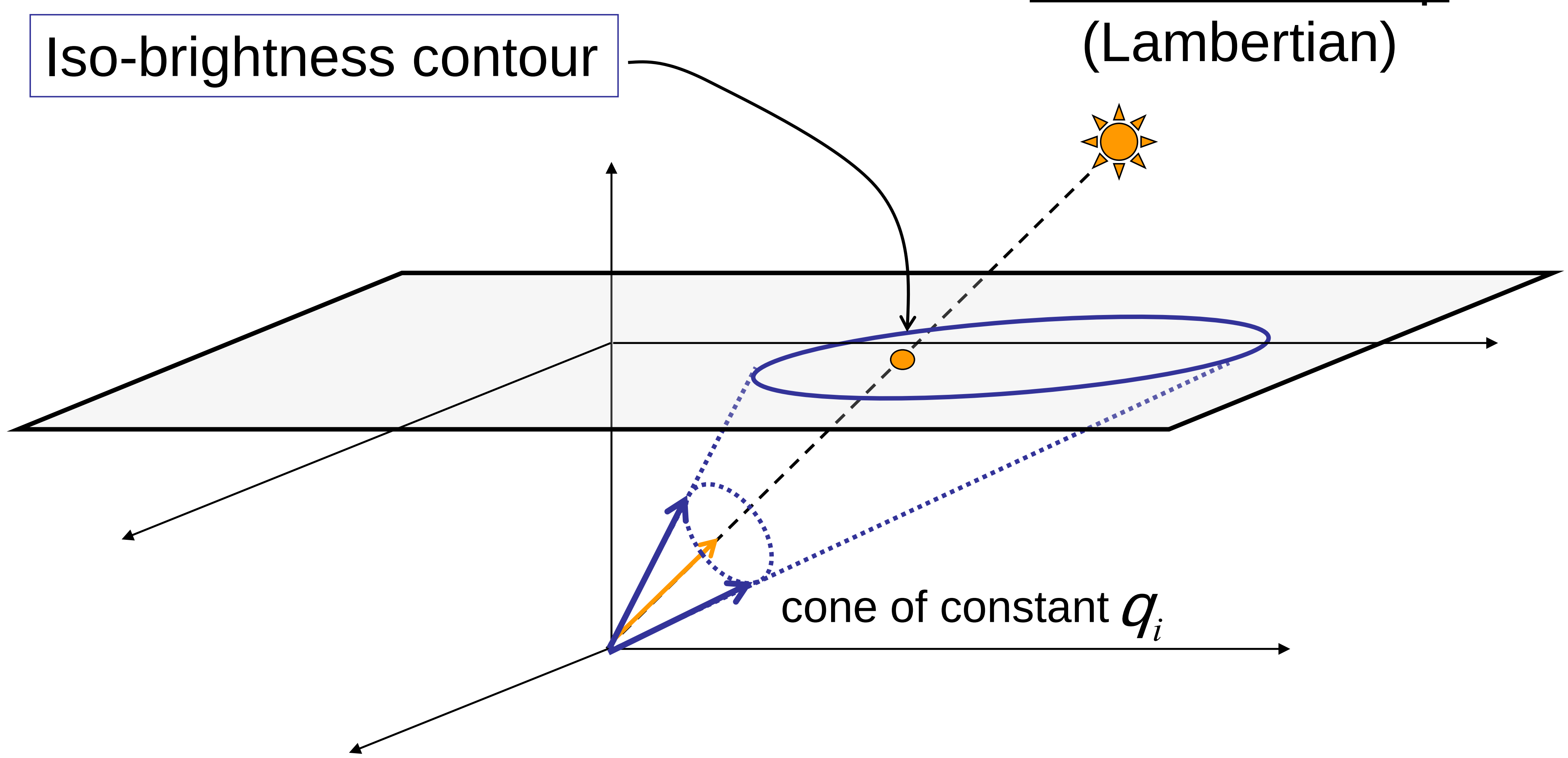
Let $\frac{r}{p} kc = 1$ then $I = \cos q_i = \mathbf{n} \times \mathbf{s}$

Reflectance Map

- Lambertian case

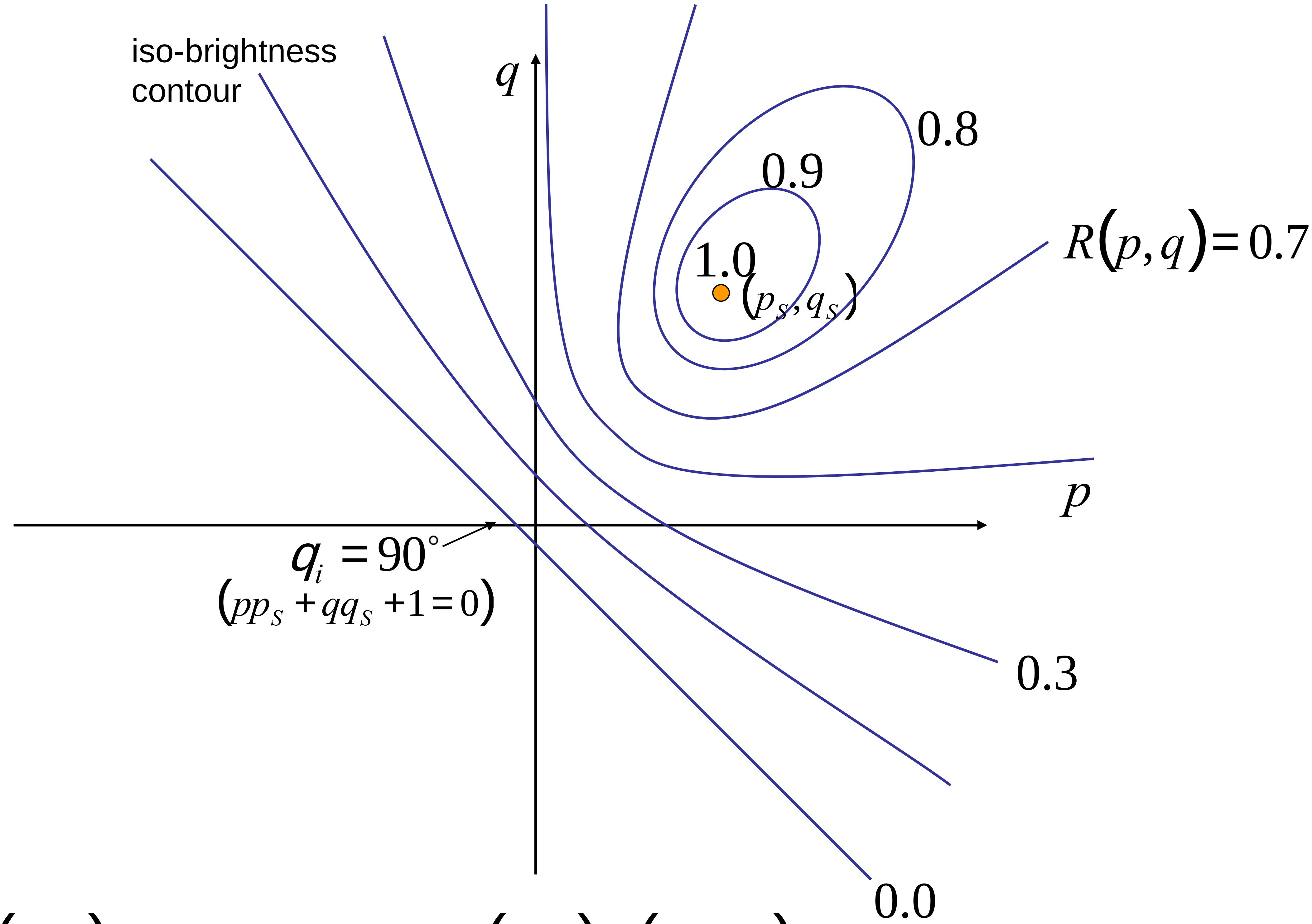
$$I = \cos q_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2 + q^2 + 1}\sqrt{p_S^2 + q_S^2 + 1}} = R(p,q)$$

Reflectance Map
(Lambertian)



Reflectance Map

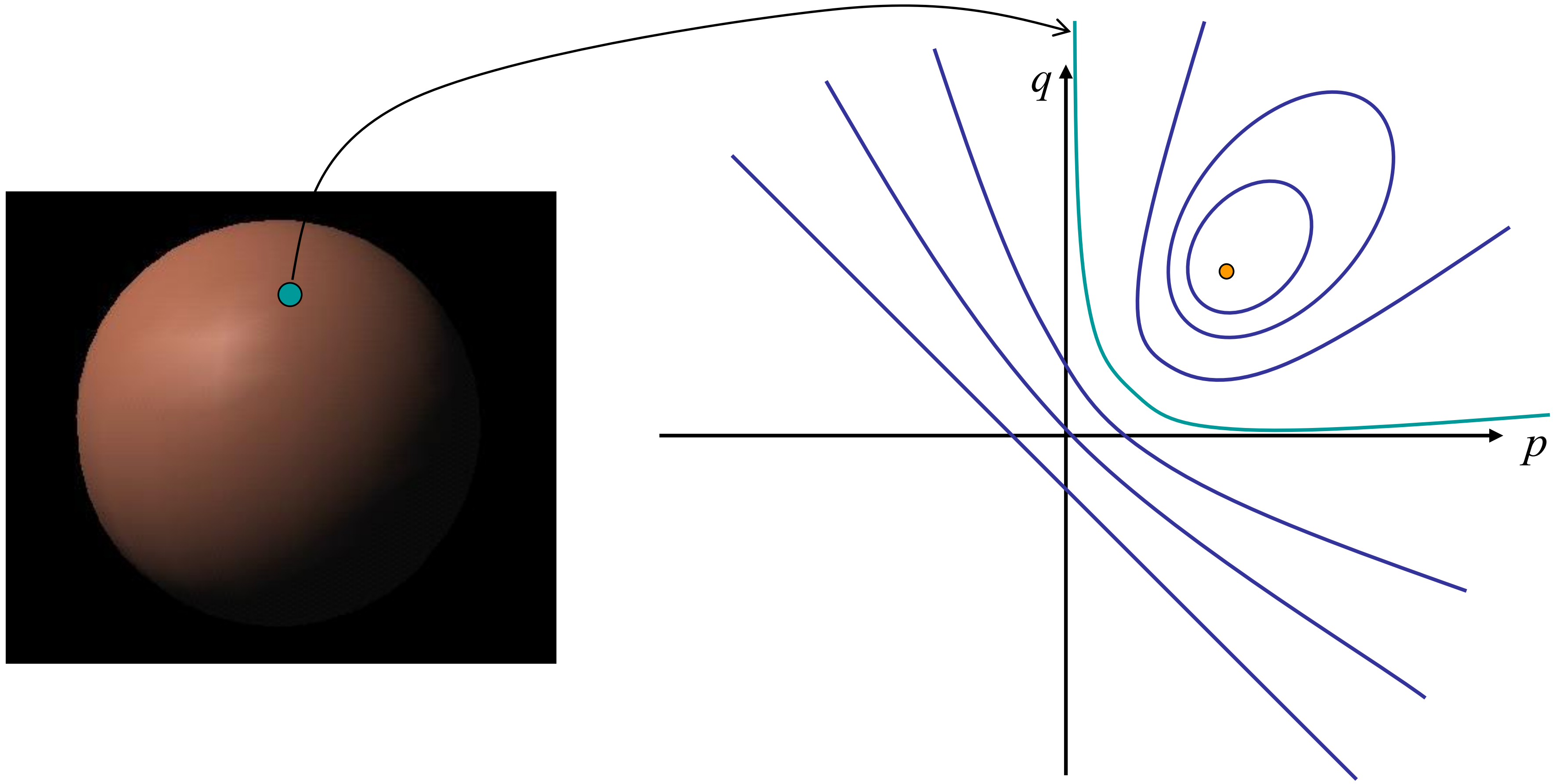
- Lambertian case



Note: $R(p,q)$ is maximum when $(p,q)=(p_S,q_S)$

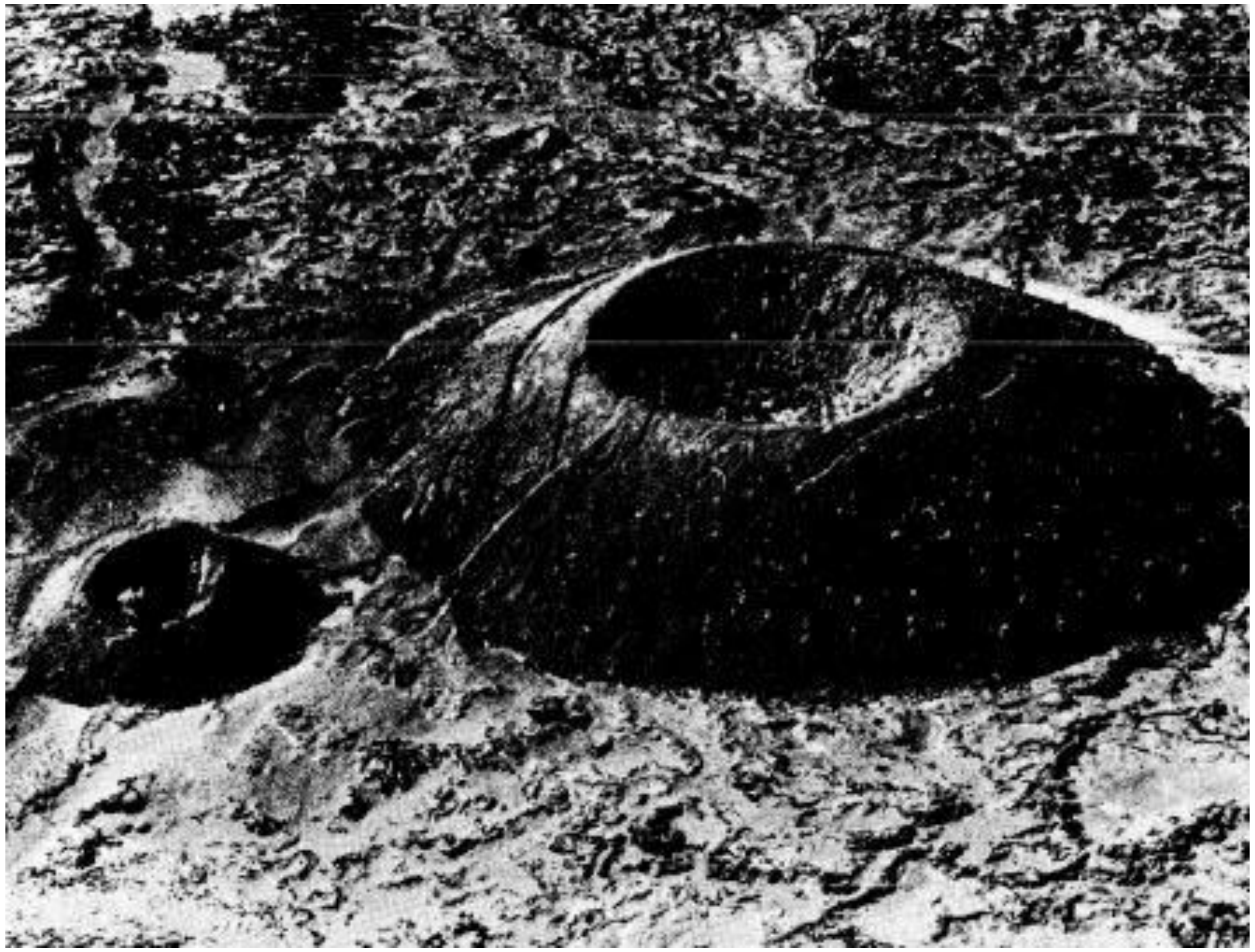
Shape from a Single Image?

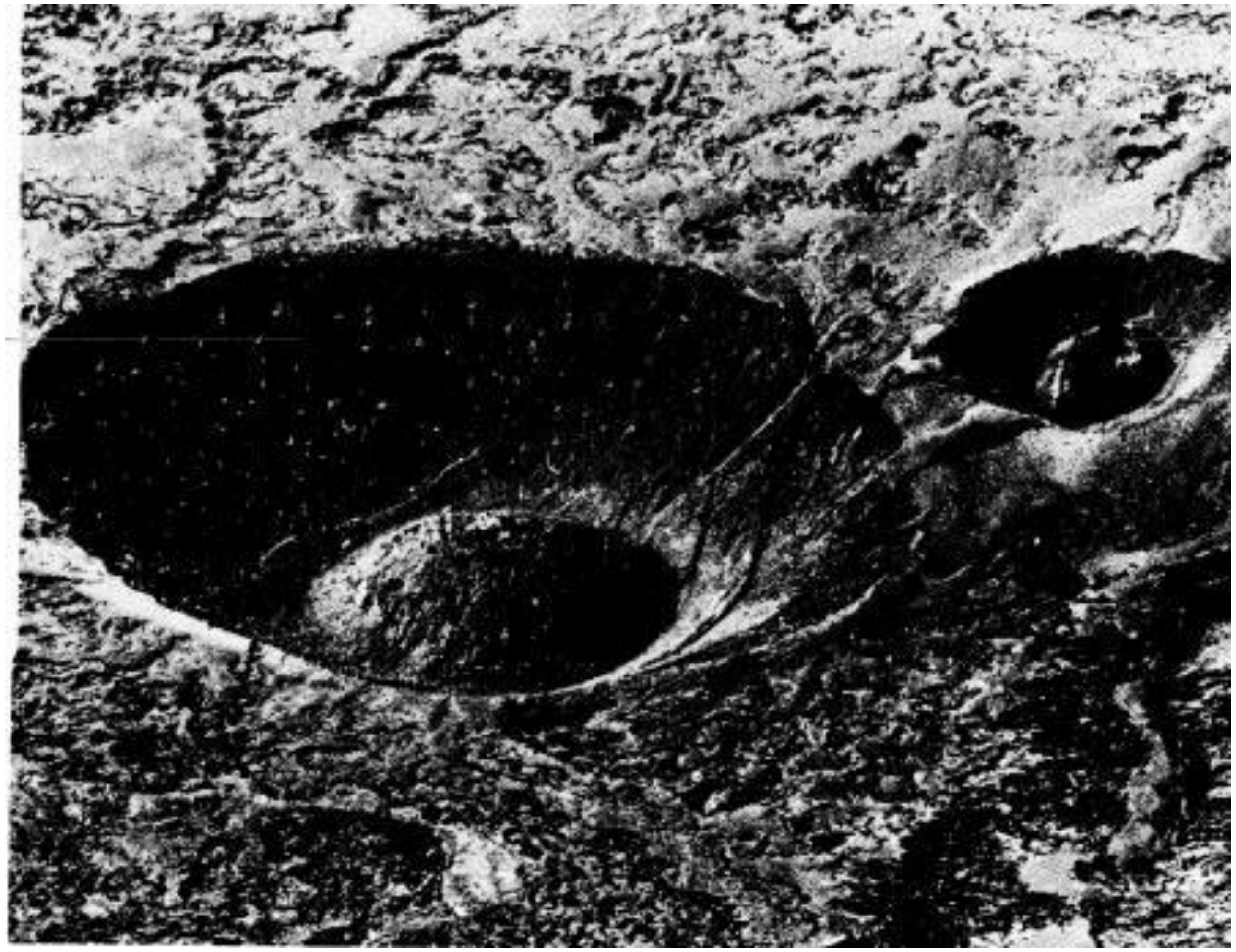
- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?
- Given $R(p,q)$ ((p_S,q_S) and surface reflectance) can we determine (p,q) uniquely for each image point?



