Extracting Lines

16-385 Computer Vision (Kris Kitani)

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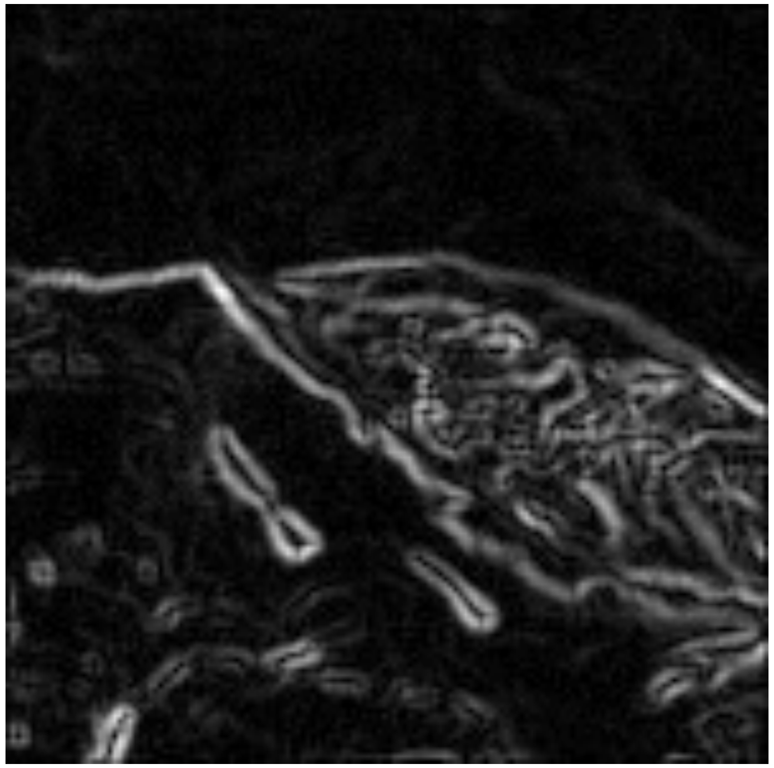
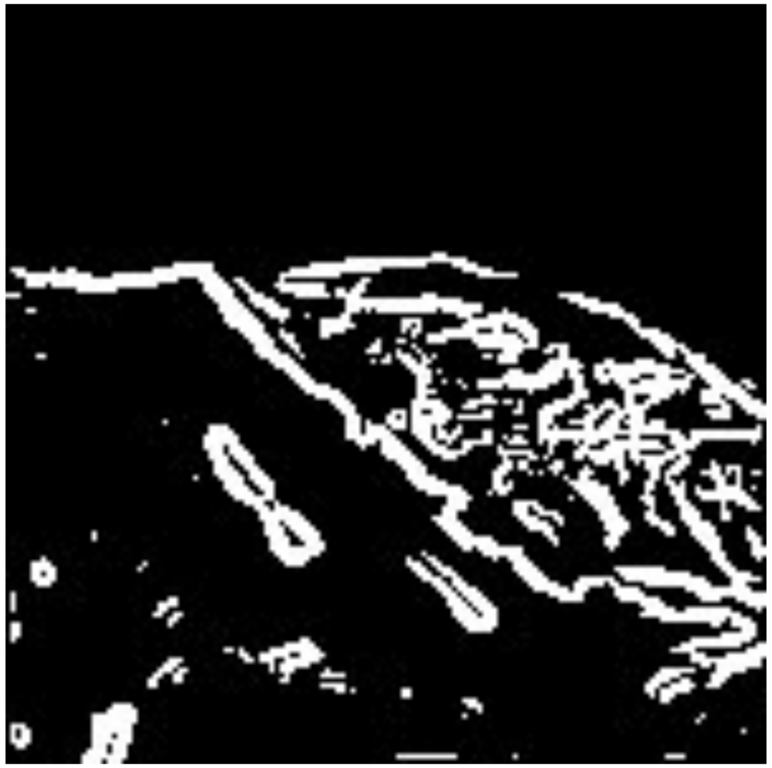
Where is the boundary of the mountain top?

