$$\mathbf{F}$$

Fundamental Matrix

16-385 Computer Vision (Kris Kitani)
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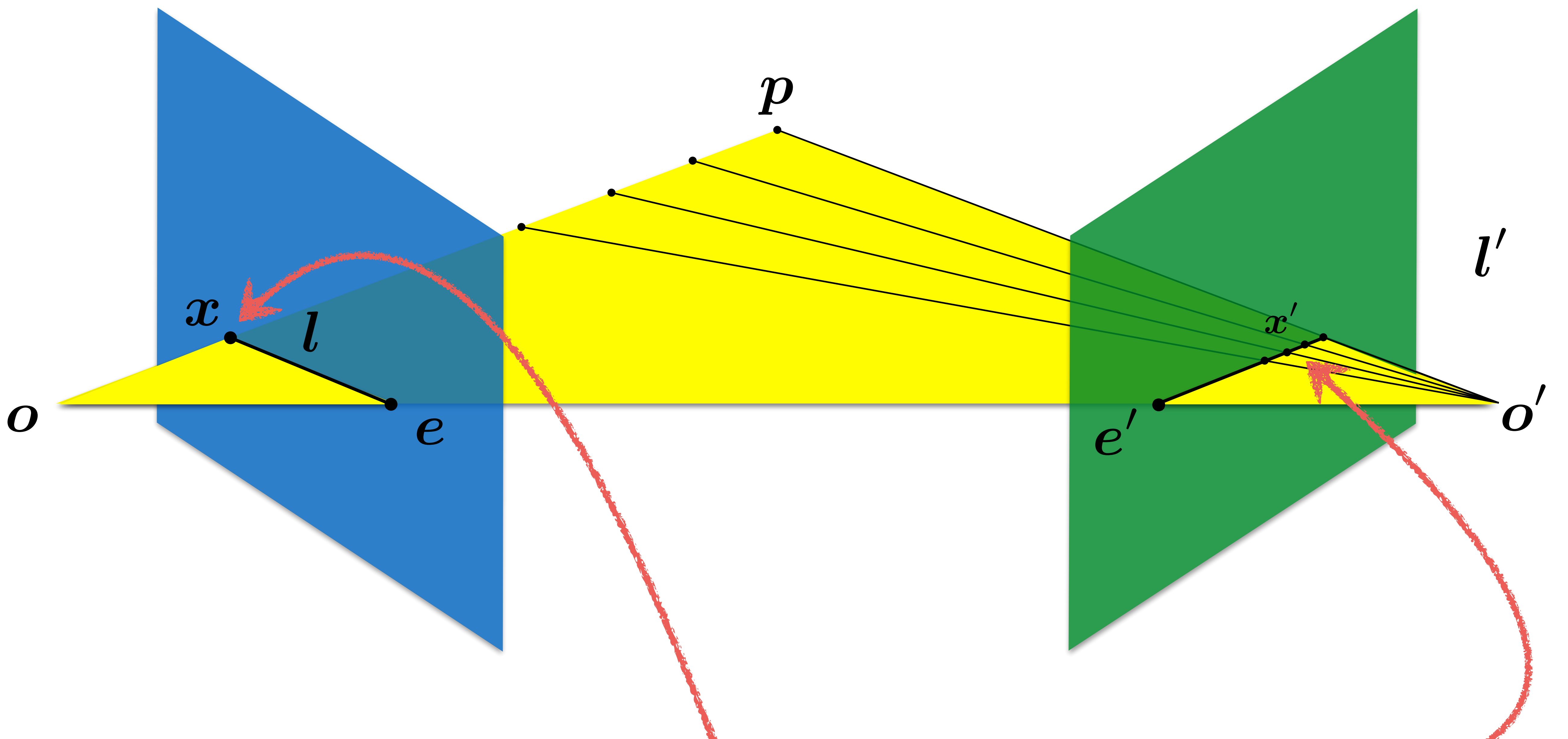
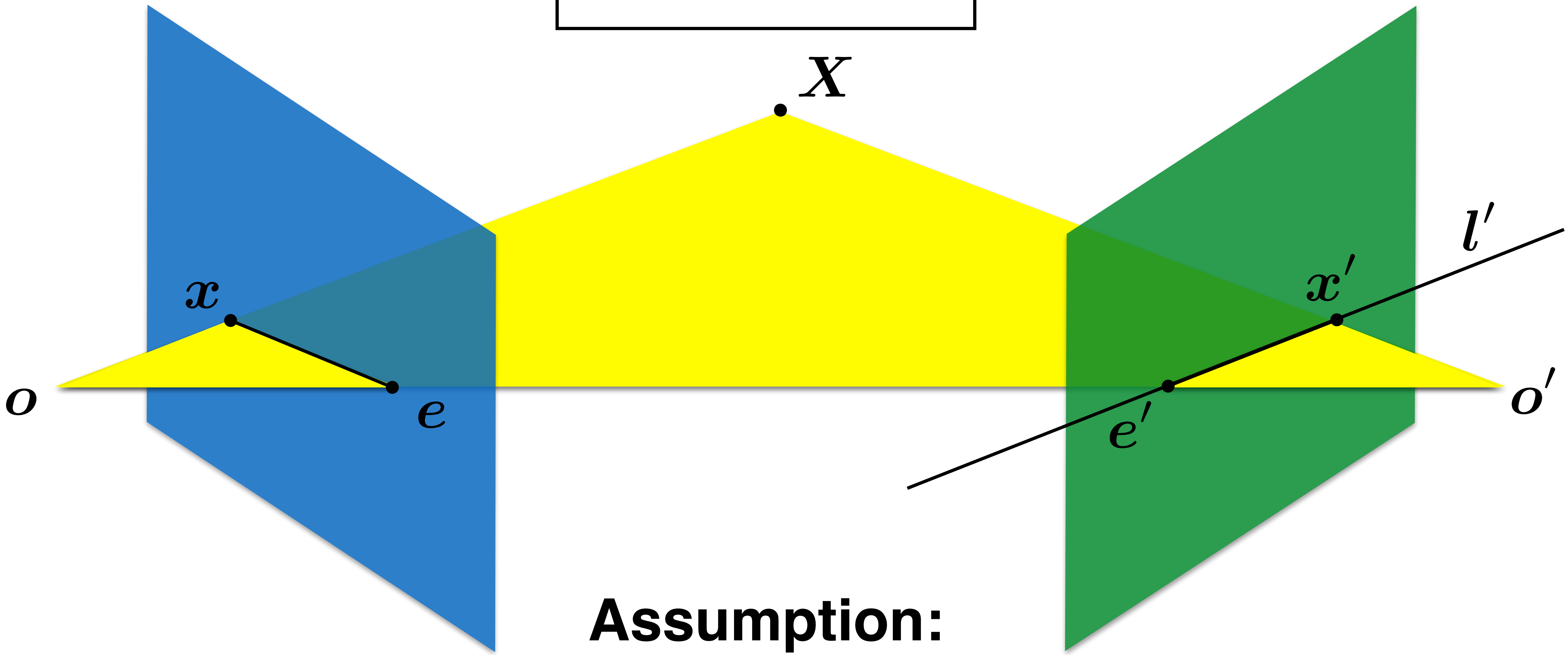
Recall:Epipolar constraint