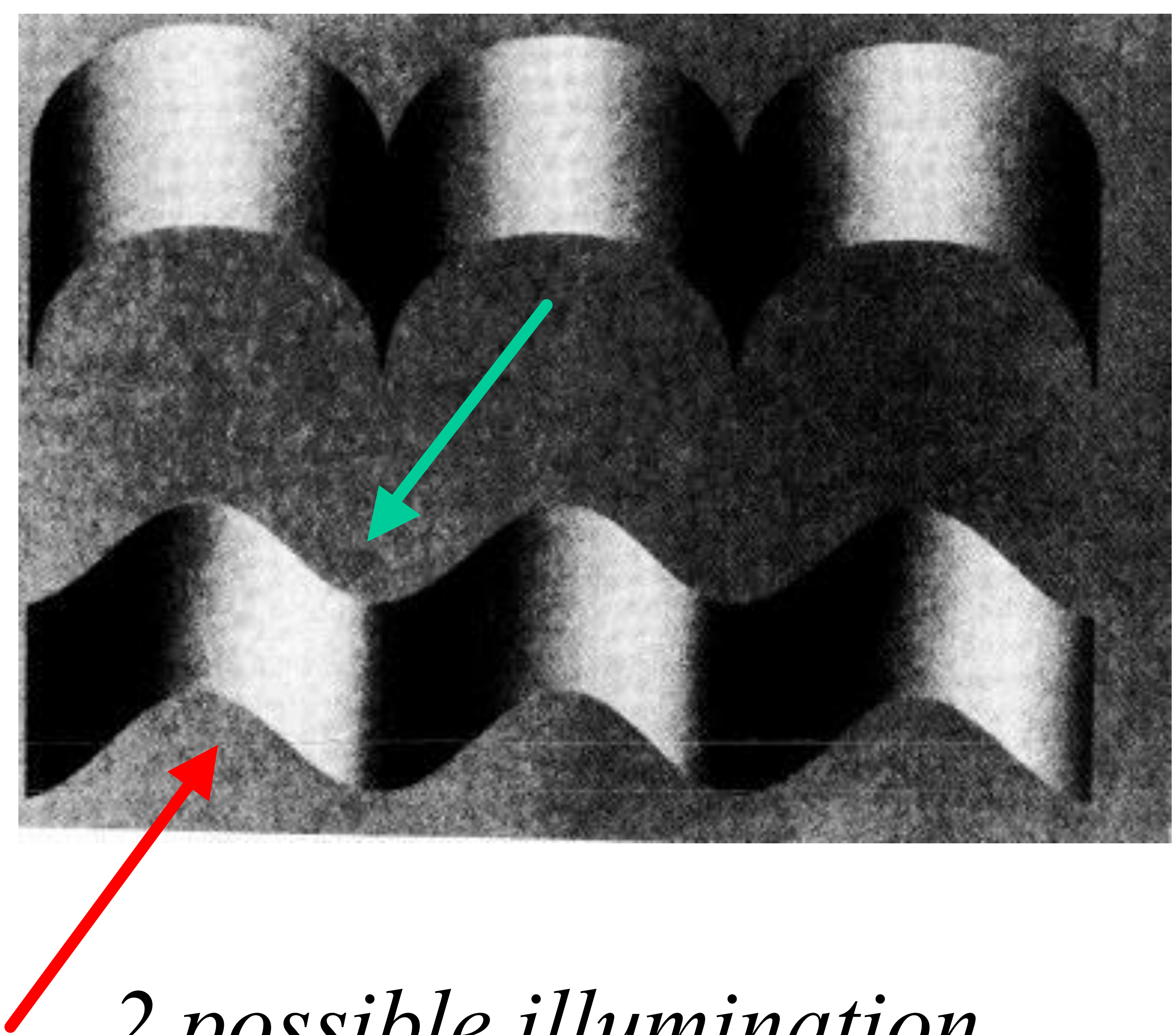
NO

Human Perception



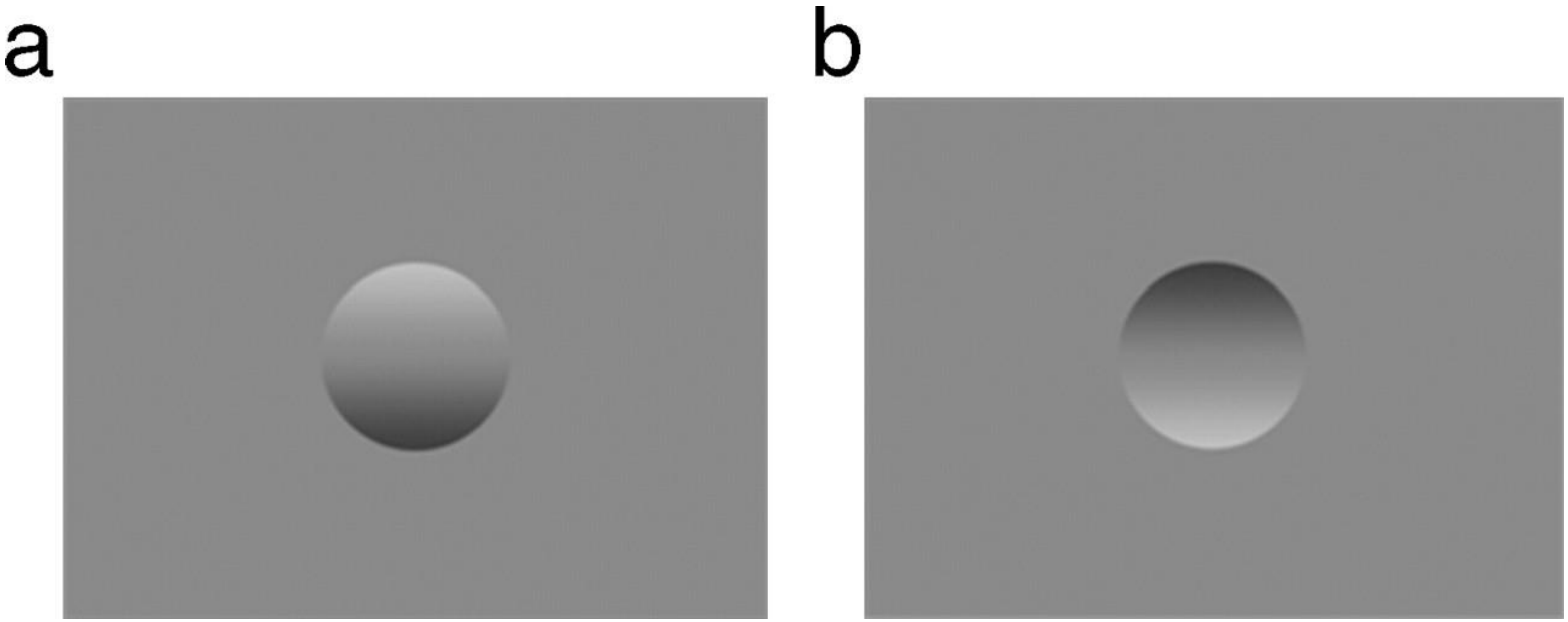


Human Perception



2 possible illumination hypotheses

Examples of the classic bump/dent stimuli used to test lighting assumptions when judging shape from shading, with shading orientations (a) 0° and (b) 180° from the vertical.



Thomas R et al. J Vis 2010;10:6

Journal of VISION

Human Perception

- Our brain often perceives shape from shading.
- Mostly, it makes many assumptions to do so.
- For example:

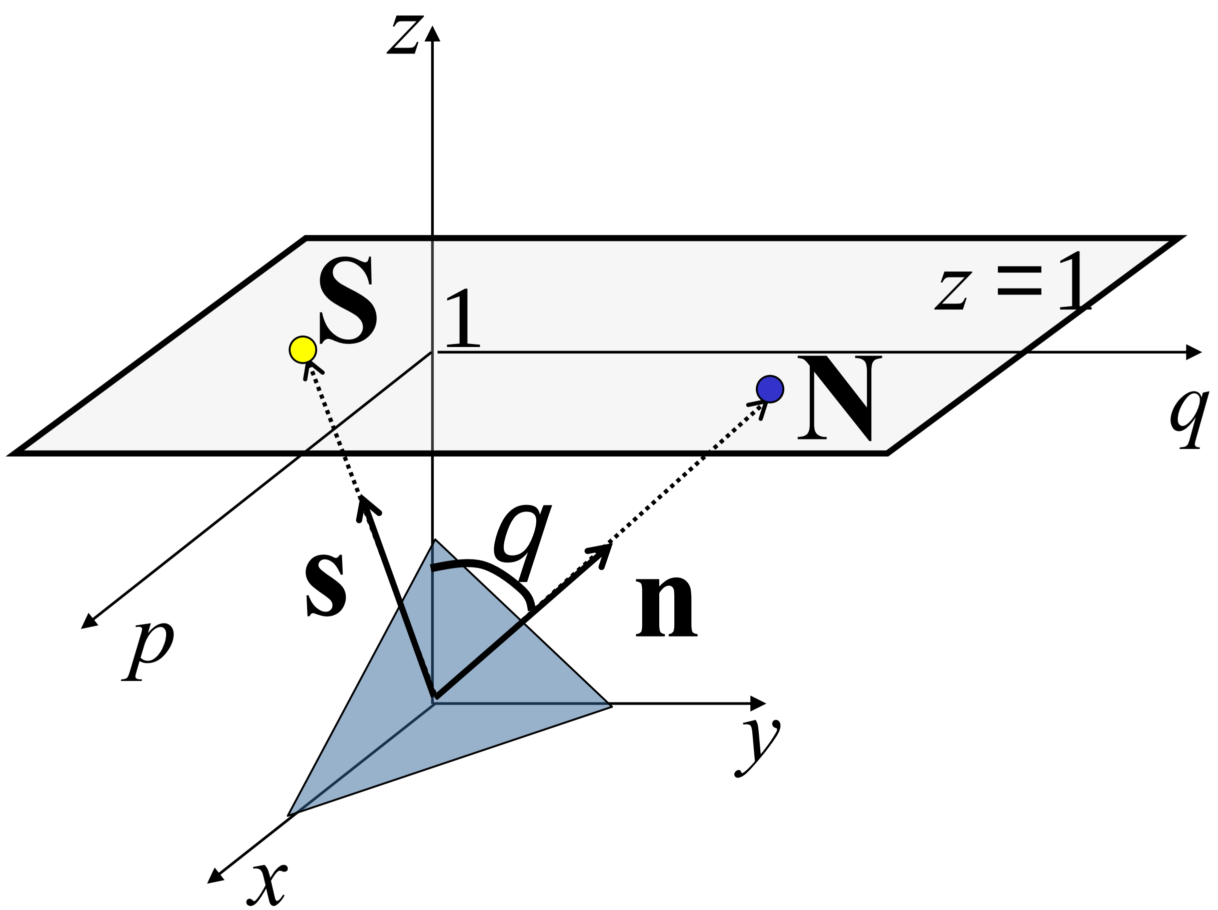
  Light is coming from above (sun).

  Biased by occluding contours.

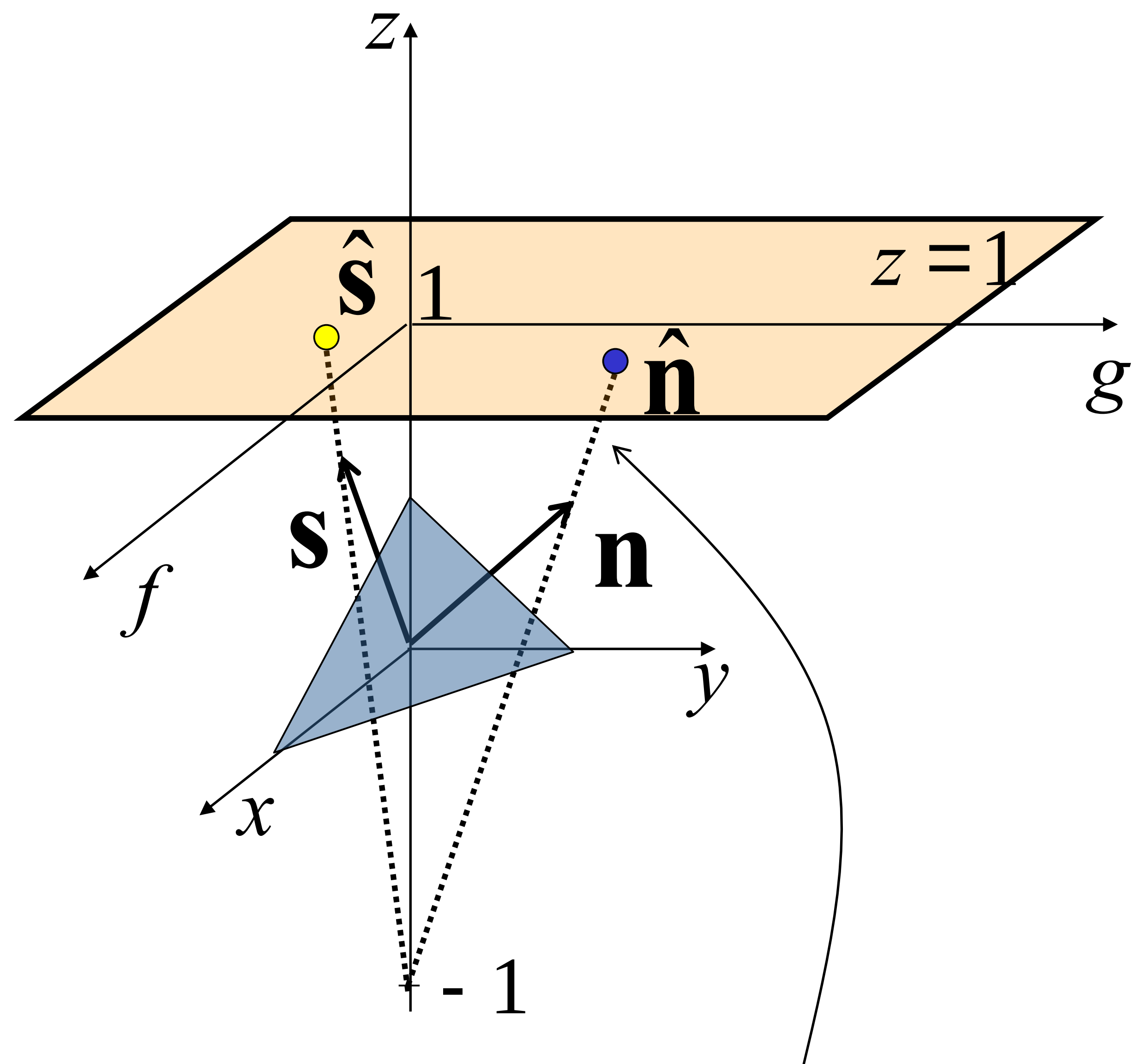
by V. Ramachandran

Stereographic Projection

(p,q)-space (gradient space)



(f,g)-space



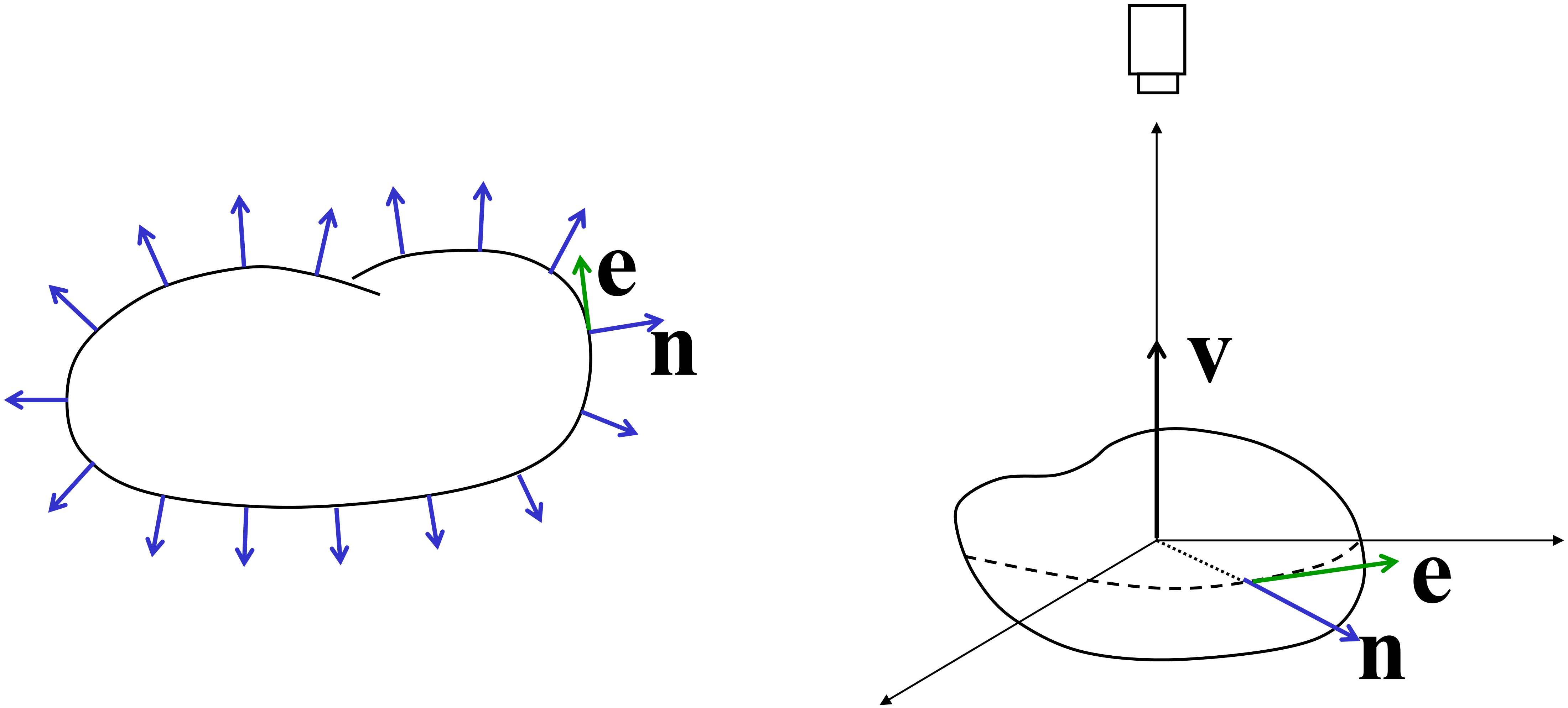
$$f = \frac{2p}{1+\sqrt{1+p^2+q^2}} \qquad g = \frac{2q}{1+\sqrt{1+p^2+q^2}}$$