Lines are hard to find

Original image



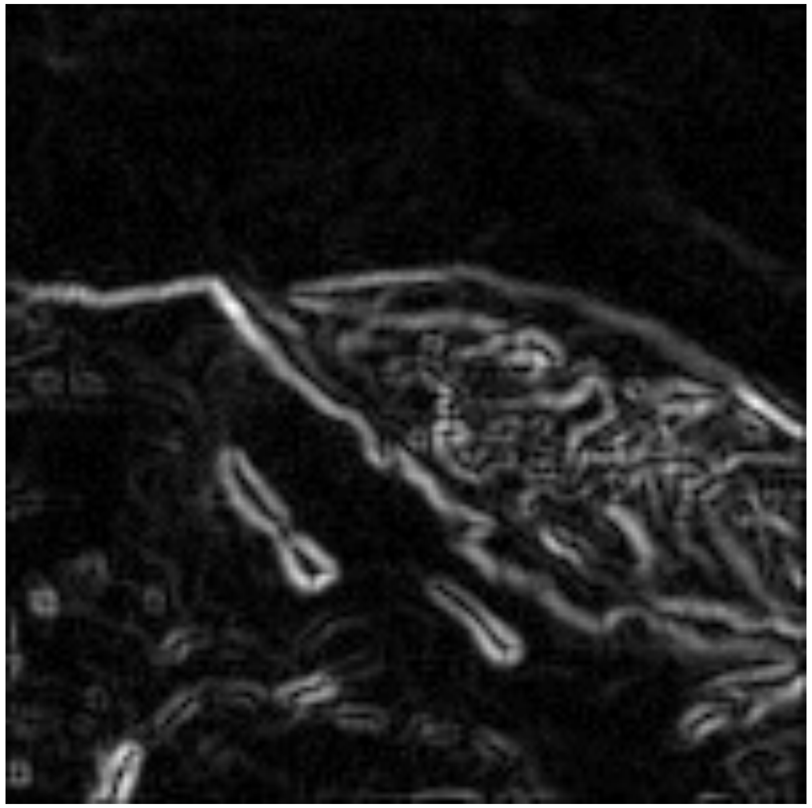
Edge detection



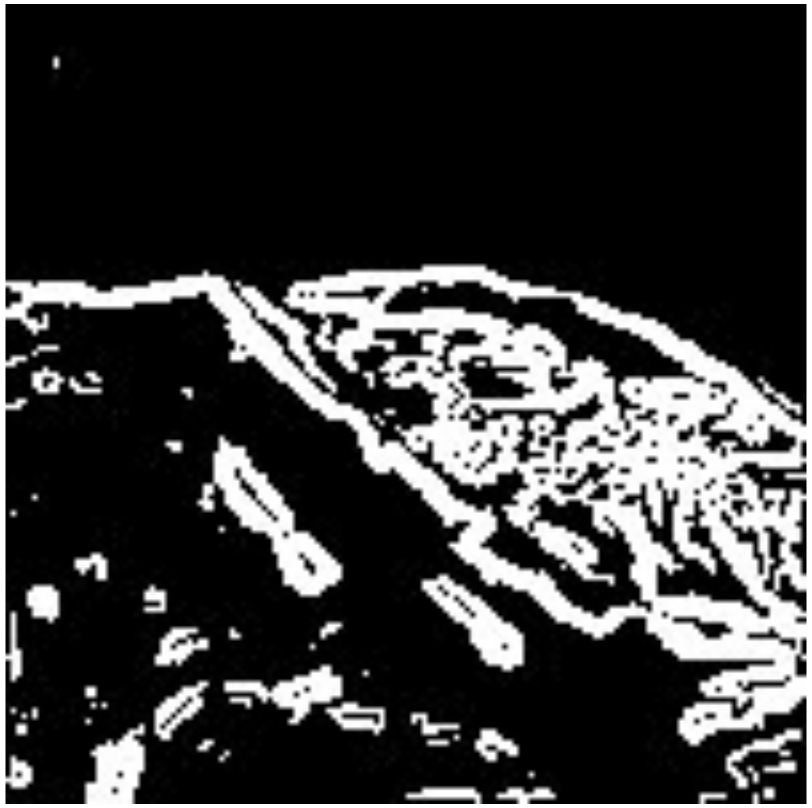
Thresholding

Noisy edge image
Incomplete boundaries

idea #1: morphology



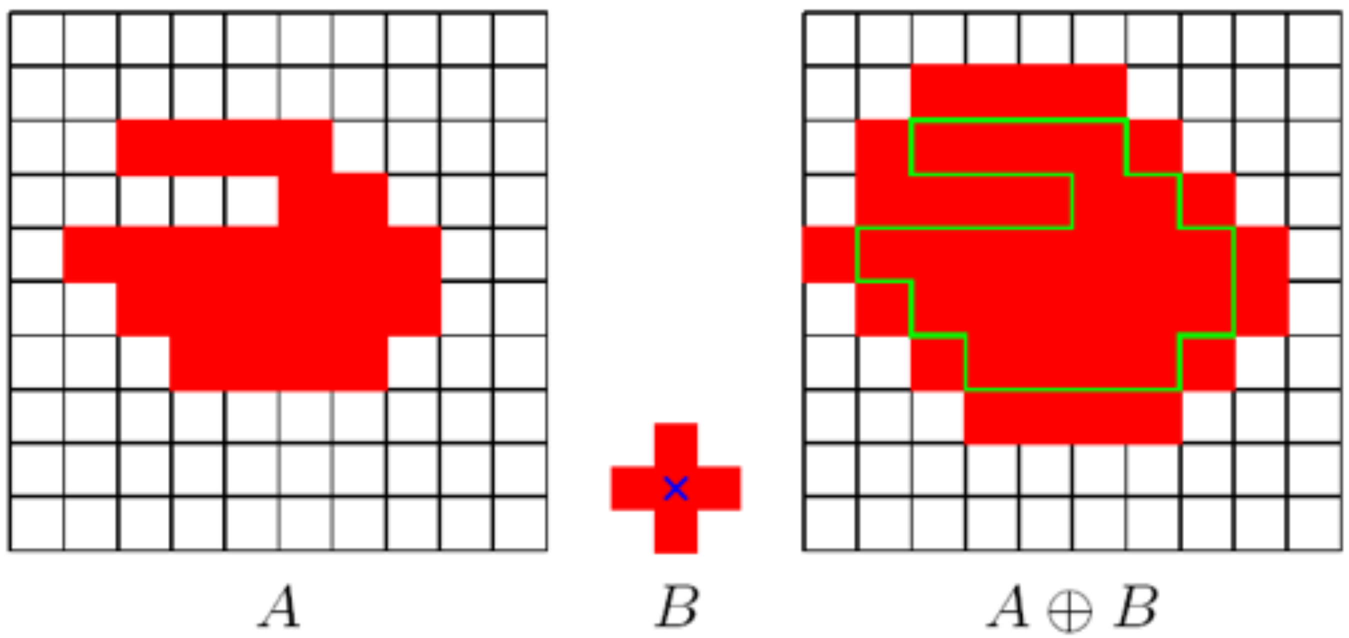
Sobel



Threshold

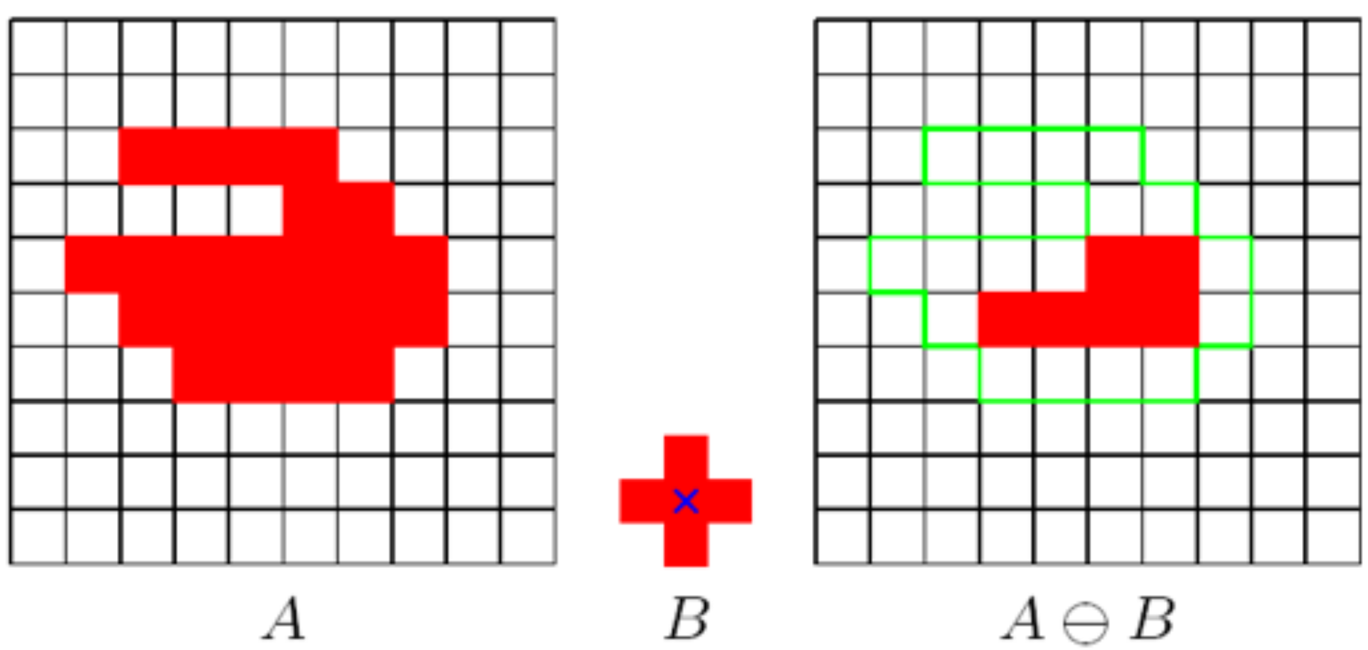
Morphology
(shrink, expand, shrink)

What are some problems with the approach?



Dilation

If filter response > 0, set to 1



Erosion

If filter response is MAX, set to 1

idea #2: breaking lines