Potential matches for $\boldsymbol{x}$ lie on the epipolar line $\boldsymbol{l'}$

Given a point in one image,
multiplying by the **essential matrix** will tell us
the **epipolar line** in the second view.

$$\mathbf{E}\boldsymbol{x} = \boldsymbol{l}'$$



Assumption:
points aligned to camera coordinate axis (calibrated camera)

How do you generalize to uncalibrated cameras?

The

Fundamental matrix

is a

generalization

of the

Essential matrix,

where the assumption of

calibrated cameras

is removed

$$\hat{\boldsymbol{x}}'^{\top} \mathbf{E} \hat{\boldsymbol{x}} = 0$$

The Essential matrix operates on image points expressed in
normalized coordinates
(points have been aligned (normalized) to camera coordinates)

$$\hat{\boldsymbol{x}}' = \mathbf{K}^{-1} \boldsymbol{x}'$$

$$\hat{\boldsymbol{x}} = \mathbf{K}^{-1} \boldsymbol{x}$$

camera point

image point

$$\hat{\boldsymbol{x}}'^{\top}\mathbf{E}\hat{\boldsymbol{x}} = 0$$

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camera point — image point

Writing out the epipolar constraint in terms of image coordinates

$$\boldsymbol{x}'^{\top}\mathbf{K}'^{-\top}\mathbf{E}\mathbf{K}^{-1}\boldsymbol{x} = 0$$

$$\boldsymbol{x}'^{\top}(\mathbf{K}'^{-\top}\mathbf{E}\mathbf{K}^{-1})\boldsymbol{x} = 0$$

$$\boldsymbol{x}'^{\top}\mathbf{F}\boldsymbol{x} = 0$$

Same equation works in image coordinates!

$$\boldsymbol{x}'^\top \mathbf{F} \boldsymbol{x} = 0$$

it maps pixels to epipolar lines

properties of the ~~E~~ F matrix

Longuet-Higgins equation	$\boldsymbol{x}'^\top \mathbf{F} \boldsymbol{x} = 0$	
Epipolar lines	$\boldsymbol{x}^\top \boldsymbol{l} = 0$	$\boldsymbol{x}'^\top \boldsymbol{l}' = 0$
	$\boldsymbol{l}' = \mathbf{F} \boldsymbol{x}$	$\boldsymbol{l} = \mathbf{F}^T \boldsymbol{x}'$
Epipoles	$\boldsymbol{e}'^\top \mathbf{F} = \mathbf{0}$	$\mathbf{F} \boldsymbol{e} = \mathbf{0}$

(points in **image** coordinates)

Breaking down the fundamental matrix

$$\mathbf{F} = \mathbf{K}'^{-\top}\mathbf{E}\mathbf{K}^{-1}$$

$$\mathbf{F} = \mathbf{K}'^{-\top}[\mathbf{t}_{\times}]\mathbf{R}\mathbf{K}^{-1}$$

Depends on both intrinsic and extrinsic parameters

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Depends on both intrinsic and extrinsic parameters

How would you solve for F?

$$\boldsymbol{x}'^{\top}_m \mathbf{F} \boldsymbol{x}_m = 0$$

The 8 Point Algorithm…