Problem

(p,q) can be infinite when $q = 90^\circ$

Redefine reflectance map as $R(f,g)$

Occluding Boundaries



n ^ **e**, **n** ^ **v** \ **n** = **e** ´ **v** **e** and **v** are known

The **n** values on the occluding boundary can be used as the boundary condition for shape-from-shading

Image Irradiance Constraint

- Image irradiance should match the reflectance map

Minimize

$$e_i = \iint_{image} \left(I(x,y) - R(f,g)\right)^2 dx\,dy$$

(minimize errors in image irradiance in the image)

Smoothness Constraint

- Used to constrain shape-from-shading
- Relates orientations *(f,g)* of neighboring surface points

Minimize

$$e_s = \iint_{\text{image}} \left(f_x^2 + f_y^2\right) + \left(g_x^2 + g_y^2\right) dxdy$$

(f, g): surface orientation under stereographic projection

$$f_x = \frac{\partial f}{\partial x}, f_y = \frac{\partial f}{\partial y}, g_x = \frac{\partial g}{\partial x}, g_y = \frac{\partial g}{\partial y}$$

(penalize rapid changes in surface orientation *f* and *g* over the image)

Shape-from-Shading

- Find surface orientations *(f,g)* at all image points that minimize

weight

$$e = e_s + \lambda e_i$$

smoothness constraint

image irradiance error

Minimize

$$e = \iint_{\text{image}} \left(f_x^2 + f_y^2 \right) + \left(g_x^2 + g_y^2 \right) + \lambda \left(I(x,y) - R(f,g) \right)^2 dx\,dy$$

Numerical Shape-from-Shading

- **Smoothness error** at image point *(i,j)*

$$s_{i,j} = \frac{1}{4}\left(\left(f_{i+1,j} - f_{i,j}\right)^2 + \left(f_{i,j+1} - f_{i,j}\right)^2 + \left(g_{i+1,j} - g_{i,j}\right)^2 + \left(g_{i,j+1} - g_{i,j}\right)^2\right)$$

Of course you can consider more neighbors (smoother results)

- **Image irradiance error** at image point *(i,j)*

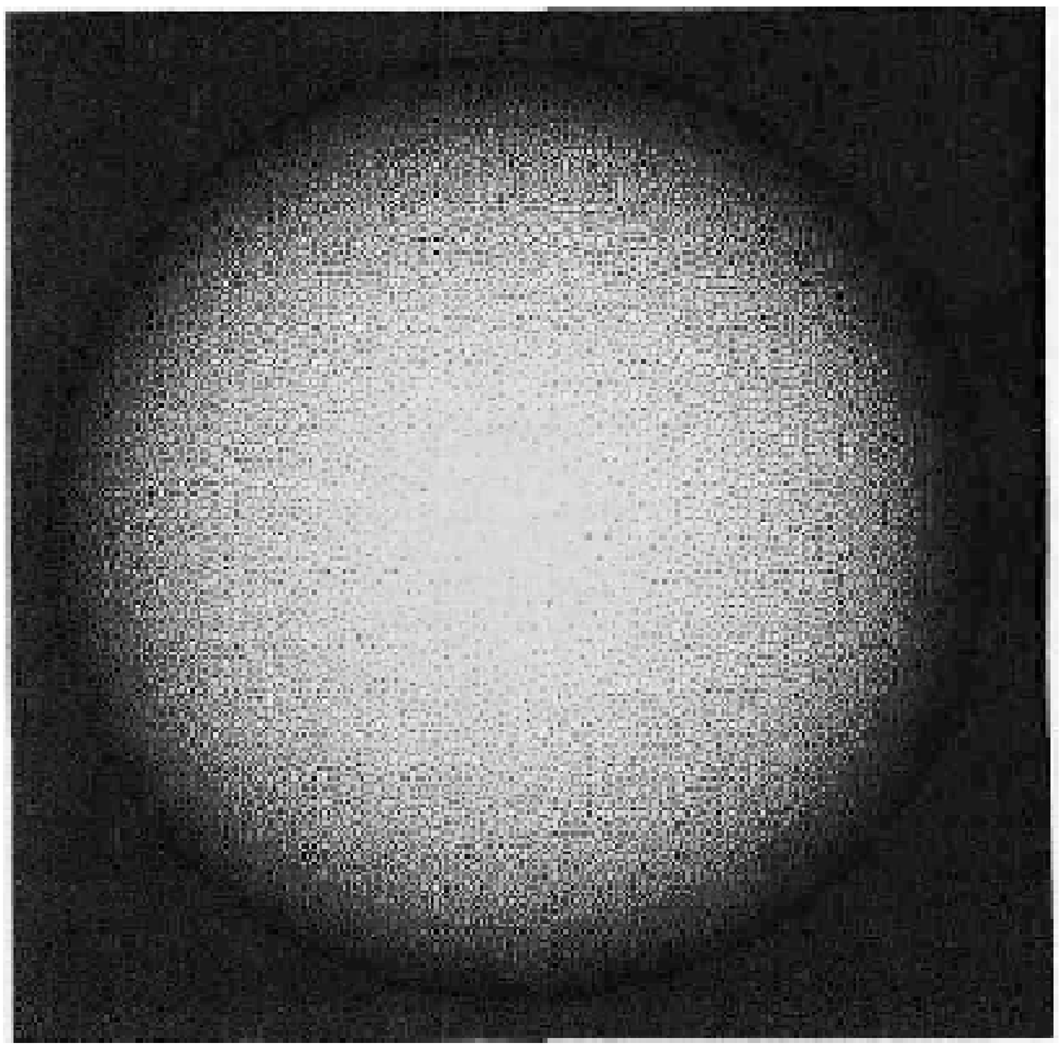
$$r_{i,j} = \left(I_{i,j} - R\left(f_{i,j}, g_{i,j}\right)\right)^2$$

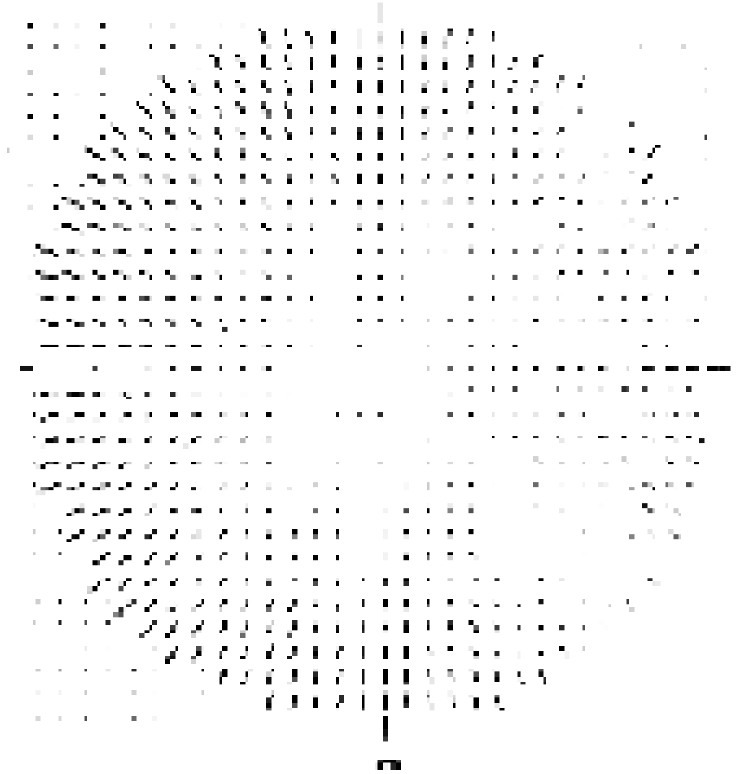
Find $\{f_{i,j}\}$ and $\{g_{i,j}\}$ that minimize

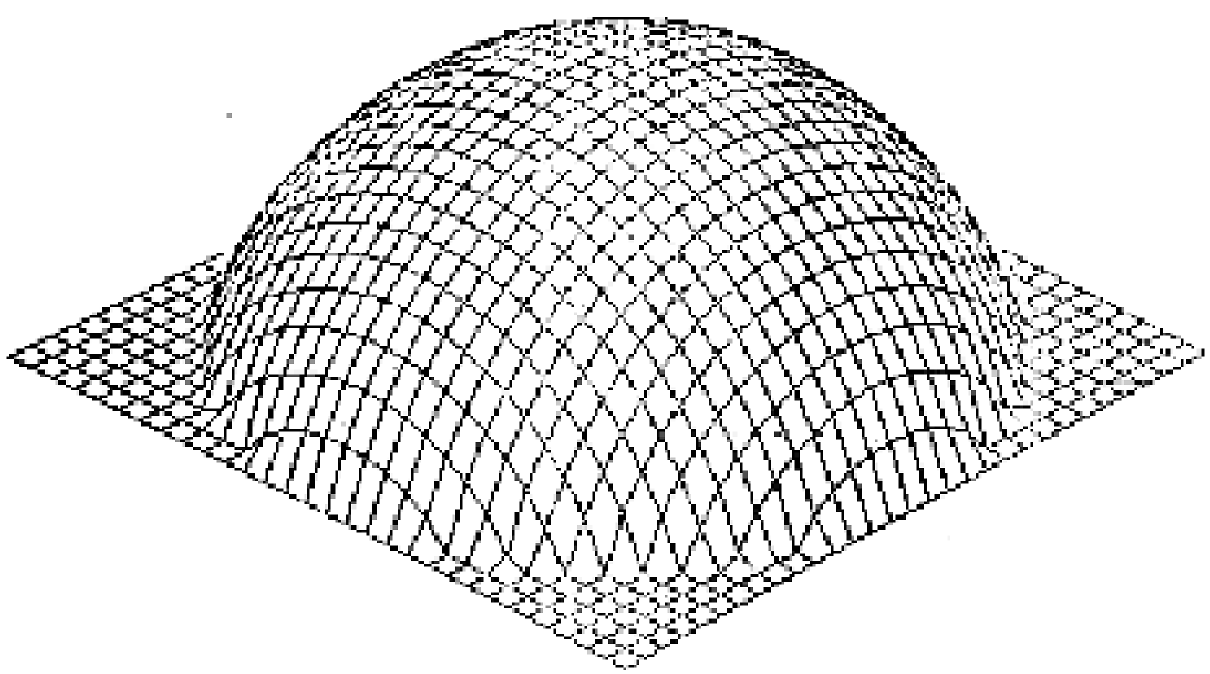
$$e = \sum_i \sum_j \left(s_{i,j} + \lambda r_{i,j}\right)$$

(Ikeuchi & Horn 89)