Divide and Conquer:

Given: Boundary lies between points A and B

Task: Find boundary

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Connect A and B with Line
  Find strongest edge along line bisector
  Use edge point as break point
Repeat
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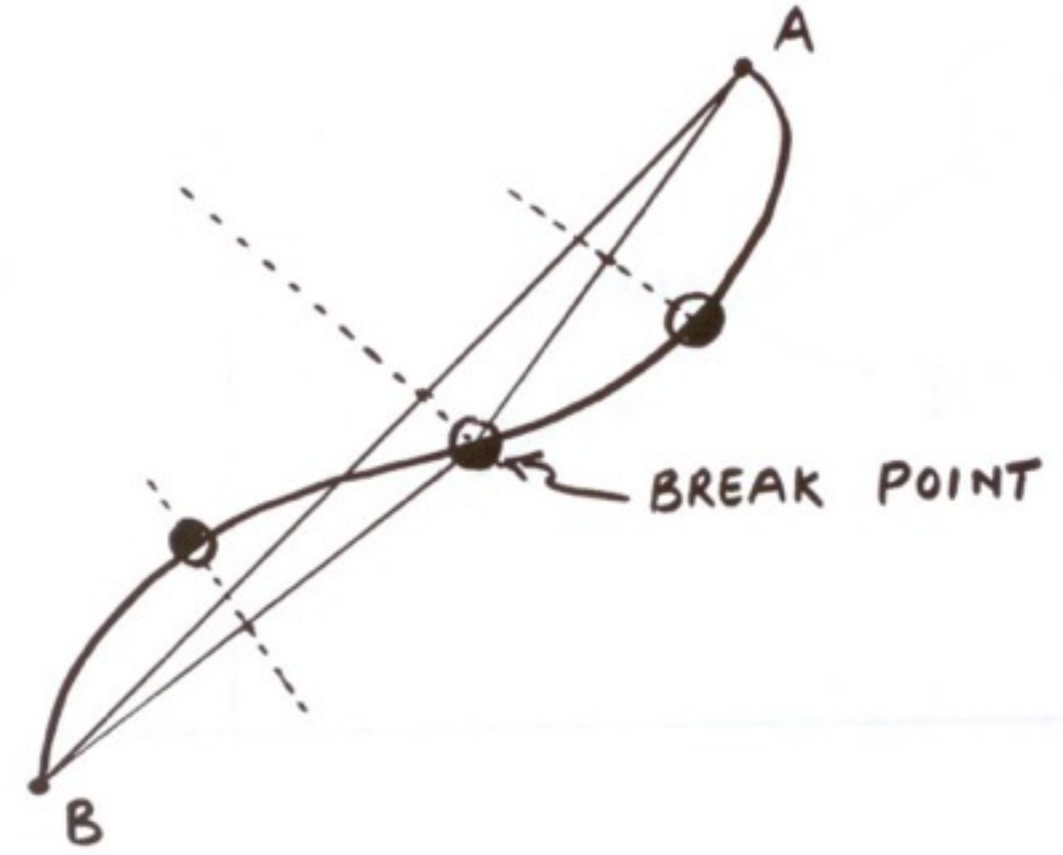


What are some problems with the approach?

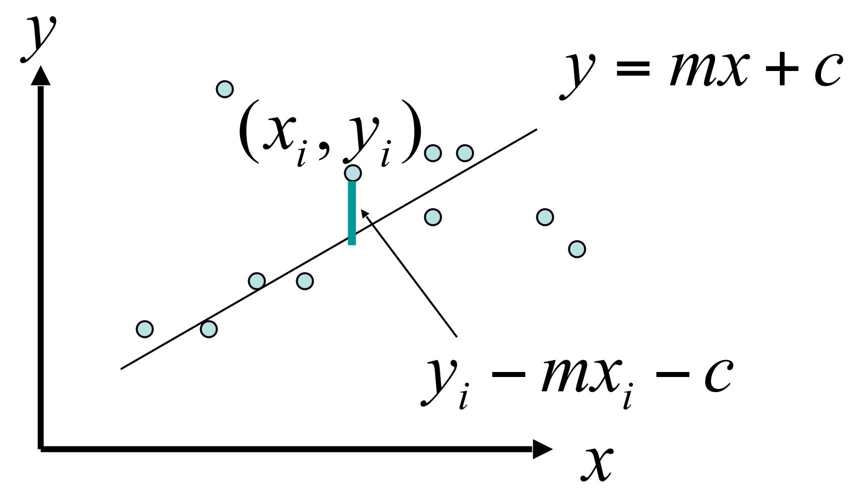
idea #3: line fitting

Given: Many (x_i, y_i) pairs

Find: Parameters (m, c)

