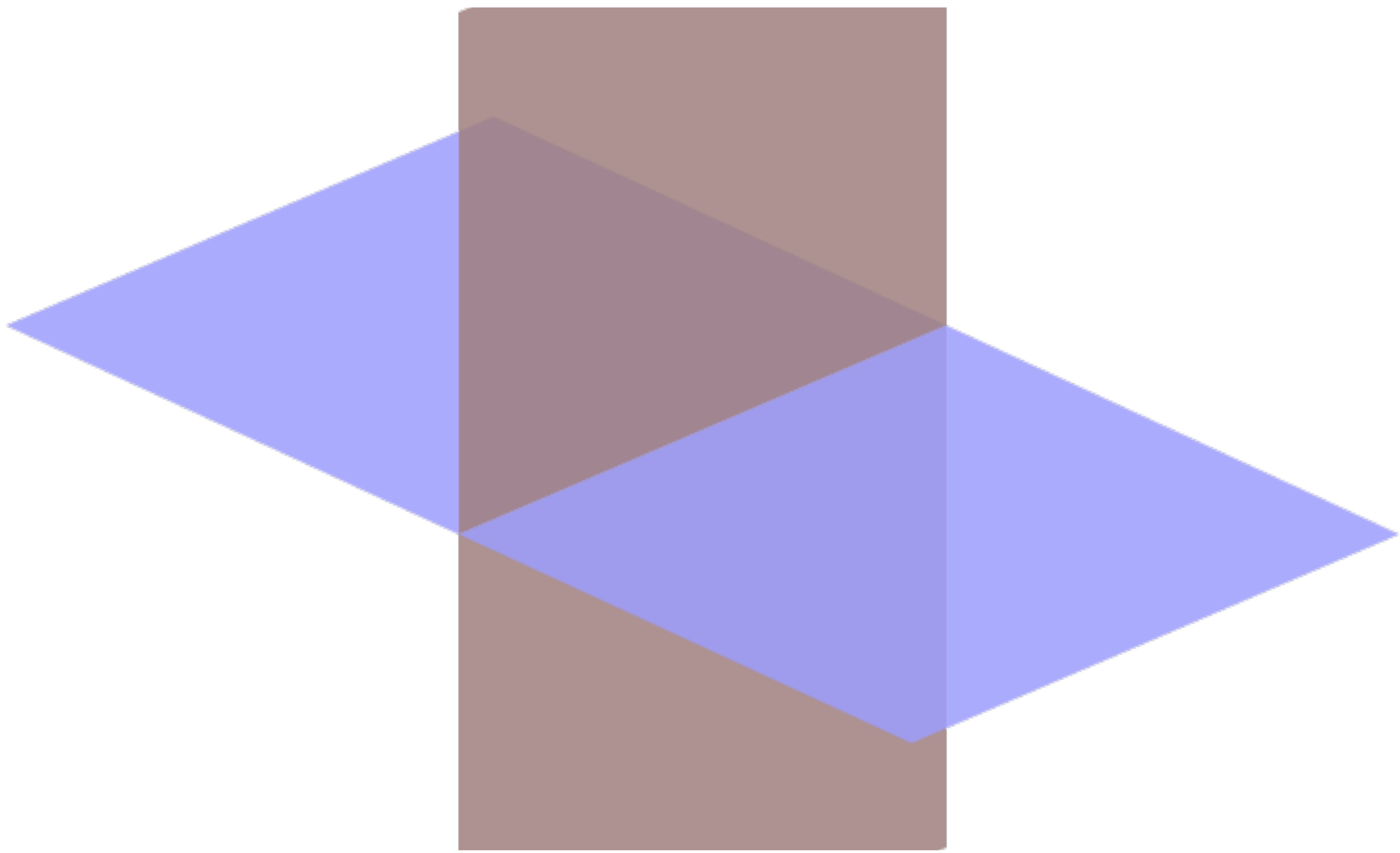

Direct Linear Transform

16-385 Computer Vision (Kris Kitani)
Carnegie Mellon University

We want to estimate the transformation between points…

Do you notice anything about these point correspondences?

We want to estimate the transformation between points…

The 3D transformation of coplanar points can be described by a **projective transform**
(will NOT work for non-coplanar points)

Projective Transform (Homography)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \mathbf{H} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

homography $\quad \mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$

parameters of the transform

Given a set of matched feature points

$$\{\boldsymbol{x}_i, \boldsymbol{x}'_i\}$$

point in one image | point in the other image

and a transformation

$$\boldsymbol{x}' = \boldsymbol{f}(\boldsymbol{x}; \boldsymbol{p})$$

projective transform (homography)

Find the best estimate of

$$\boldsymbol{p}$$

Given

$$\{\boldsymbol{x}_i, \boldsymbol{x}'_i\}$$

how do you solve for the parameters?

parameters

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

How do you deal with this?