Results

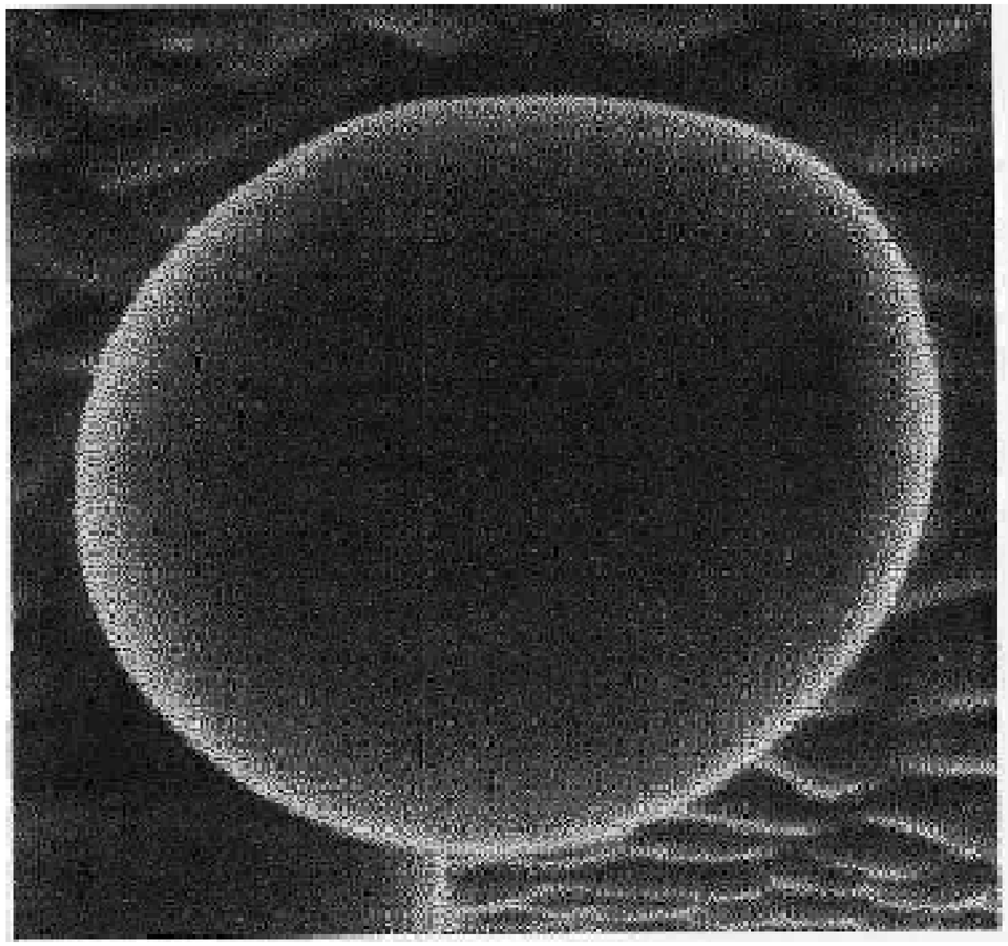




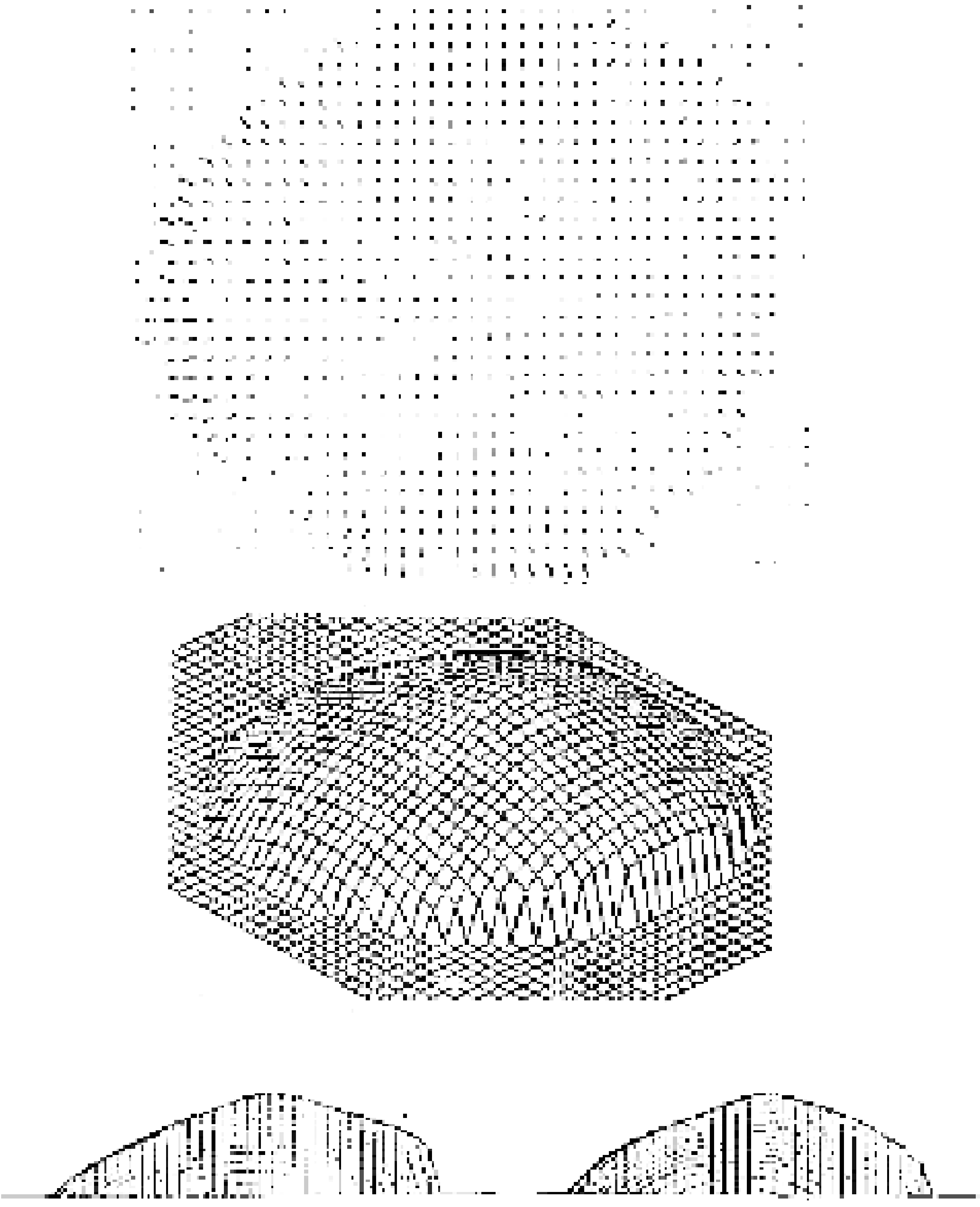


by Ikeuchi and Horn

Results

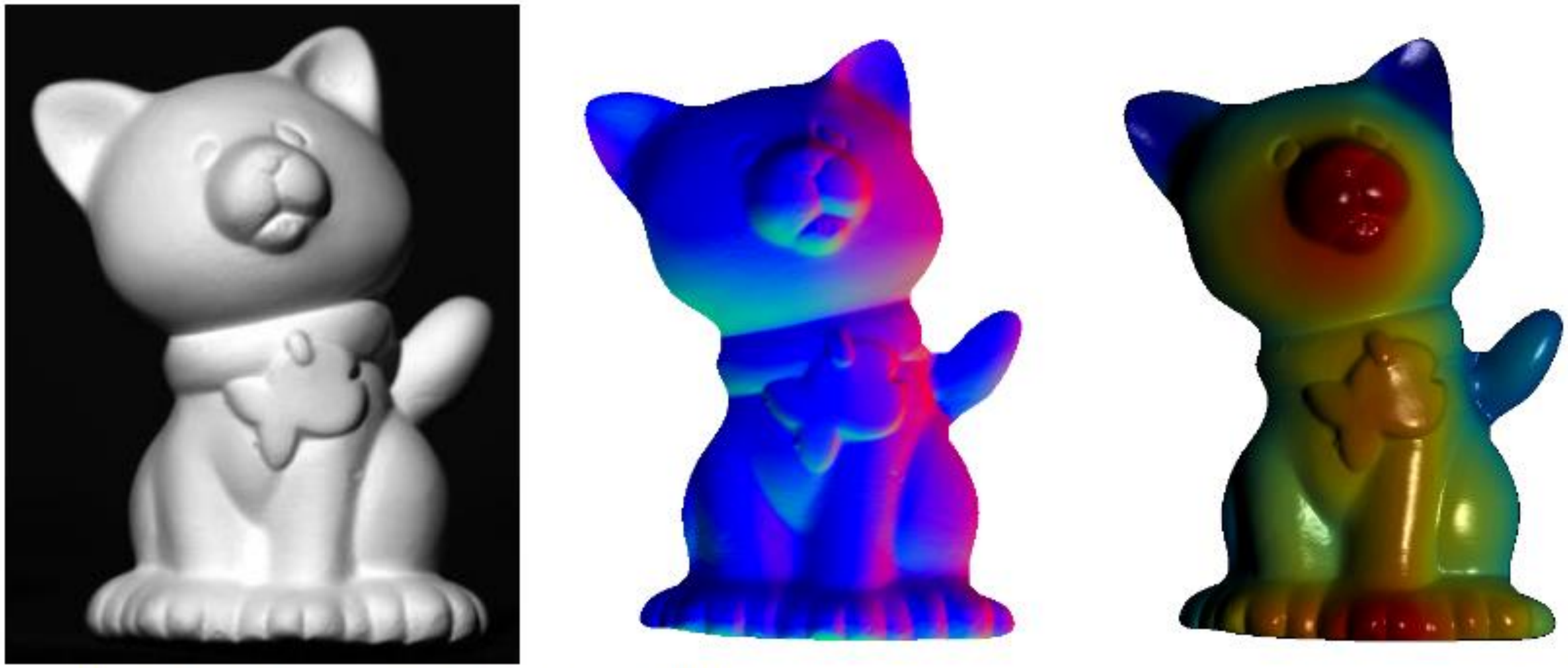


Scanning Electron Microscope image
(inverse intensity)

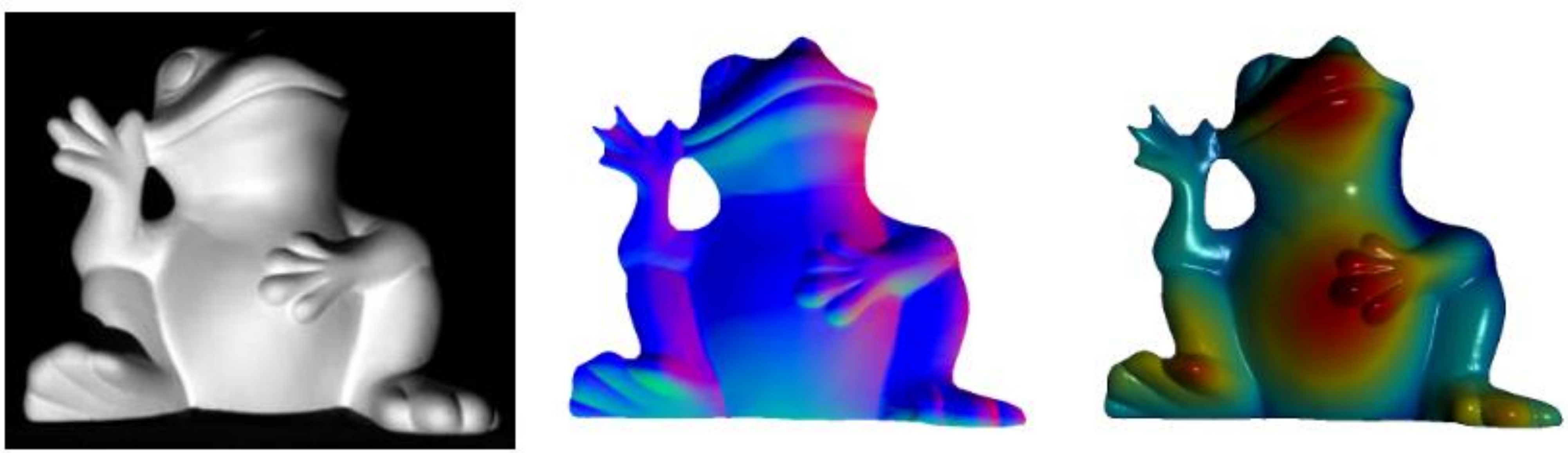


by Ikeuchi and Horn

More modern results



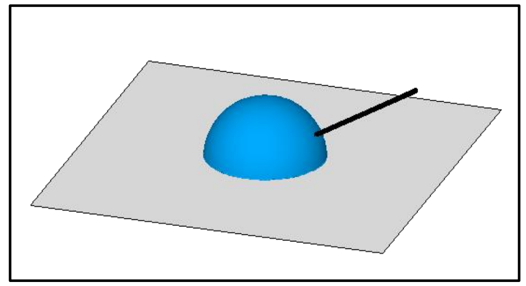
Resolution: 640 x 500; Re-rendering Error: 0.0075.



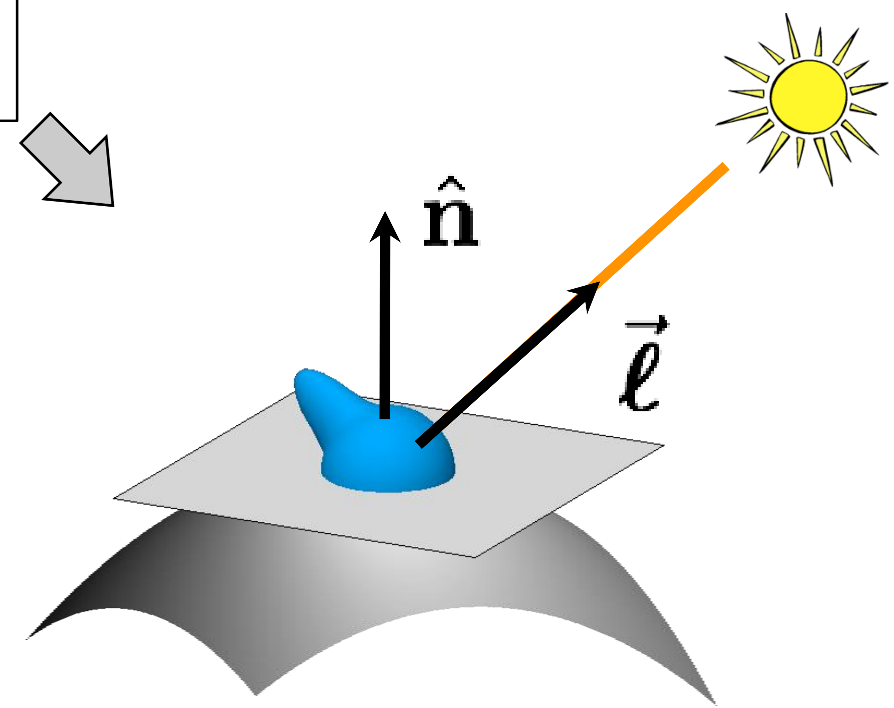
Resolution: 590 x 690; Re-rendering Error: 0.0083.

Single-lighting is ambiguous

ASSUMPTION 1:
LAMBERTIAN

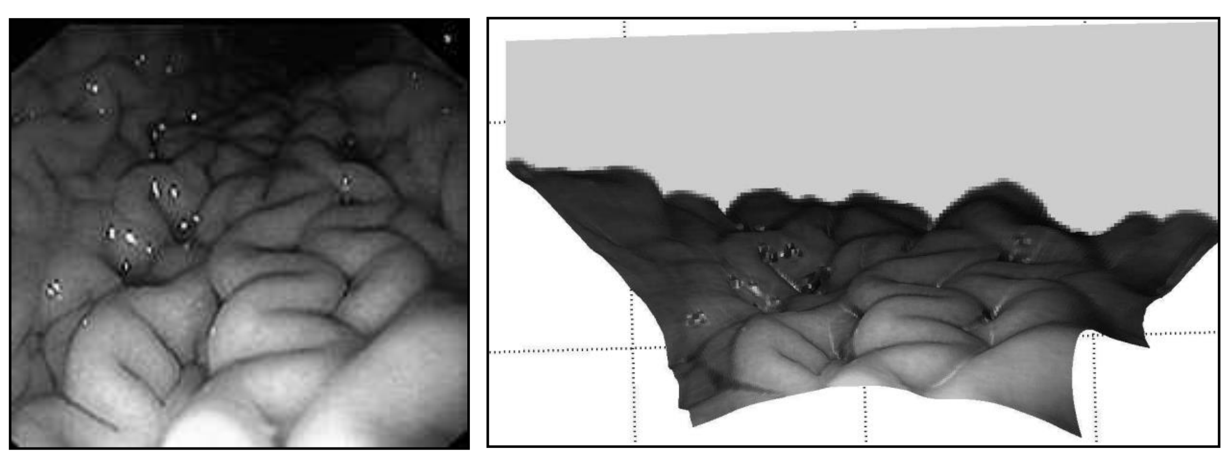


ASSUMPTION 2:
DIRECTIONAL LIGHTING



$$L^{\text{out}}(\hat{\boldsymbol{\omega}}) = \int_{\Omega_{\text{in}}} f(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) L^{\text{in}}(\hat{\boldsymbol{\omega}}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\boldsymbol{\omega}}_{\text{in}}$$

$$I = a\hat{\mathbf{n}}^\top \vec{\boldsymbol{\ell}}$$

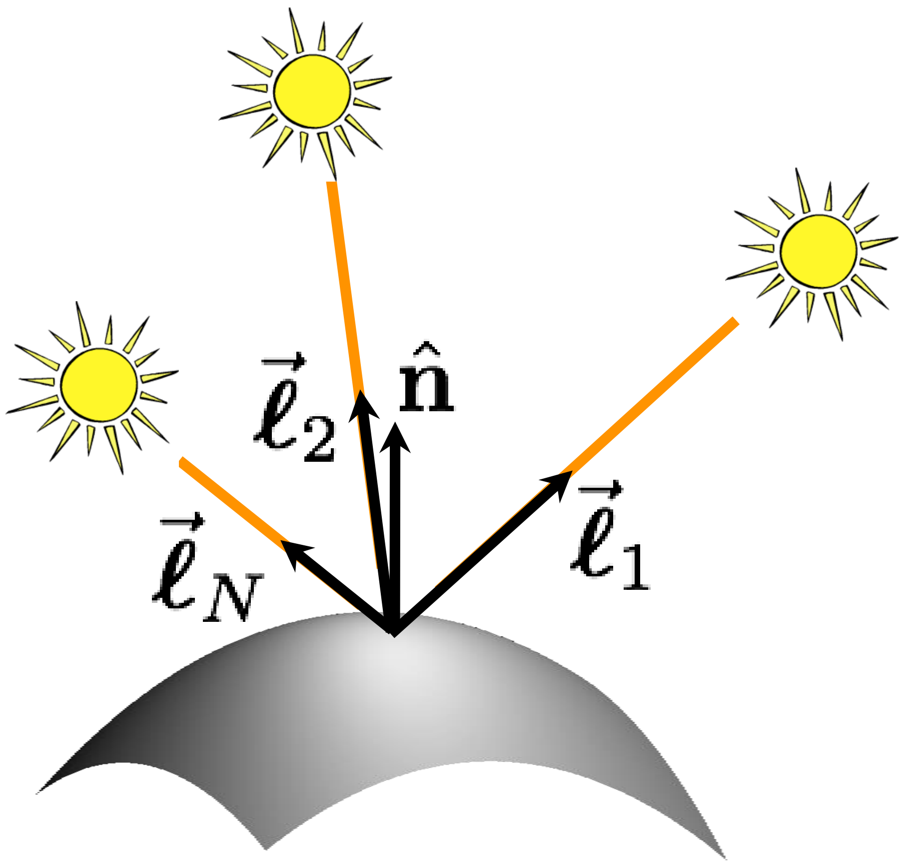


[Prados, 2004]

Photometric stereo

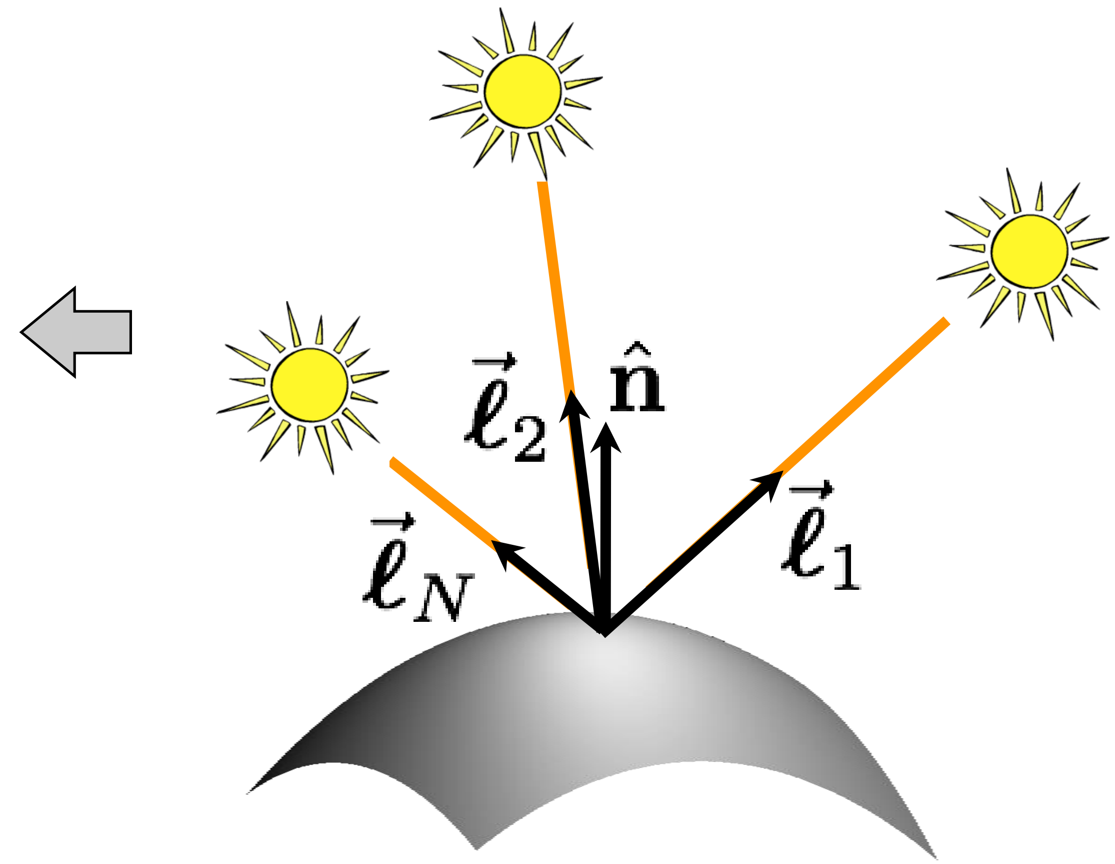
Lambertian photometric stereo

$$I_1 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1$$
$$I_2 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2$$
$$\vdots$$
$$I_N = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N$$



Lambertian photometric stereo

$$I_1 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1$$
$$I_2 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2$$
$$\vdots$$
$$I_N = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N$$