Minimize: Average square distance:

$$E = \sum_i \frac{(y_i - mx_i - c)^2}{N}$$



What are some problems with the approach?

idea #3: line fitting

Given: Many (x_i, y_i) pairs

Find: Parameters (m, c)

Minimize: Average square distance:

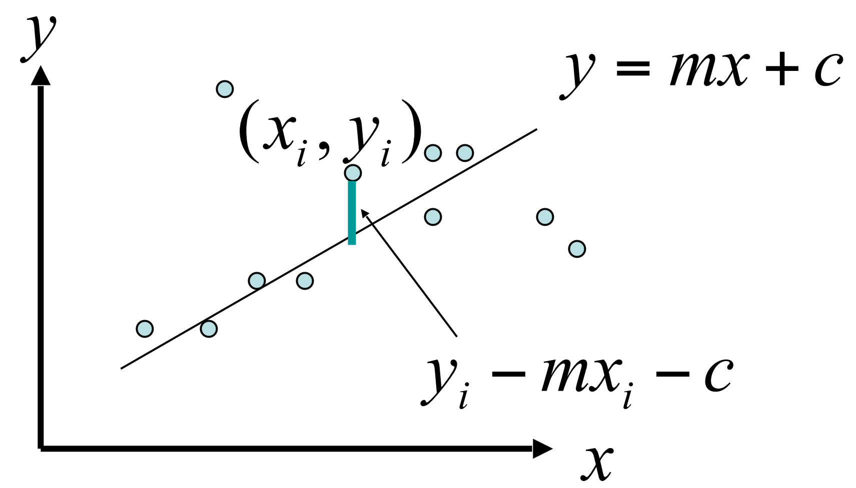
$$E = \sum_i \frac{(y_i - mx_i - c)^2}{N}$$

Using:

$$\frac{\partial E}{\partial m} = 0 \quad \& \quad \frac{\partial E}{\partial c} = 0$$

Note:

$$\bar{y} = \frac{\sum_i y_i}{N} \quad \bar{x} = \frac{\sum_i x_i}{N}$$



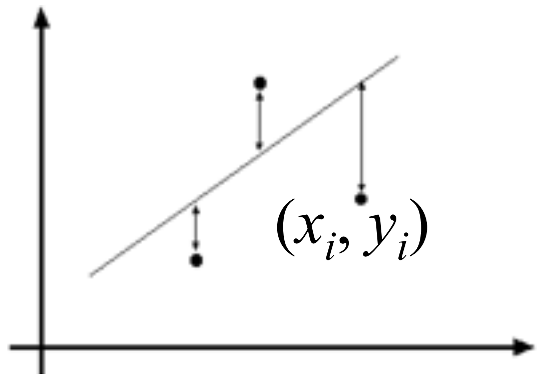
$$c = \bar{y} - m\bar{x}$$

$$m = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

What are some problems with the approach?

Data: $(x_1, y_1), \ldots, (x_n, y_n)$

Line equation: $y_i = m\, x_i + b$



Find (m, b) to minimize $E = \sum_{i=1}^{n}(y_i - mx_i - b)^2$

$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \qquad X = \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \qquad B = \begin{bmatrix} m \\ b \end{bmatrix}$$

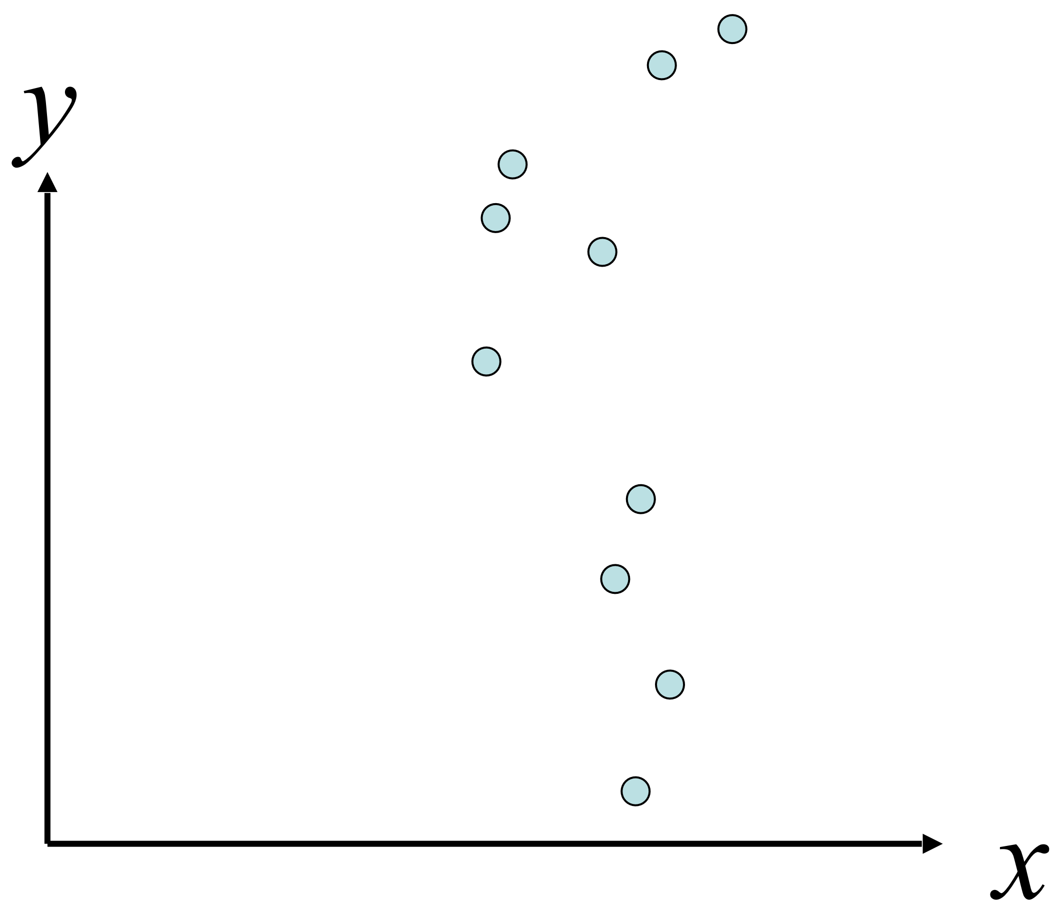
$$E = \|Y - XB\|^2 = (Y - XB)^T (Y - XB) = Y^T Y - 2(XB)^T Y + (XB)^T (XB)$$

$$\frac{dE}{dB} = 2X^T XB - 2X^T Y = 0 \qquad\qquad X^T XB = X^T Y$$

Normal equations: least squares solution to $XB=Y$

Problems with parameterizations

Where is the line that minimizes E? $E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$