Given

$$\{\boldsymbol{x}_i, \boldsymbol{x}'_i\}$$

how do you solve for the parameters?

parameters

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

How do you deal with this?

Direct Linear Transform

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Remove scale factor and get it in a linear form

(rewrite similarity equations as homogenous linear equation and solve → DLT)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Remove scale factor and get it in a linear form

(rewrite similarity equations as homogenous linear equation and solve → DLT)

Multiplied out

$$x' = \alpha(h_1 x + h_2 y + h_3)$$
$$y' = \alpha(h_4 x + h_5 y + h_6)$$
$$1 = \alpha(h_7 x + h_8 y + h_9)$$

Divide out unknown scale factor (divide line 1 and 2 by 3)

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Divide out unknown scale factor (divide line 1 and 2 by 3)

$$x'(h_7 x + h_8 y + h_9) = (h_1 x + h_2 y + h_3)$$

$$y'(h_7 x + h_8 y + h_9) = (h_4 x + h_5 y + h_6)$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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How do you rearrange terms to make it a linear system of equations?

$$x'(h_7x + h_8y + h_9) = (h_1x + h_2y + h_3)$$
$$y'(h_7x + h_8y + h_9) = (h_4x + h_5y + h_6)$$

Just rearrange the terms

$$h_7xx' + h_8yx' + h_9x' - h_1x - h_2y - h_3 = 0$$
$$h_7xy' + h_8yy' + h_9y' - h_4x - h_5y - h_6 = 0$$

$$h_7xx' + h_8yx' + h_9x' - h_1x - h_2y - h_3 = 0$$
$$h_7xy' + h_8yy' + h_9y' - h_4x - h_5y - h_6 = 0$$

In matrix form:

$$\mathbf{A}_i \boldsymbol{h} = \mathbf{0}$$

$$\mathbf{A}_i = \left[\begin{array}{ccccccccc} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{array} \right]$$

$$\boldsymbol{h} = \left[\begin{array}{ccccccccc} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 & h_9 \end{array} \right]^\top$$

How many equations from one point correspondence?

$$\mathbf{A}\boldsymbol{h} = \mathbf{0}$$

$$\begin{bmatrix} \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \\ \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \\ \vdots \\ \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

'Homogeneous Linear Least Squares' problem

How do we solve this?

(usually have more constraints than variables)

$$\mathbf{A}\boldsymbol{h} = \mathbf{0}$$

$$\begin{bmatrix} \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \\ \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \\ \vdots \\ \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

‘Homogeneous Linear Least Squares’ problem

Solve with SVD!

Singular Value Decomposition

ortho-normal diagonal ortho-normal

unit norm constraint

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top$$

n x m   n x n   n x m   m x m

$$= \sum_{i=1}^{9} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^\top$$

n x 1   1 x m

Each column of V represents a solution for $\mathbf{A}\boldsymbol{h} = \mathbf{0}$

where the eigenvalues represents the reprojection error

Solving for H using DLT

Given $\{\boldsymbol{x}_i, \boldsymbol{x}'_i\}$ solve for H such that $\boldsymbol{x}' = \mathbf{H}\boldsymbol{x}$

1. For each correspondence, create 2x9 matrix $\mathbf{A}_i$

2. Concatenate into single 2n x 9 matrix $\mathbf{A}$

3. Compute SVD of $\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\top$

4. Store Eigenvector of the smallest Eigenvalue $\boldsymbol{h} = \boldsymbol{v}_{\hat{i}}$

5. Reshape to get $\mathbf{H}$

General form of total least squares

(**Warning:** change of notation. x is a vector of parameters!)

$$E_{\text{TLS}} = \sum_i (\boldsymbol{a}_i \boldsymbol{x})^2$$

$$= \|\mathbf{A}\boldsymbol{x}\|^2 \quad \text{(matrix form)}$$

$$\|\boldsymbol{x}\|^2 = 1 \quad \text{constraint}$$

minimize $\|\mathbf{A}\boldsymbol{x}\|^2$

subject to $\|\boldsymbol{x}\|^2 = 1$

→

(Rayleigh quotient)

minimize $\dfrac{\|\mathbf{A}\boldsymbol{x}\|^2}{\|\boldsymbol{x}\|^2}$

Solution is the eigenvector corresponding to smallest eigenvalue of

$$\mathbf{A}^\top \mathbf{A}$$

(equivalent)

Solution is the column of **V** corresponding to smallest singular value

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\top$$

LLS/SVD

doesn’t

deal well

with

outliers