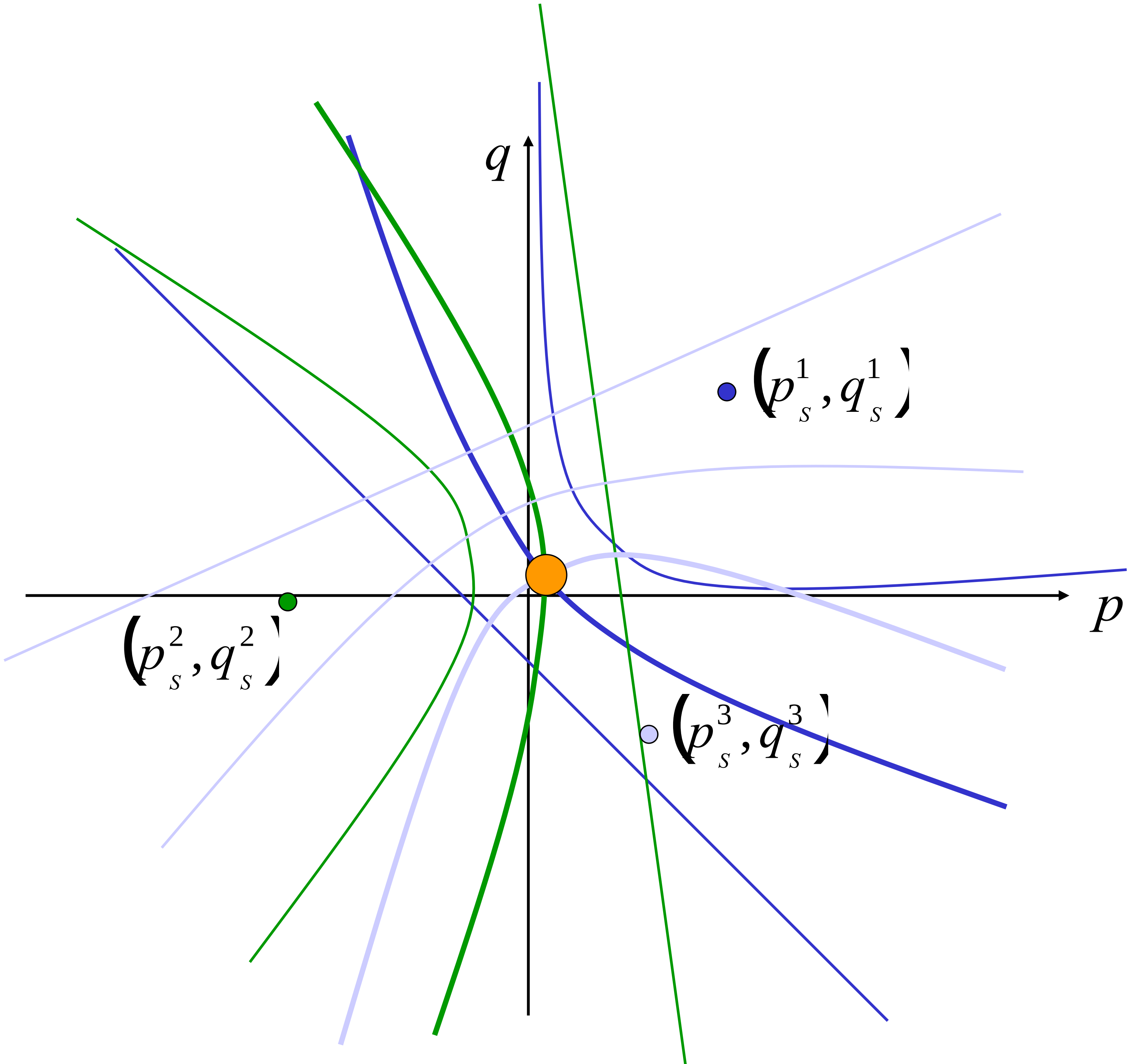
define "pseudo-normal" $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

solve linear system for pseudo-normal

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix}_{N\times 3} \begin{bmatrix} \vec{\boldsymbol{b}} \end{bmatrix}_{3\times 1}$$

once system is solved, b gives normal direction and albedo

Photometric Stereo



Solving the Equations

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_2 \end{bmatrix}}_{\mathbf{I}_{3\times 1}} = \underbrace{\begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix}}_{\mathbf{S}_{3\times 3}} \underbrace{\rho\,\mathbf{n}}_{\tilde{\mathbf{n}}_{3\times 1}}$$

$$\tilde{\mathbf{n}} = \mathbf{S}^{-1}\mathbf{I}$$

inverse

$$\rho = \left|\tilde{\mathbf{n}}\right|$$

$$\mathbf{n} = \frac{\tilde{\mathbf{n}}}{\left|\tilde{\mathbf{n}}\right|} = \frac{\tilde{\mathbf{n}}}{\rho}$$

More than Three Light Sources

- Get better results by using more lights

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1^T \\ \vdots \\ \mathbf{s}_N^T \end{bmatrix} \rho\, \mathbf{n}$$

- Least squares solution:

$$\mathbf{I} = \mathbf{S}\tilde{\mathbf{n}} \qquad \longleftarrow \qquad N \times 1 = (N \times 3)(3 \times 1)$$

$$\mathbf{S}^T\mathbf{I} = \mathbf{S}^T\mathbf{S}\tilde{\mathbf{n}}$$

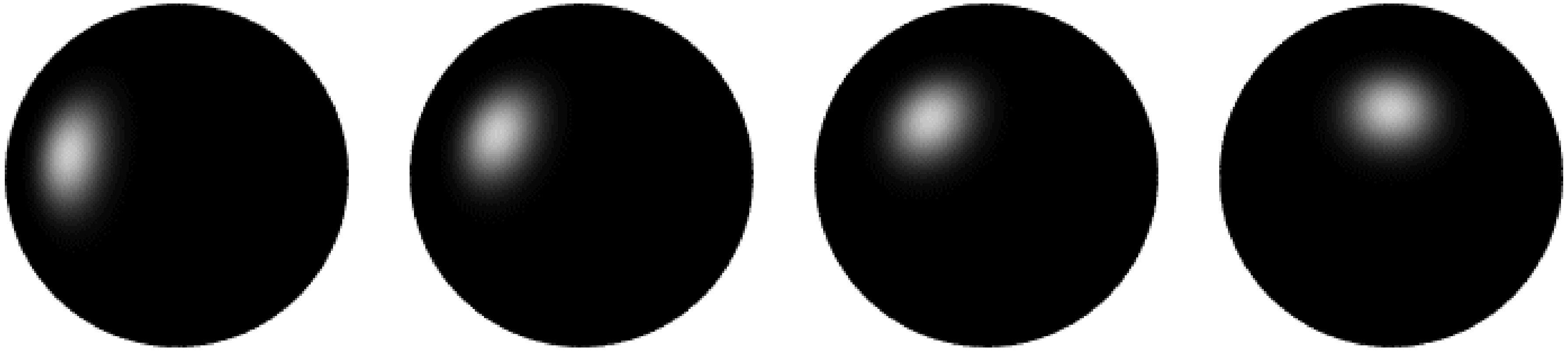
$$\tilde{\mathbf{n}} = \left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T\mathbf{I}$$

Moore-Penrose pseudo inverse

- Solve for $\rho, \mathbf{n}$ as before

Computing light source directions

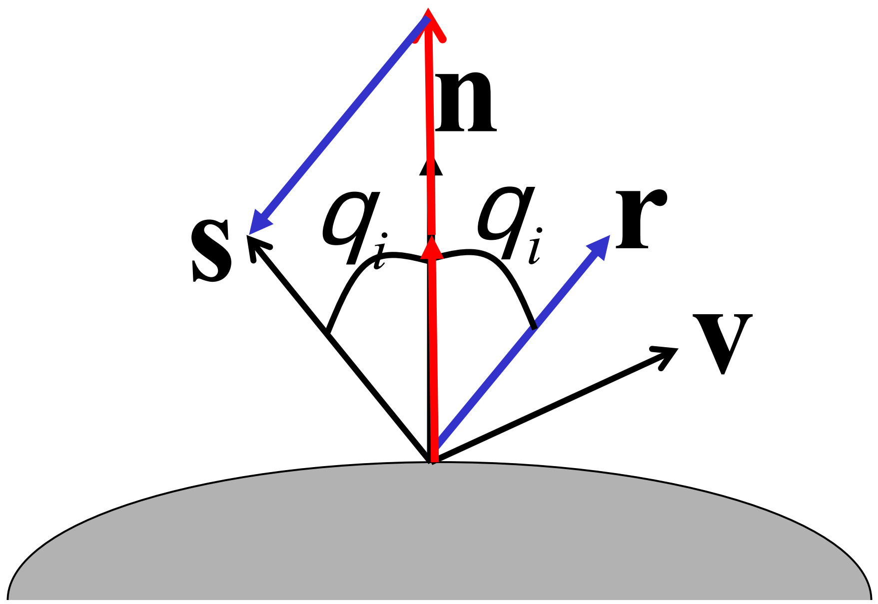
- Trick: place a chrome sphere in the scene



  - the location of the highlight tells you the source direction

Specular Reflection - Recap

- For a perfect mirror, light is reflected about **N**



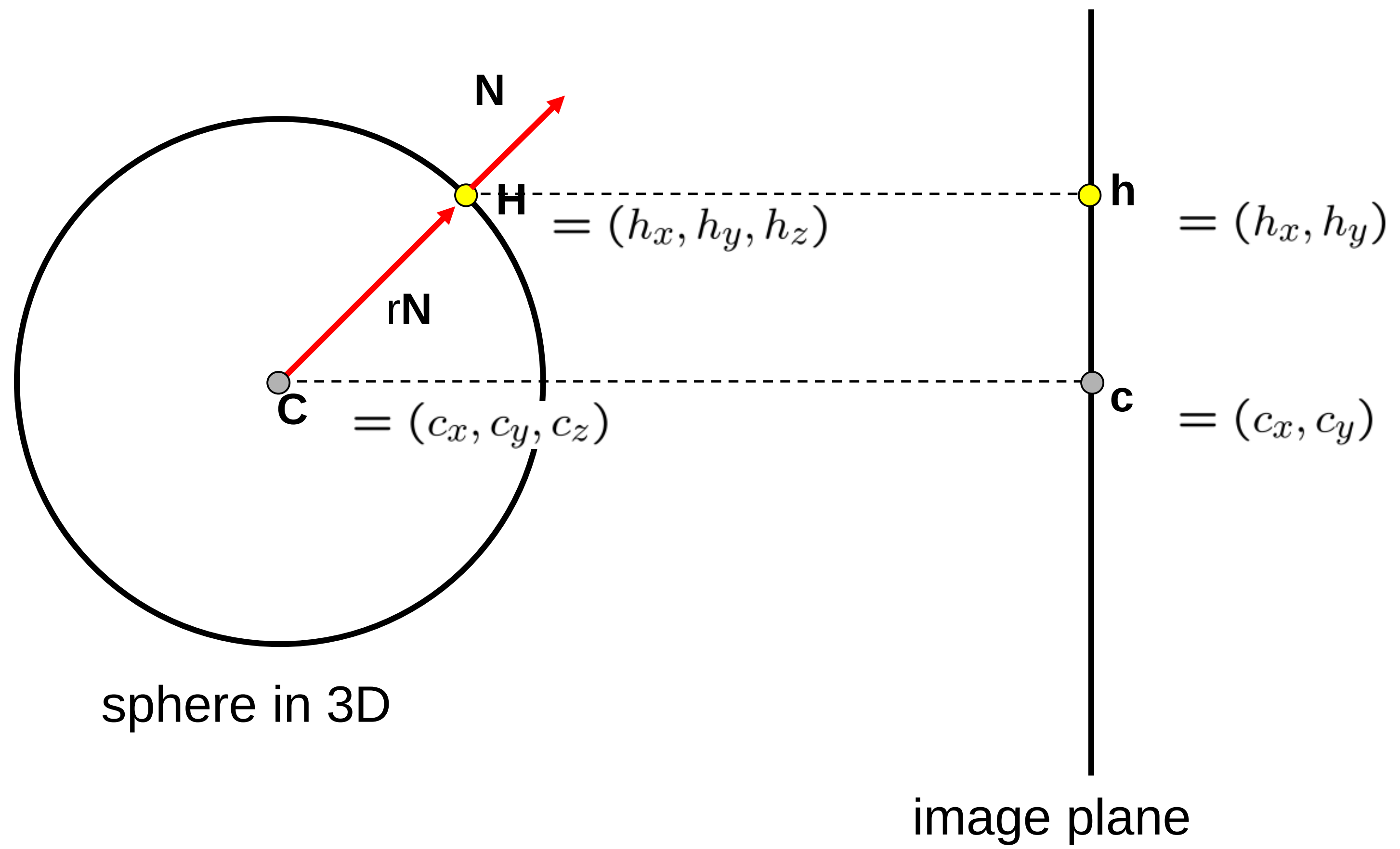
$$R_e = \begin{cases} R_i & \text{if } \mathbf{v} = \mathbf{r} \\ 0 & \text{otherwise} \end{cases}$$

- We see a highlight when $\mathbf{v} = \mathbf{r}$
- Then $\mathbf{s}$ is given as follows:

$$\mathbf{s} = 2(\mathbf{n} \times \mathbf{r})\mathbf{n} - \mathbf{r}$$

Computing the Light Source Direction

Chrome sphere that has a highlight at position **h** in the image



- Can compute **N** by studying this figure
  - Hints:
    - use this equation: $||H - C|| = r$
    - can measure **c**, **h**, and r in the image

Limitations

- Big problems
  - Doesn't work for shiny things, semi-translucent things
  - Shadows, inter-reflections
- Smaller problems
  - Camera and lights have to be distant
  - Calibration requirements
    - measure light source directions, intensities
    - camera response function

Trick for Handling Shadows

- Weight each equation by the pixel brightness:

$$I_i\left(I_i\right) = I_i\left(\rho\, \mathbf{n} \cdot \mathbf{s}_i\right)$$

- Gives weighted least-squares matrix equation:

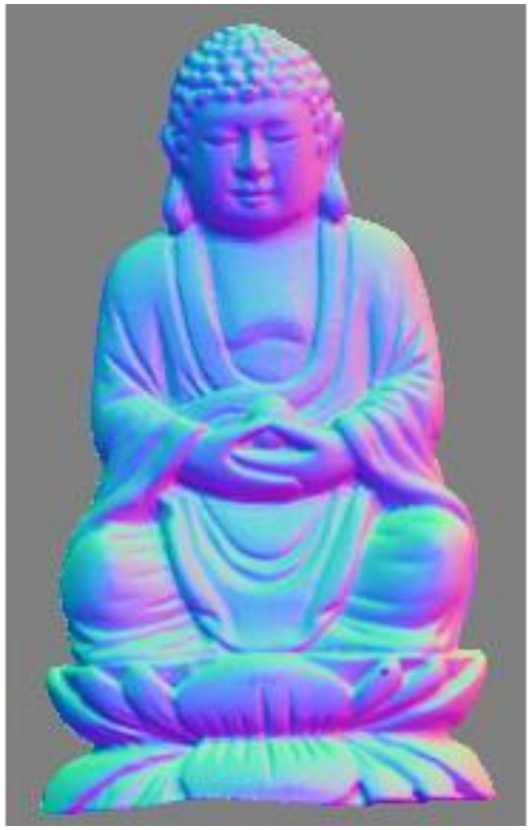
$$\begin{bmatrix} I_1^2 \\ \vdots \\ I_N^2 \end{bmatrix} = \begin{bmatrix} I_1 \mathbf{s}_1^T \\ \vdots \\ I_N \mathbf{s}_N^T \end{bmatrix} \rho\, \mathbf{n}$$

- Solve for $\rho, \mathbf{n}$ as before

Depth from normals

- Solving the linear system per-pixel gives us an estimated surface normal for each pixel

Input photo



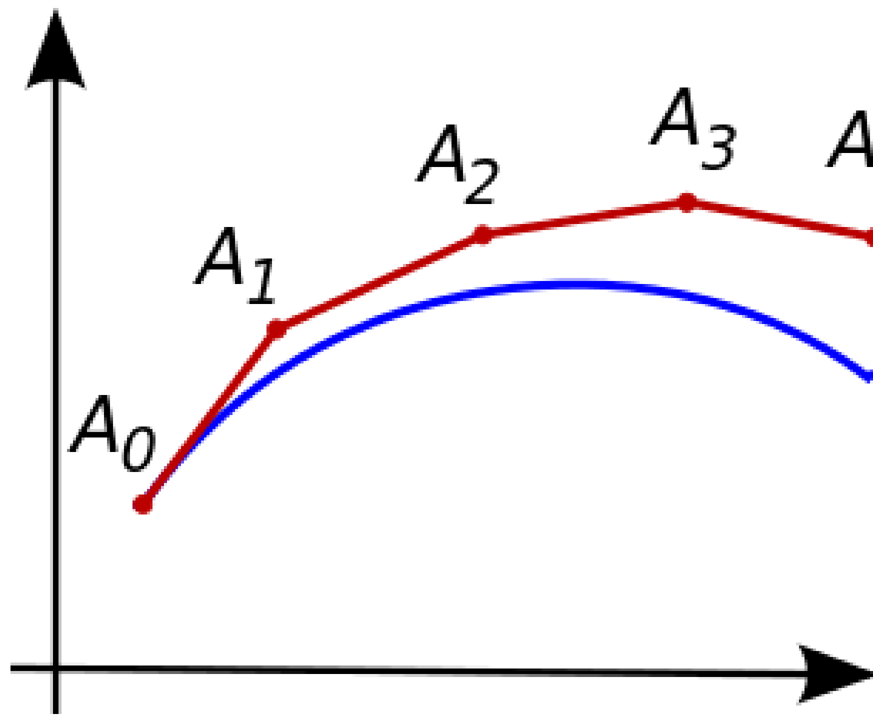
Estimated normals

Estimated normals (needle diagram)

- How can we compute depth from normals?
  - Normals are like the “derivative” of the true depth

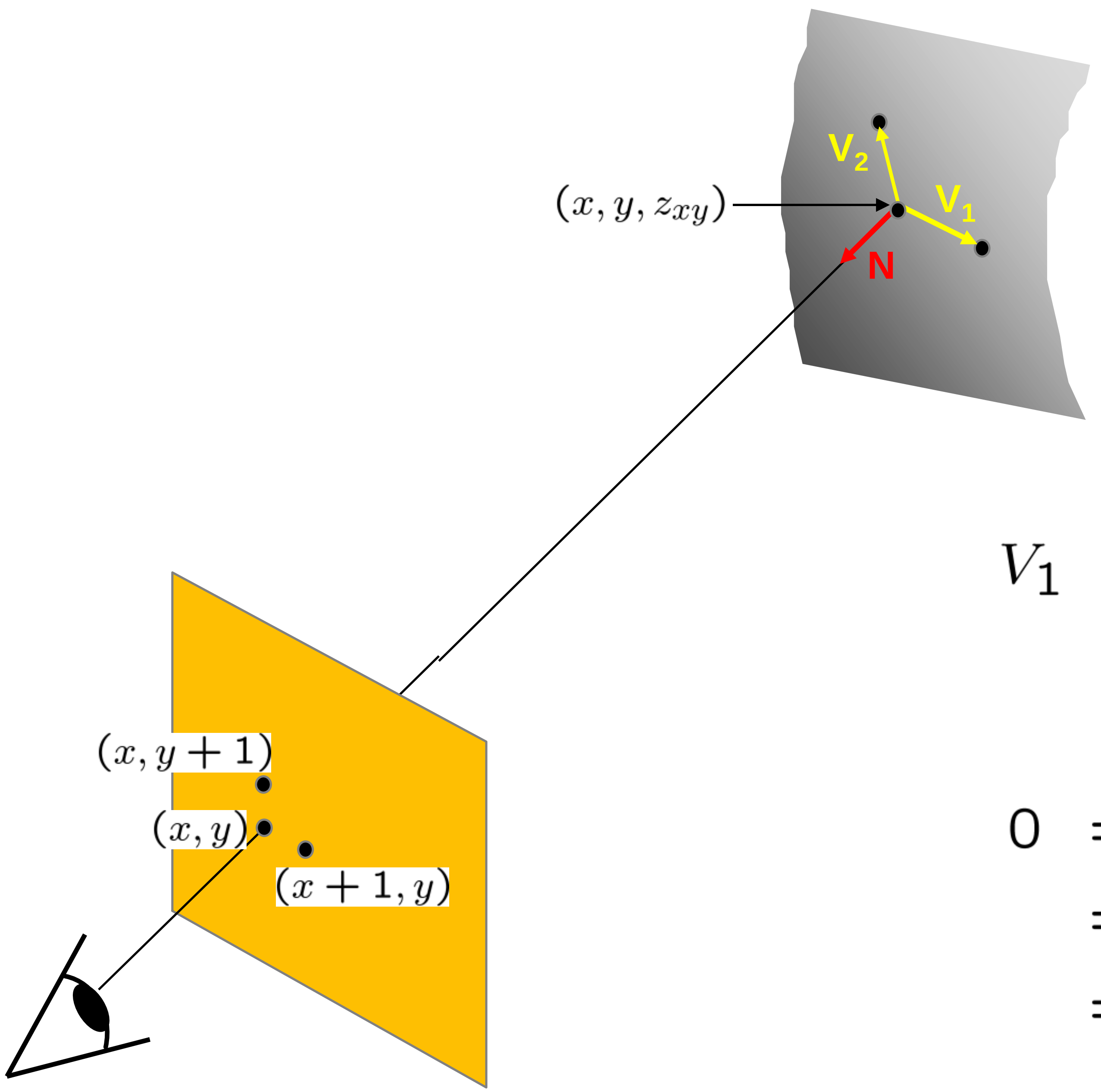
Normal Integration

- Integrating a set of derivatives is easy in 1D
  - (similar to Euler’s method from diff. eq. class)