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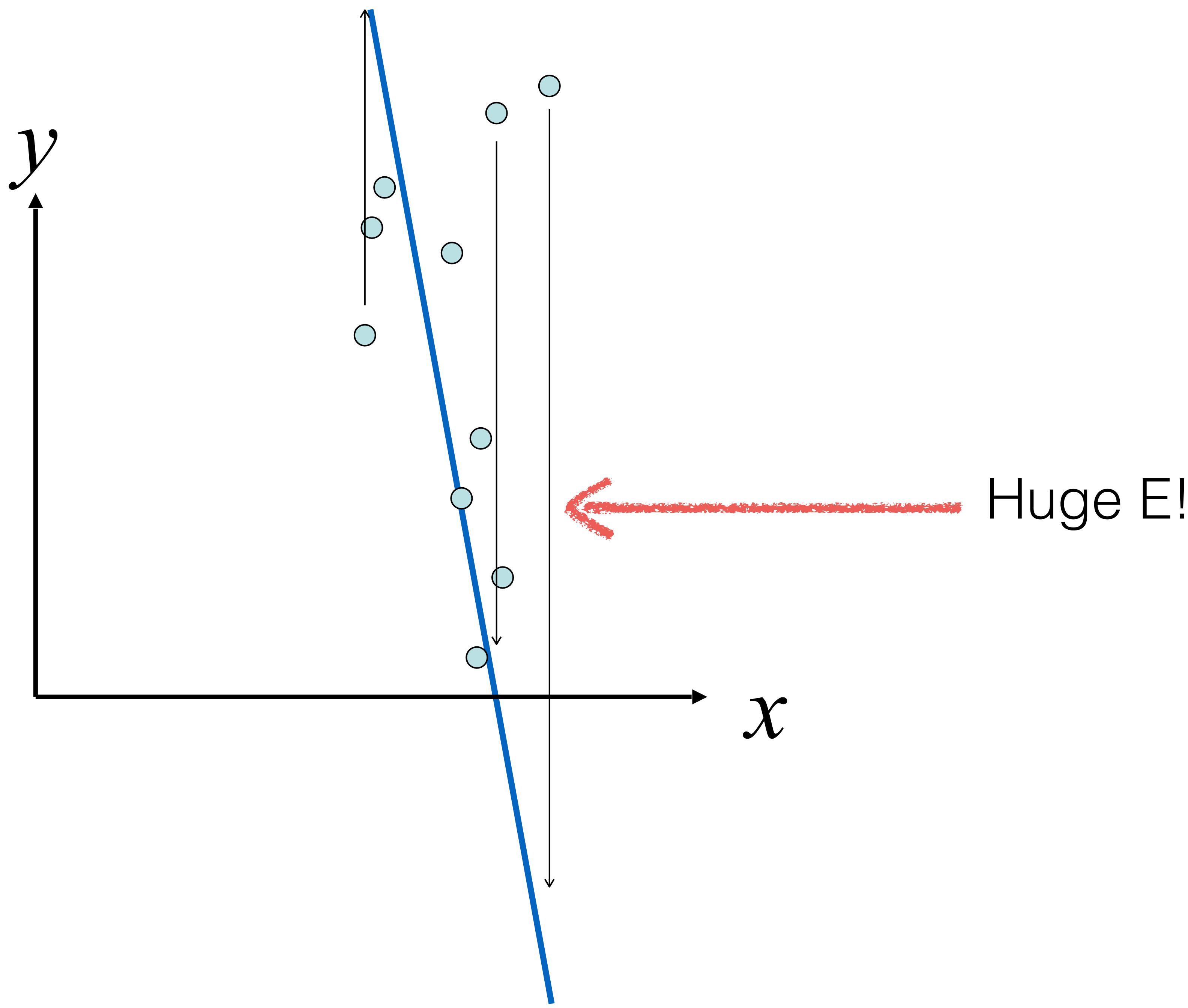