- Could just integrate normals in each column / row separately
- Instead, we formulate as a linear system and solve for depths that *best agree with the surface normals*

Depth from normals



$$\begin{aligned} V_1 &= (x+1, y, z_{x+1,y}) - (x, y, z_{xy}) \\ &= (1, 0, z_{x+1,y} - z_{xy}) \end{aligned}$$

$$\begin{aligned} 0 &= N \cdot V_1 \\ &= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\ &= n_x + n_z(z_{x+1,y} - z_{xy}) \end{aligned}$$

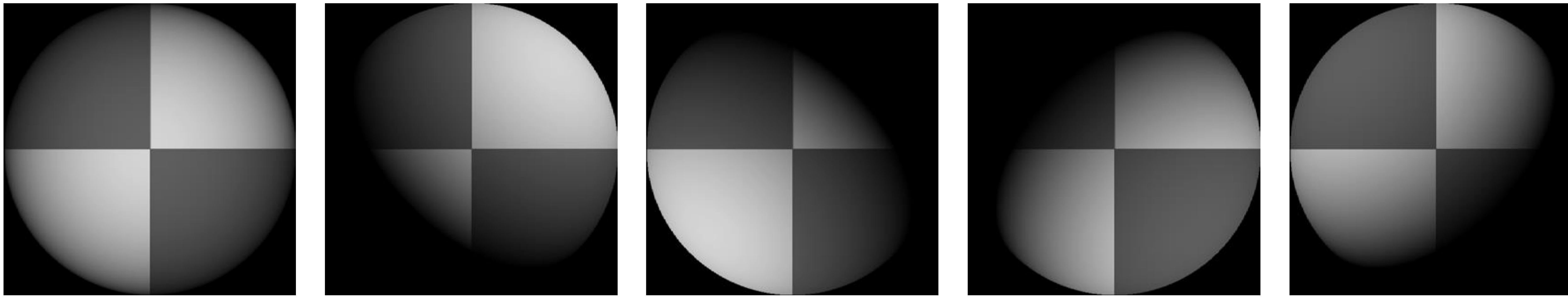
Get a similar equation for $\mathbf{V_2}$

- Each normal gives us two linear constraints on z
- compute z values by solving a matrix equation

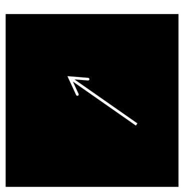
Results

1. Estimate light source directions
2. Compute surface normals
3. Compute albedo values
4. Estimate depth from surface normals
5. Relight the object (with original texture and uniform albedo)

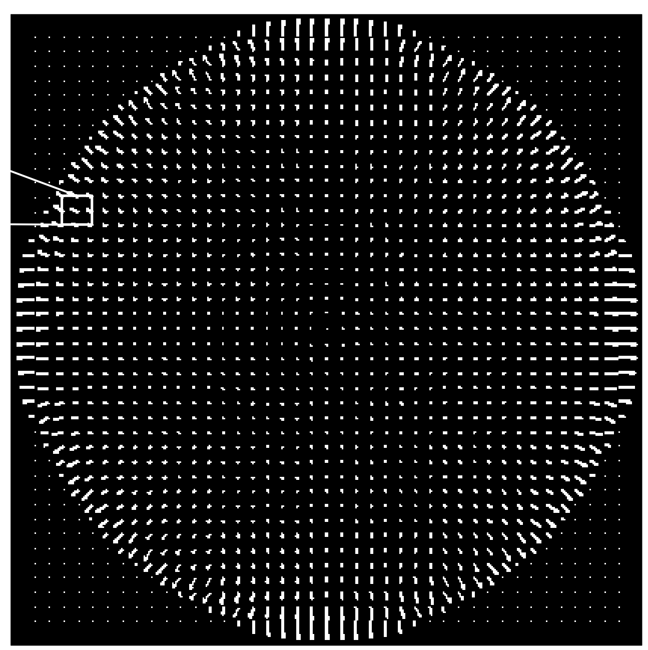
Results: Lambertian Sphere



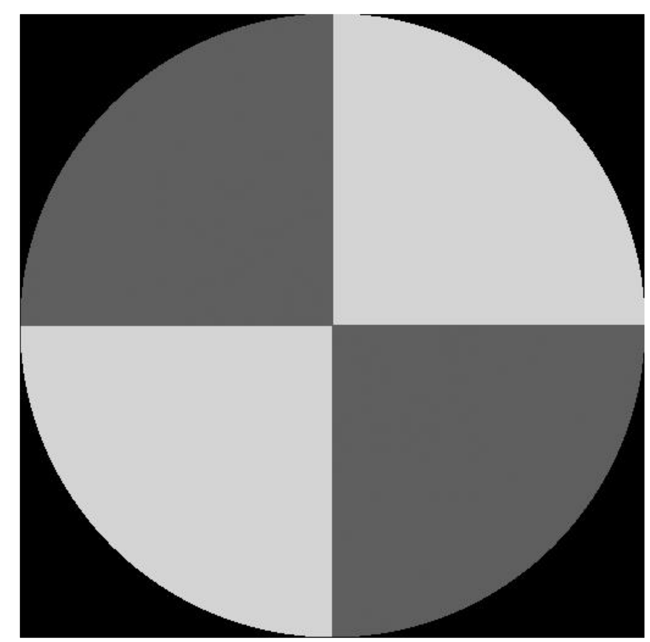
Input Images



Needles are projections of surface normals on image plane



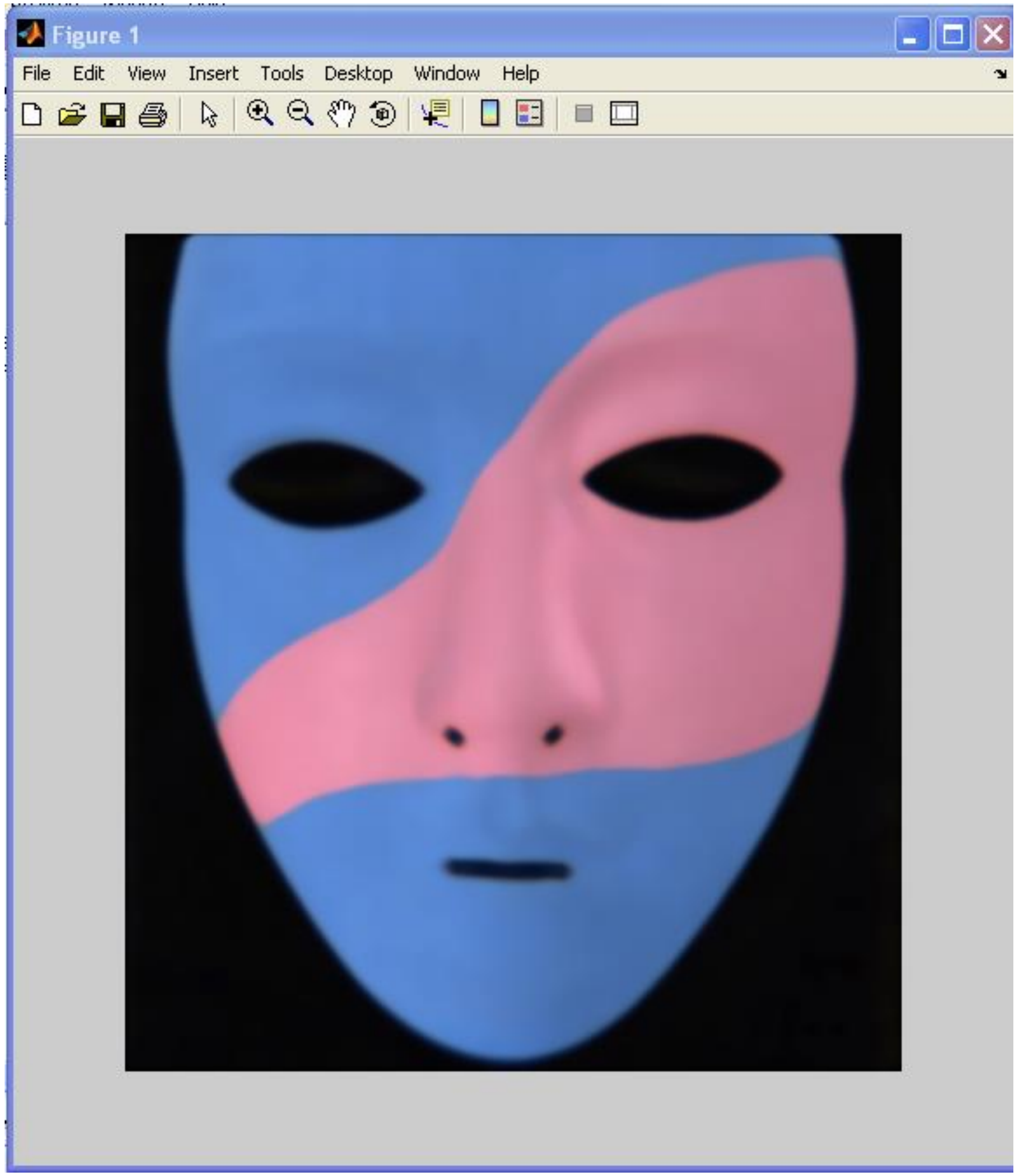
Estimated Surface Normals

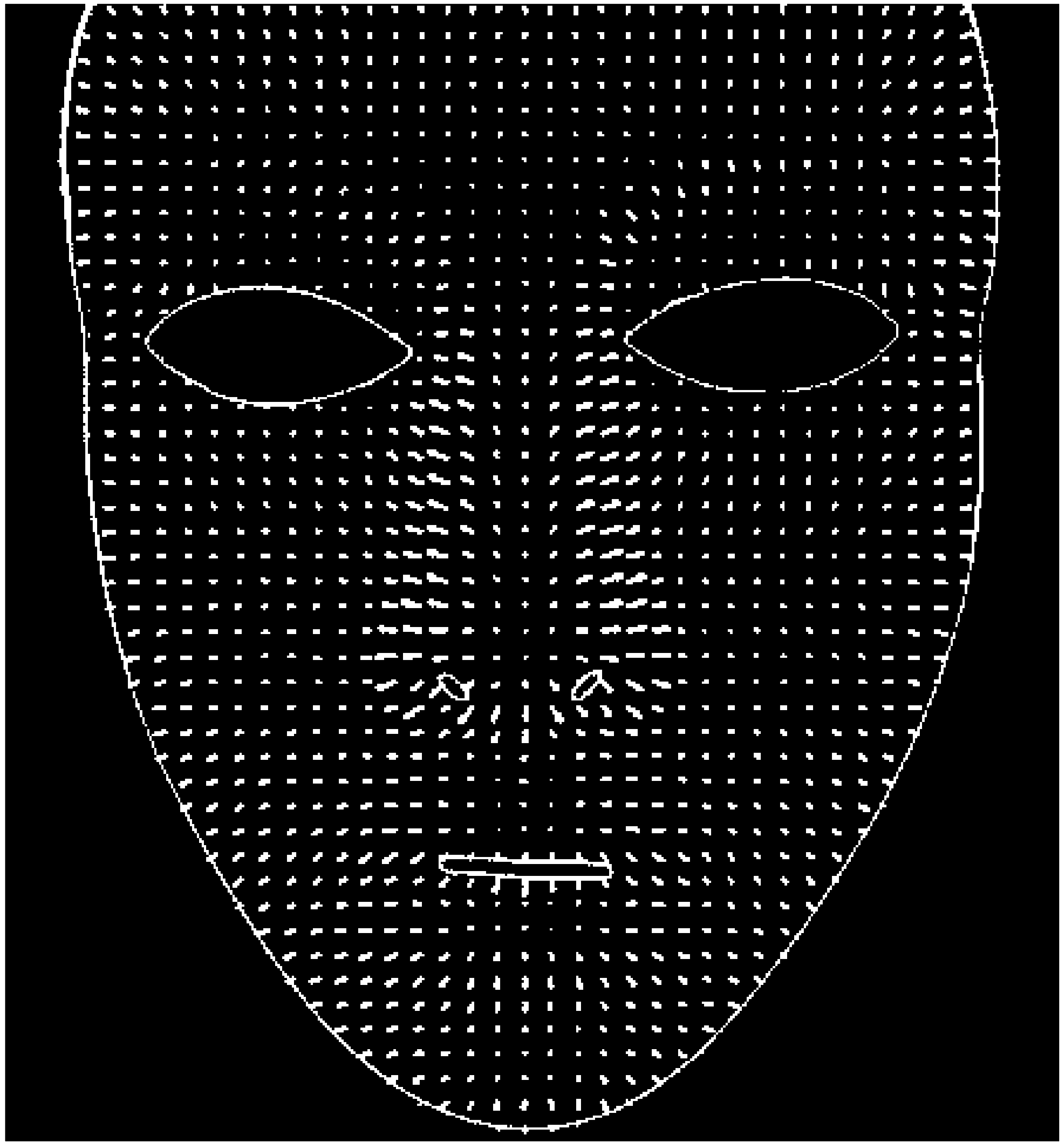


Estimated Albedo

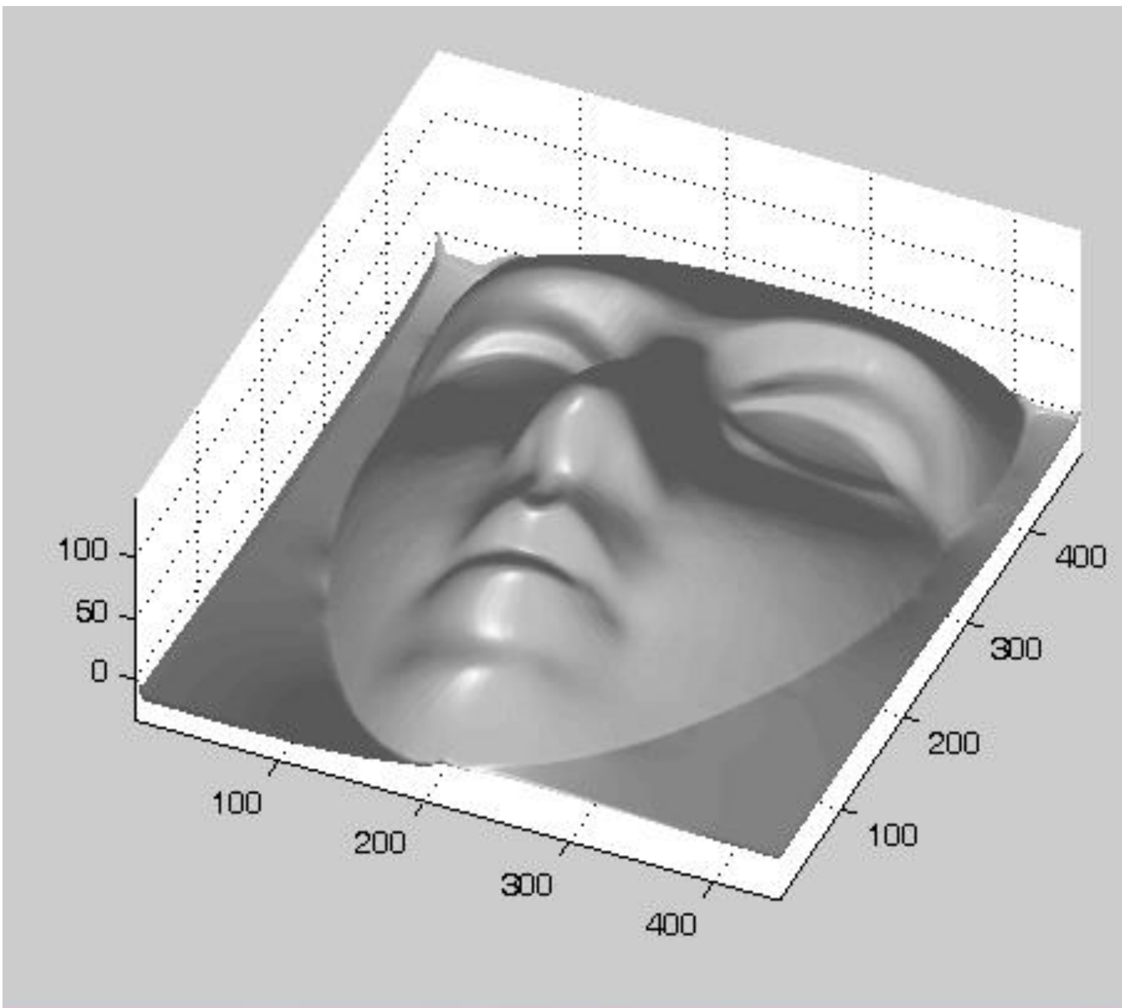
Lambertain Mask

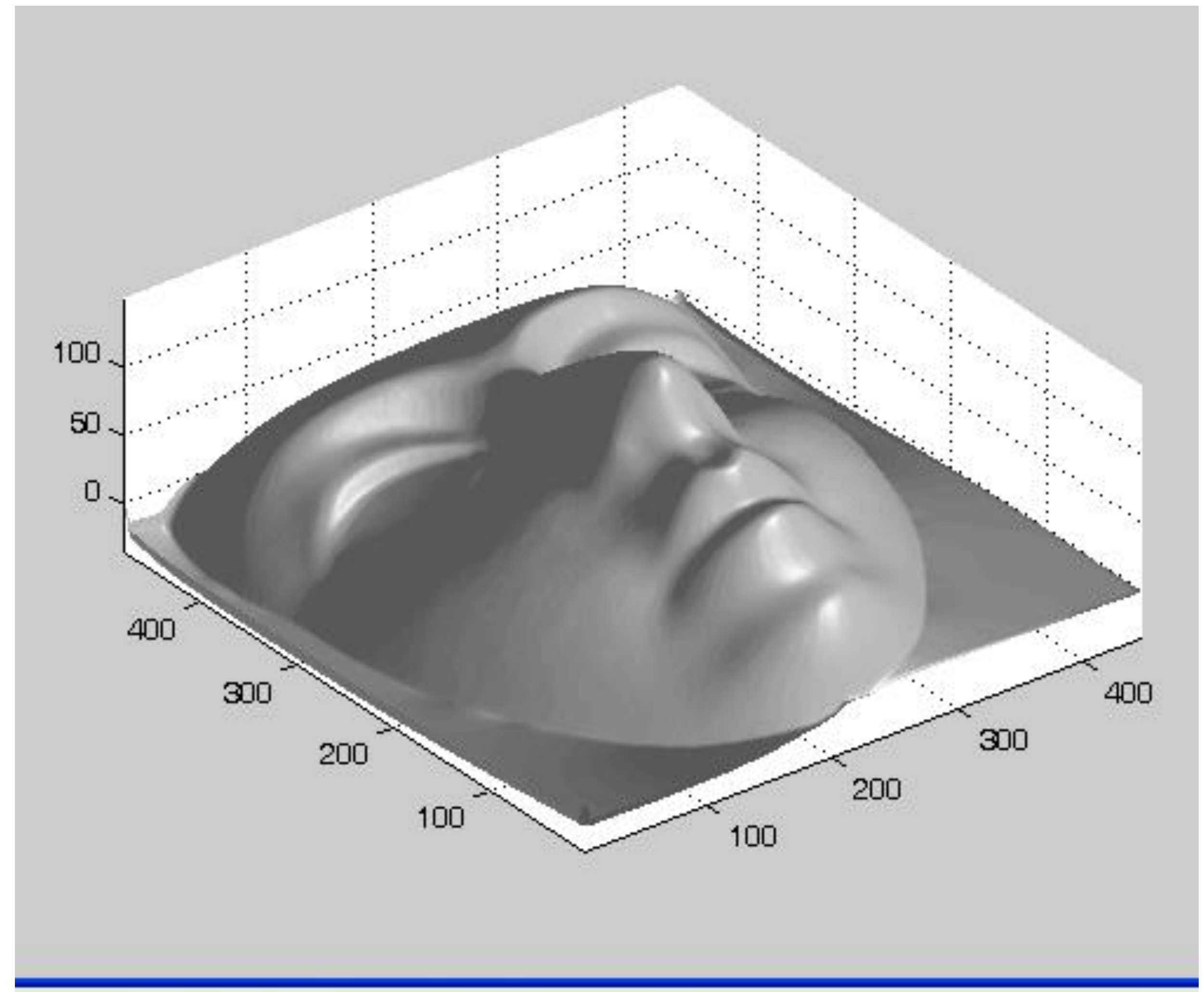
Results – Albedo and Surface Normal





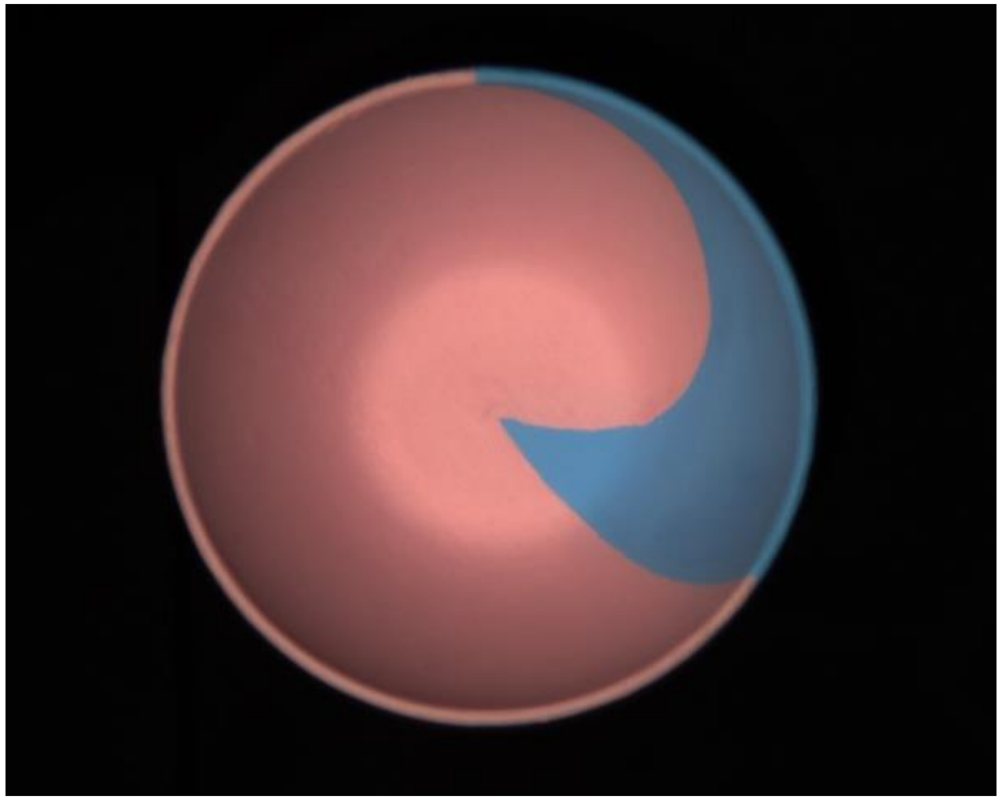
Results – Shape of Mask

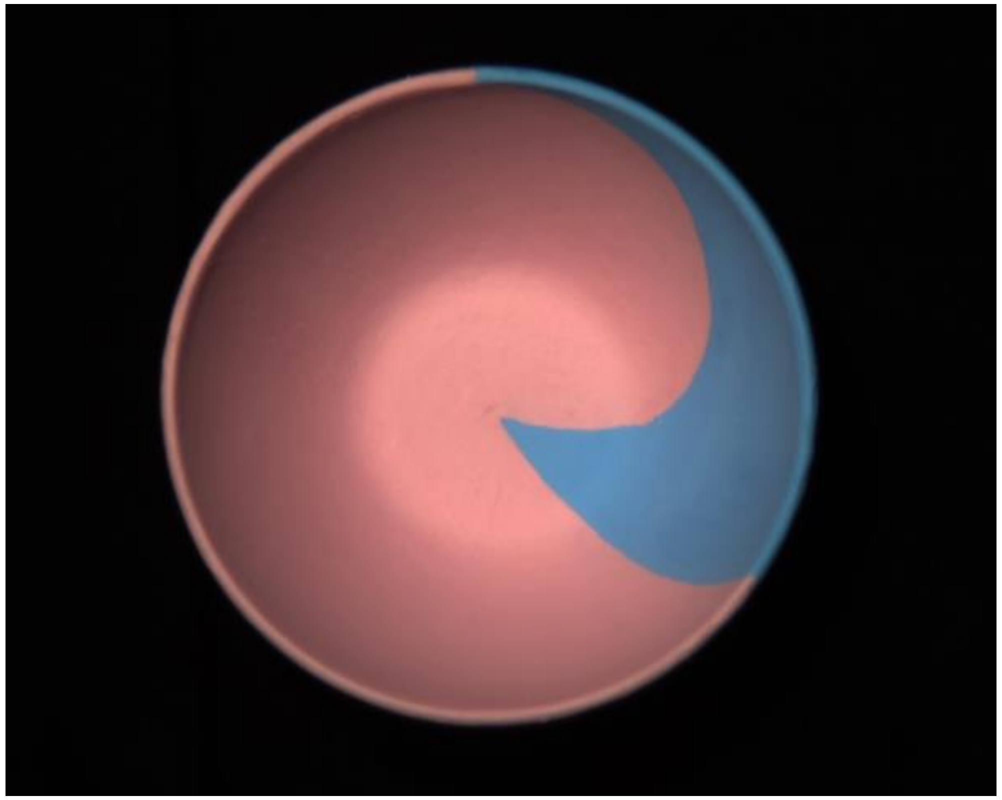


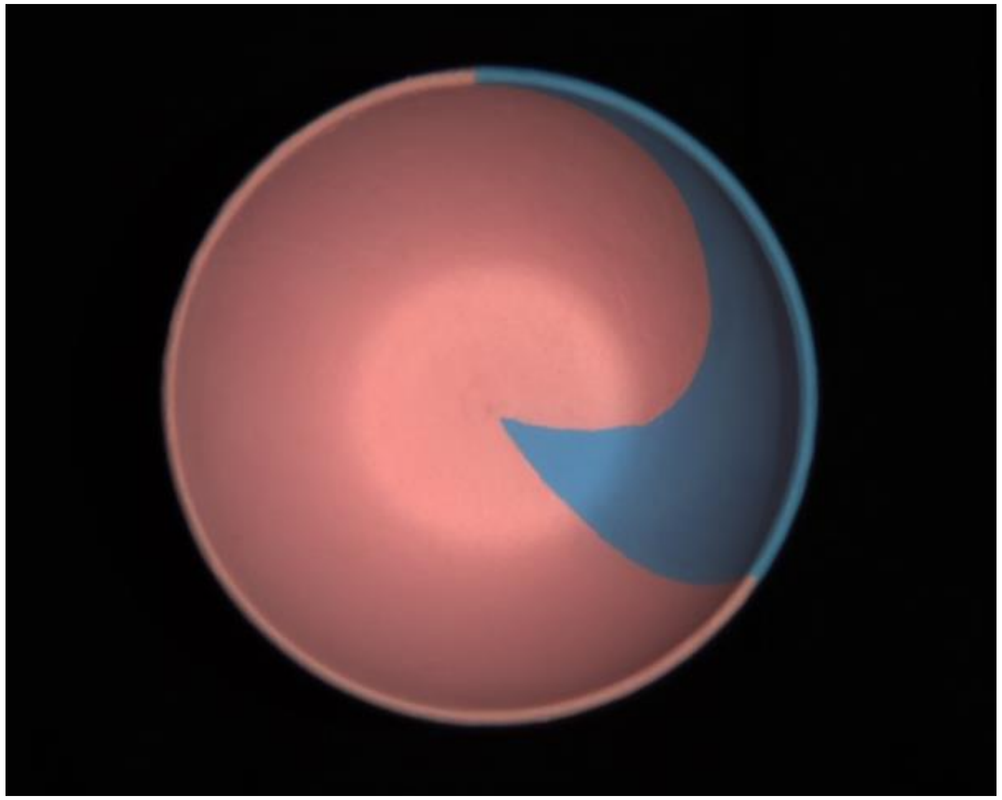


Results: Lambertian Toy

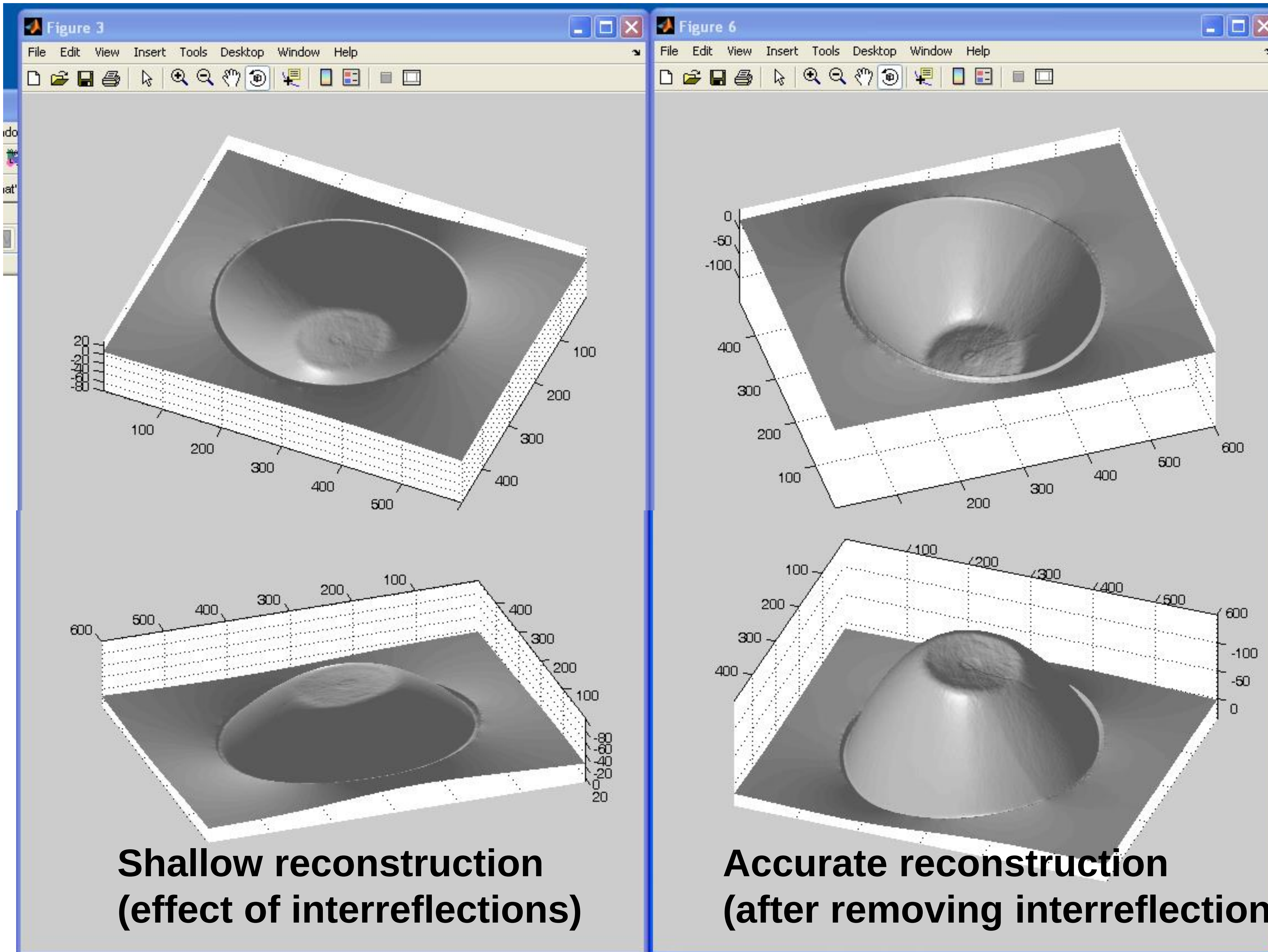
Original Images







Results - Shape



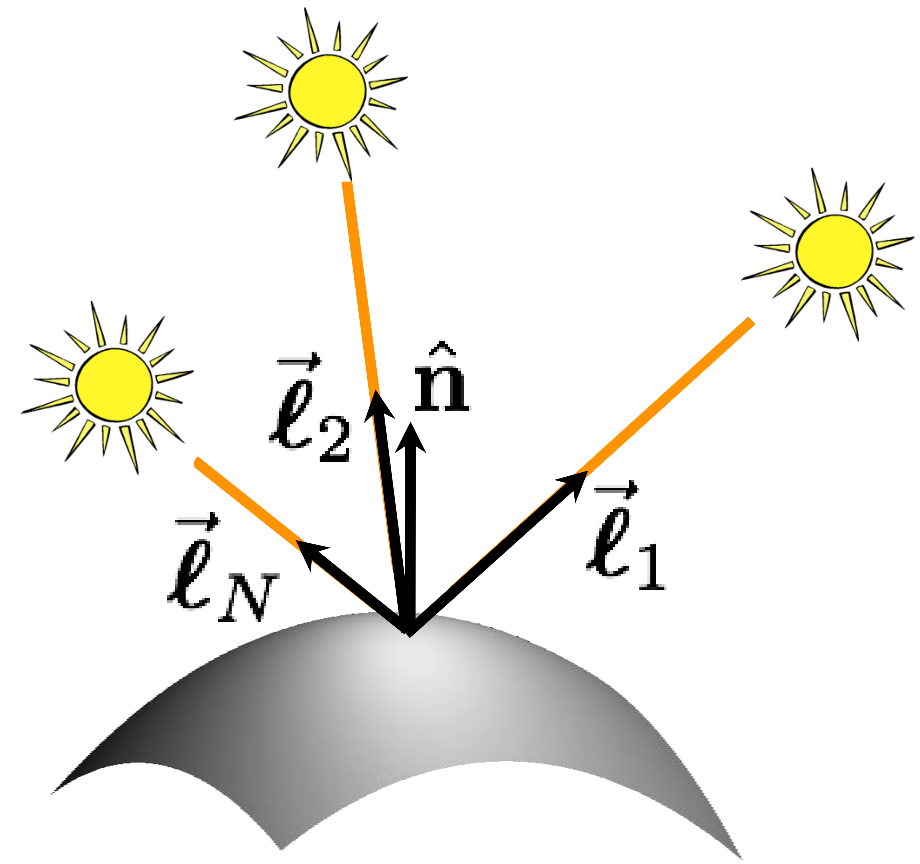
Shallow reconstruction (effect of interreflections)

Accurate reconstruction (after removing interreflections)

What if the light directions are unknown?