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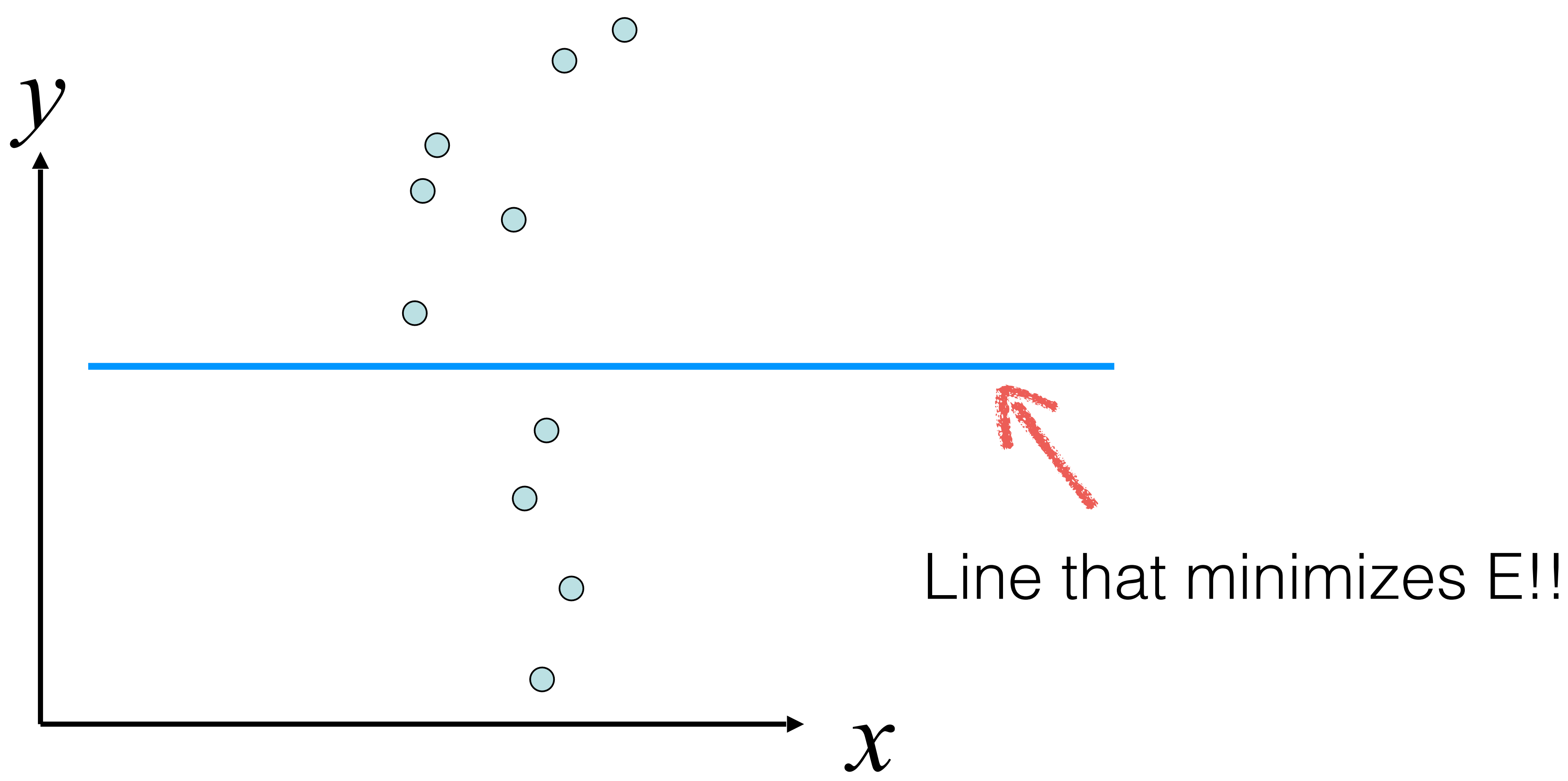

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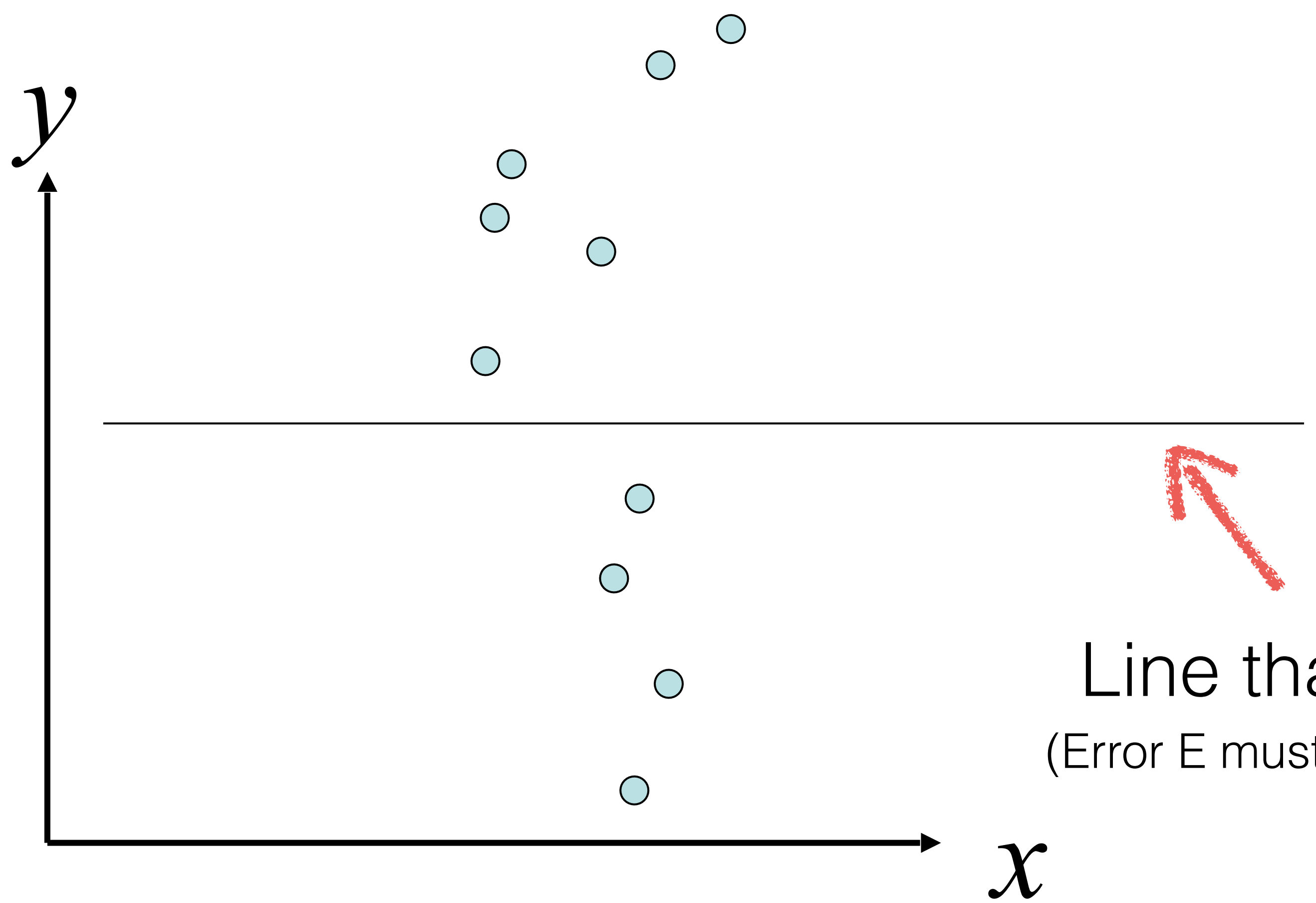


Line that minimizes E!!

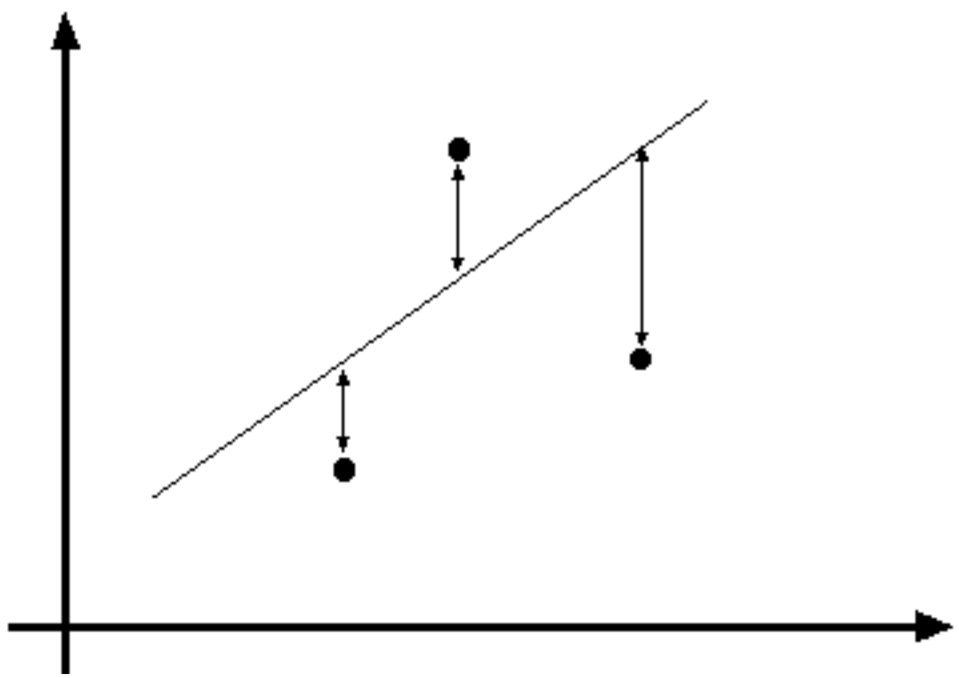
(Error E must be formulated carefully!)

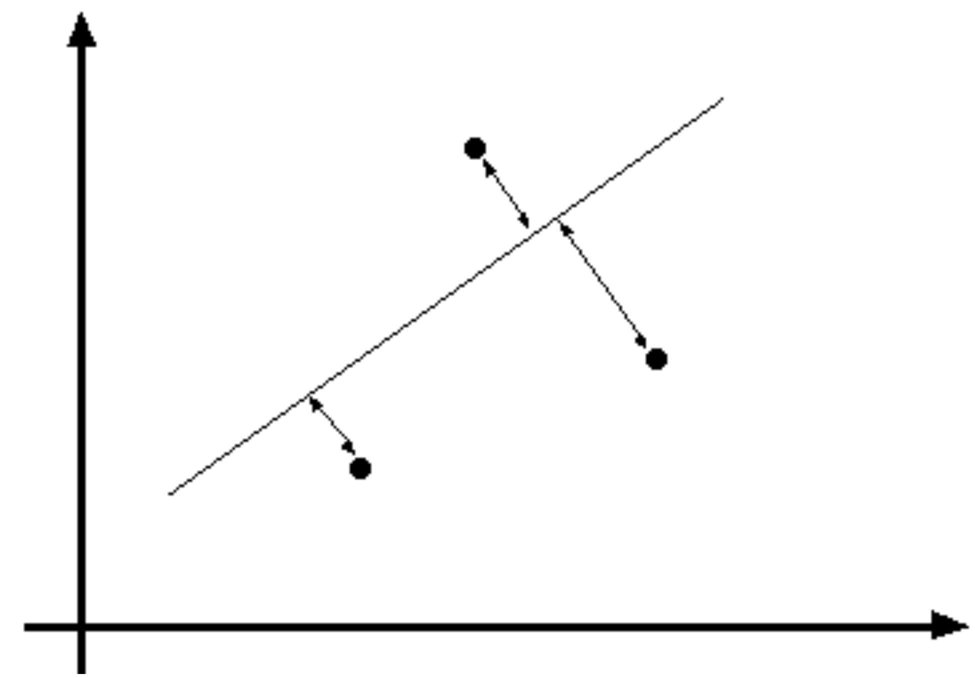
Use this instead:

$$E = \frac{1}{N} \sum_i (\rho - x_i \cos\theta + y_i \sin\theta)^2$$

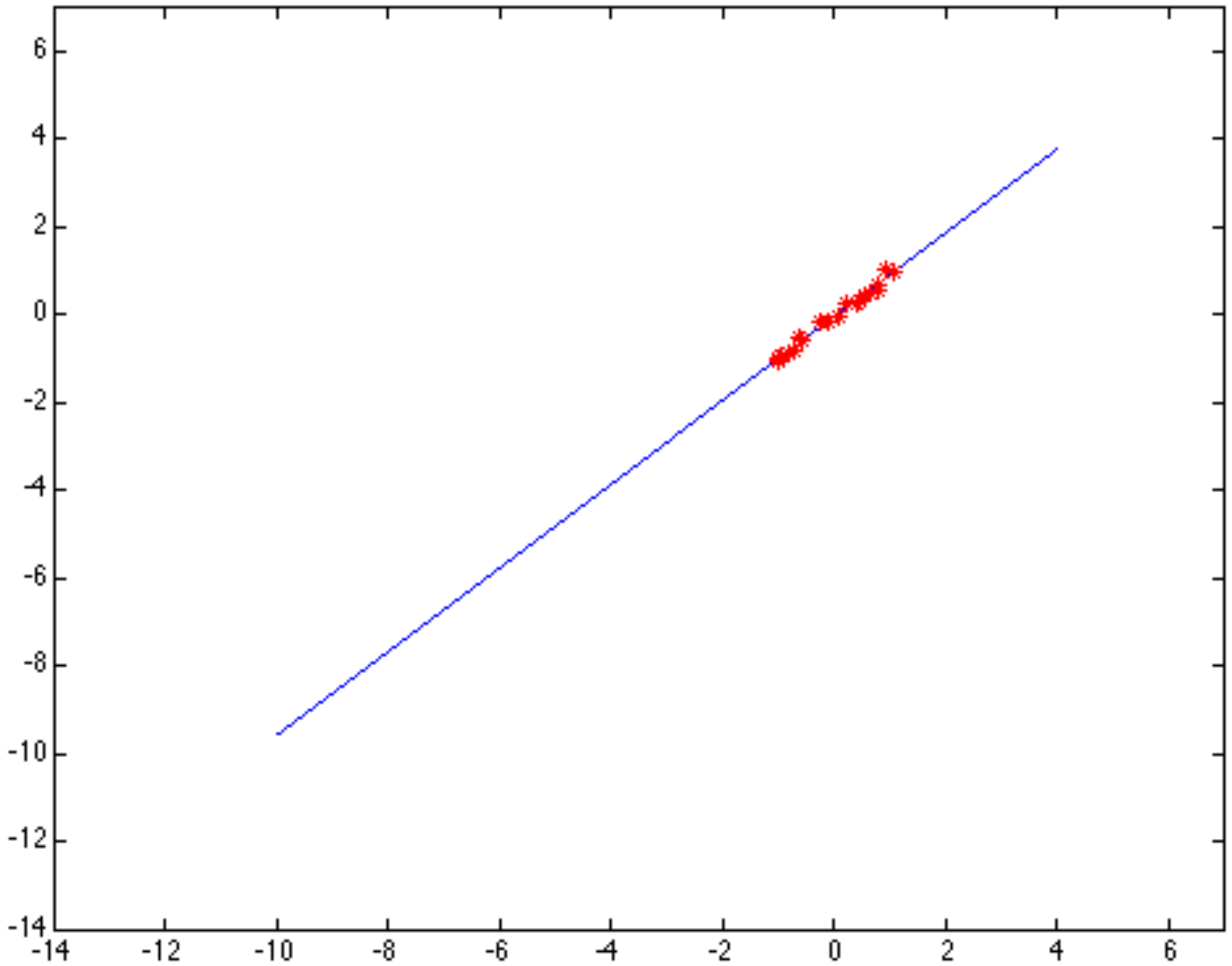
I'll explain this later …

Line fitting is easily setup as a maximum likelihood problem
… but choice of model is important

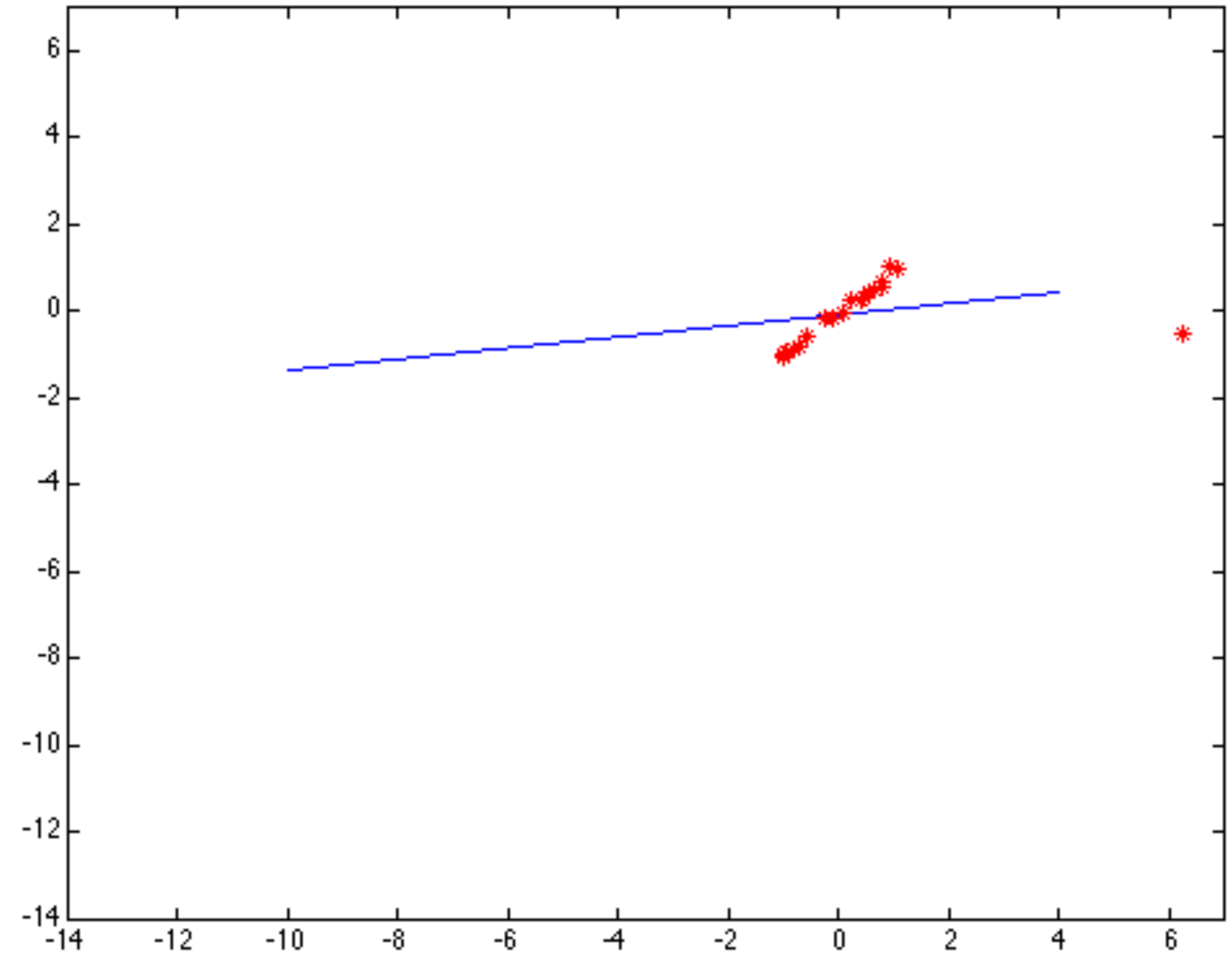




Problems with noise



Least-squares error fit



Squared error heavily penalizes outliers

Model fitting is difficult because…

- **Extraneous data:** clutter or multiple models
  - We do not know what is part of the model?
  - Can we pull out models with a few parts from much larger amounts of background clutter?
- **Missing data:** only some parts of model are present
- **Noise**

- Cost:
  - It is not feasible to check all combinations of features by fitting a model to each possible subset

So what can we do?