What if the light directions are unknown?

$$I_1 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1$$
$$I_2 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2$$
$$\vdots$$
$$I_N = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N$$



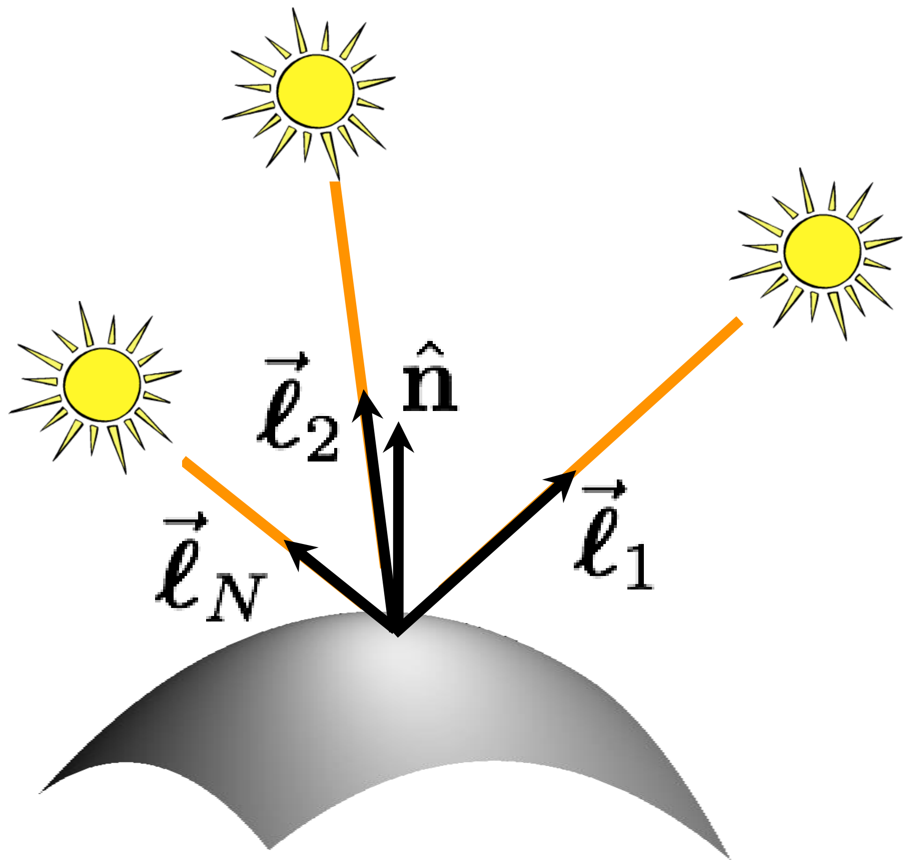
define "pseudo-normal" $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

solve linear system for pseudo-normal

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix}_{N\times 3} \begin{bmatrix} \vec{\boldsymbol{b}} \end{bmatrix}_{3\times 1}$$

What if the light directions are unknown?

$$I_1 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1$$
$$I_2 = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2$$
$$\vdots$$
$$I_N = a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N$$



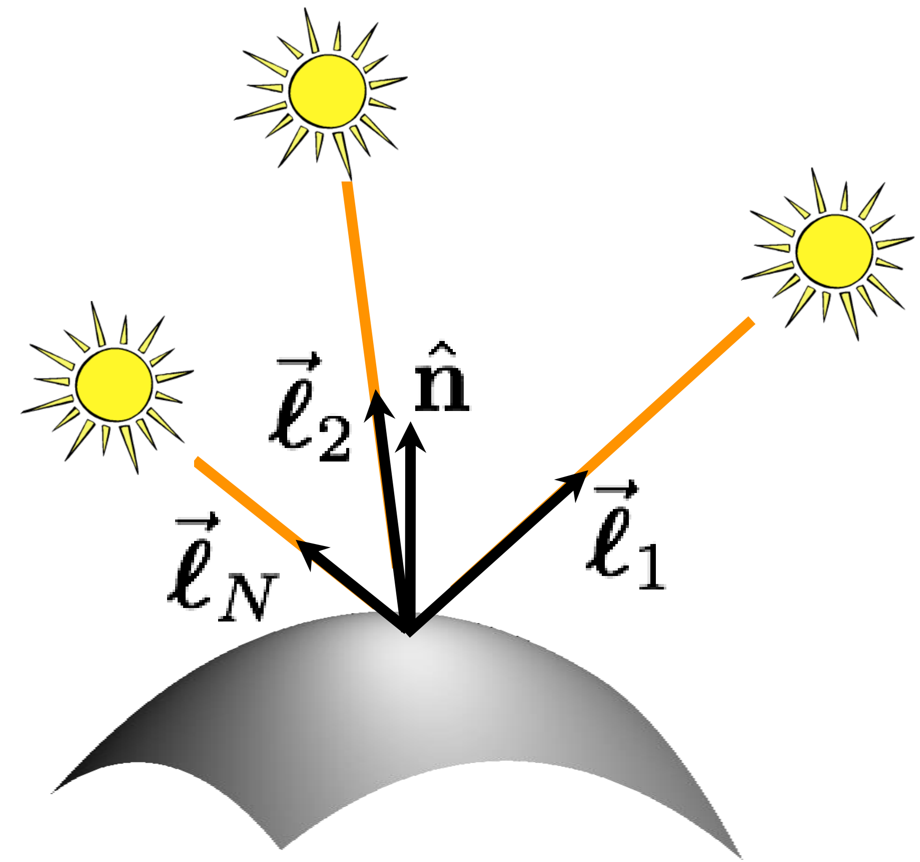
define "pseudo-normal" $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

solve linear system for pseudo-normal at each image pixel

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}_{N \times M} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix}_{N \times 3} \begin{bmatrix} B \end{bmatrix}_{3 \times M}$$

What if the light directions are unknown?

$$
\begin{aligned}
I_1 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_1 \\
I_2 &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_2 \\
&\vdots \\
I_N &= a\hat{\boldsymbol{n}}^\top \vec{\boldsymbol{\ell}}_N
\end{aligned}
$$



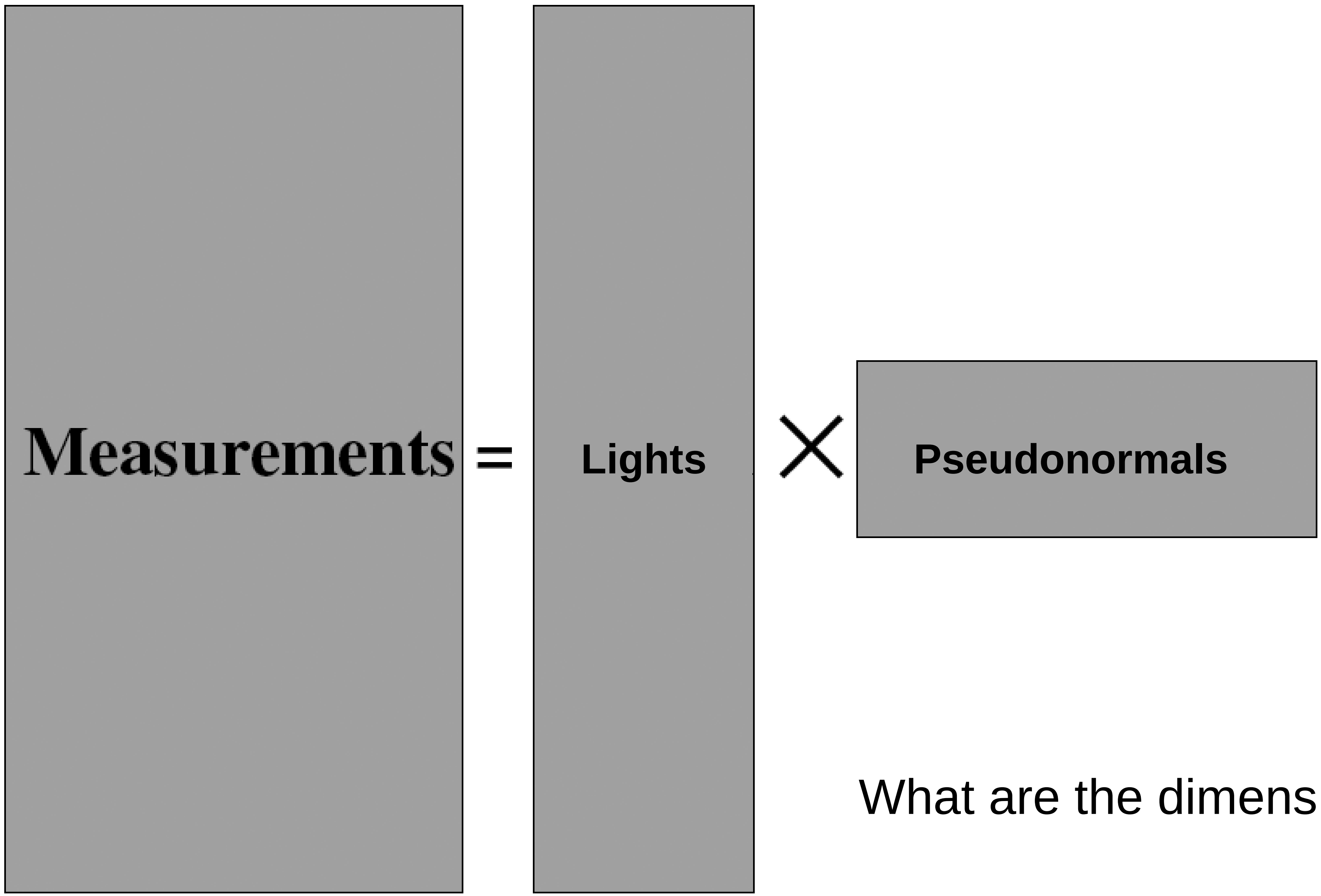
define "pseudo-normal" $\vec{\boldsymbol{b}} \triangleq a\hat{\boldsymbol{n}}$

solve linear system for pseudo-normal at each image pixel

$$
\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}_{N\times M} = \begin{bmatrix} \vec{\boldsymbol{\ell}}_1^\top \\ \vec{\boldsymbol{\ell}}_2^\top \\ \vdots \\ \vec{\boldsymbol{\ell}}_N^\top \end{bmatrix}_{N\times 3} \begin{bmatrix} B \end{bmatrix}_{3\times M}
$$

How do we solve this system without knowing light matrix L?

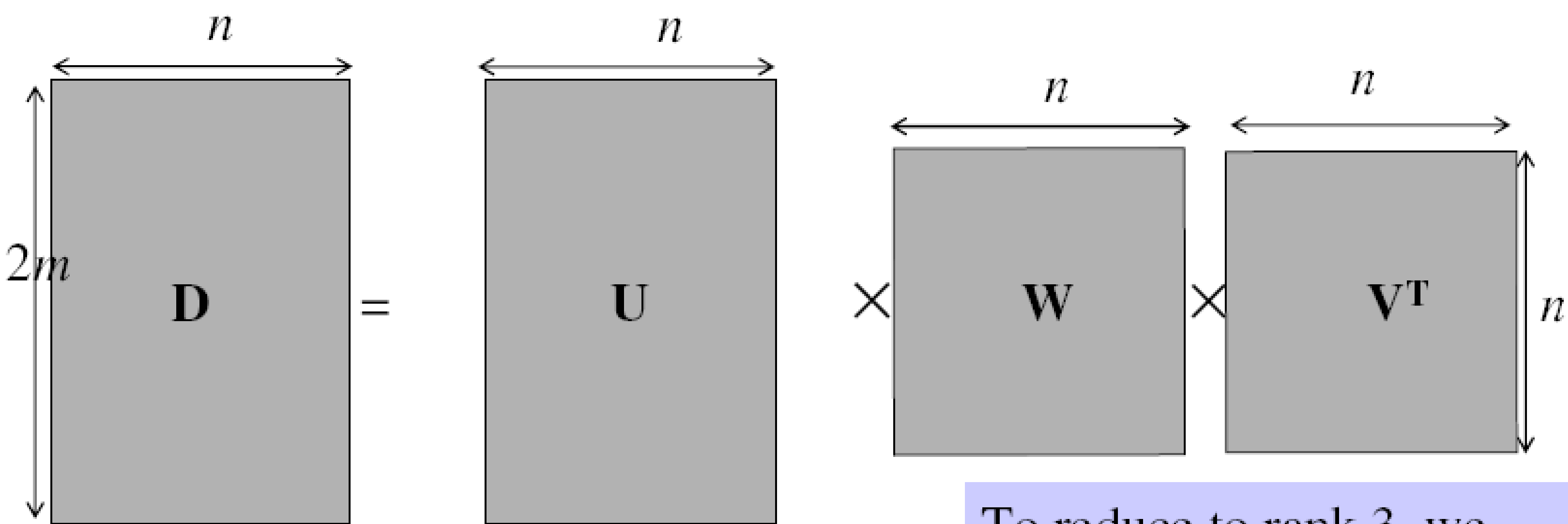
Factorizing the measurement matrix



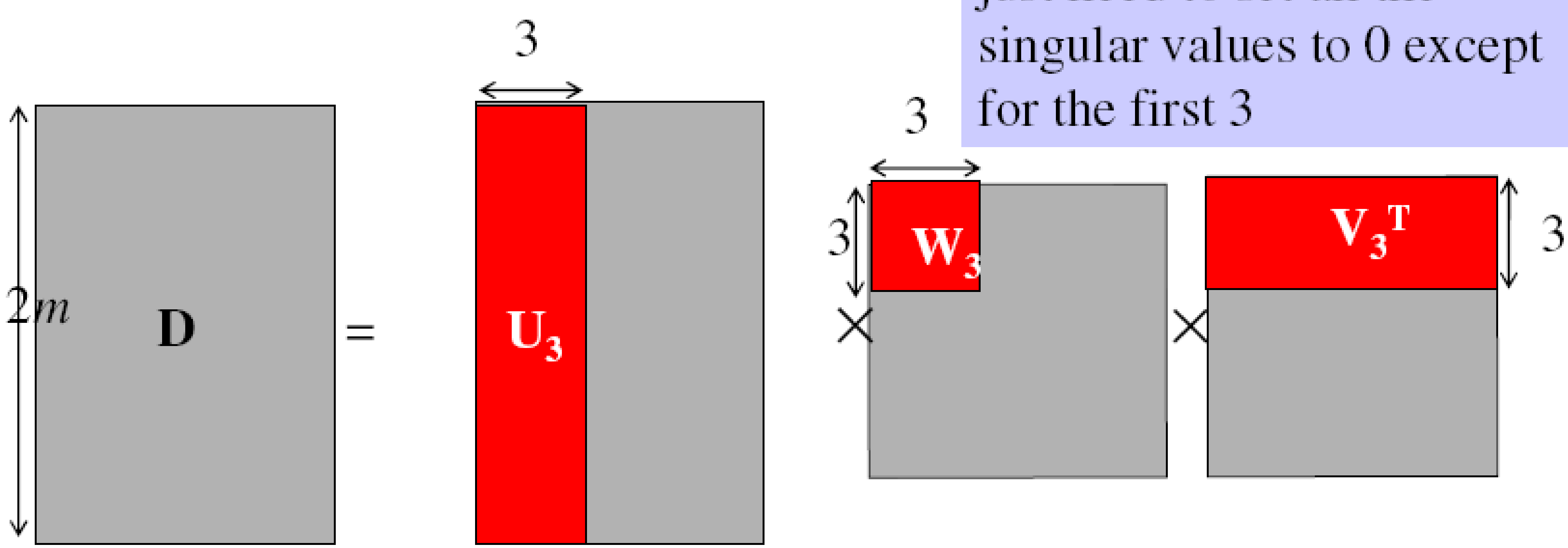
What are the dimensions?

Factorizing the measurement matrix

- Singular value decomposition:



To reduce to rank 3, we just need to set all the singular values to 0 except for the first 3



This decomposition minimizes $|\mathbf{I}\text{-}\mathbf{LB}|^2$

Are the results unique?

Bas-relief ambiguity

$$\mathbf{I = L\,B = (L\,Q^{-1})\,(Q\,B)}$$

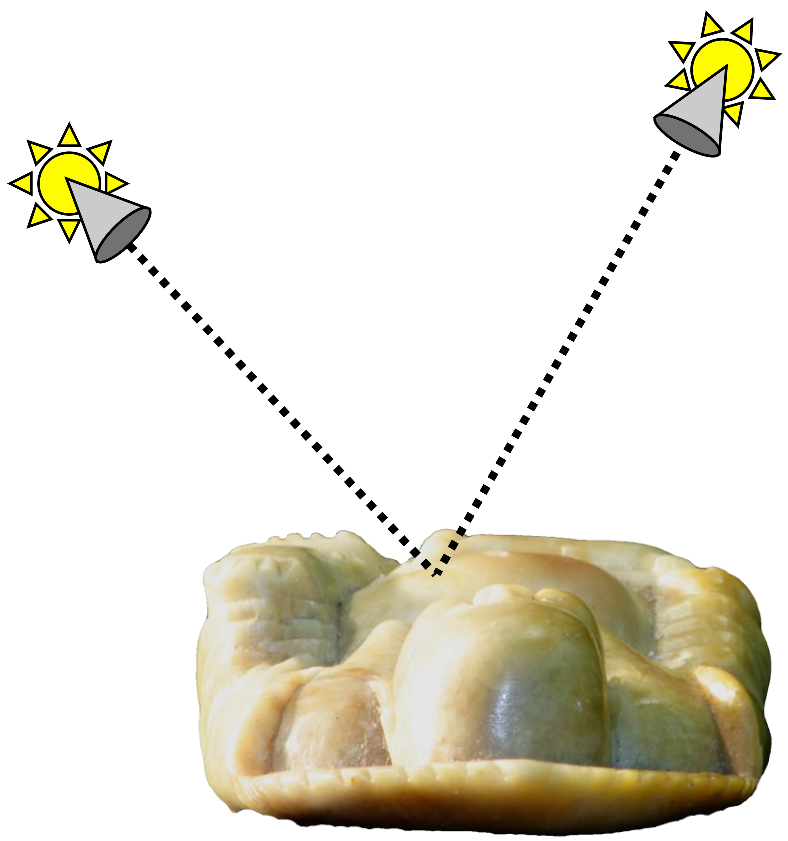
(Belhumeur et al., 1999)

What assumptions have we made for all this?

What assumptions have we made for all this?

- Lambertian BRDF
- Directional lighting
- No interreflections or scattering

Shape independent of BRDF via reciprocity: "Helmholtz Stereopsis"



$I = f($shape, ~~illumination~~, ~~reflectance~~$)$

$f^{-1} =$

[Zickler et al., 2002]

What assumptions have we made for all this?

- Lambertian BRDF
- Directional lighting
- No interreflections or scattering

References

Basic reading:

- Szeliski, Section 2.2, 12.1.
- Gortler, Chapter 21.