

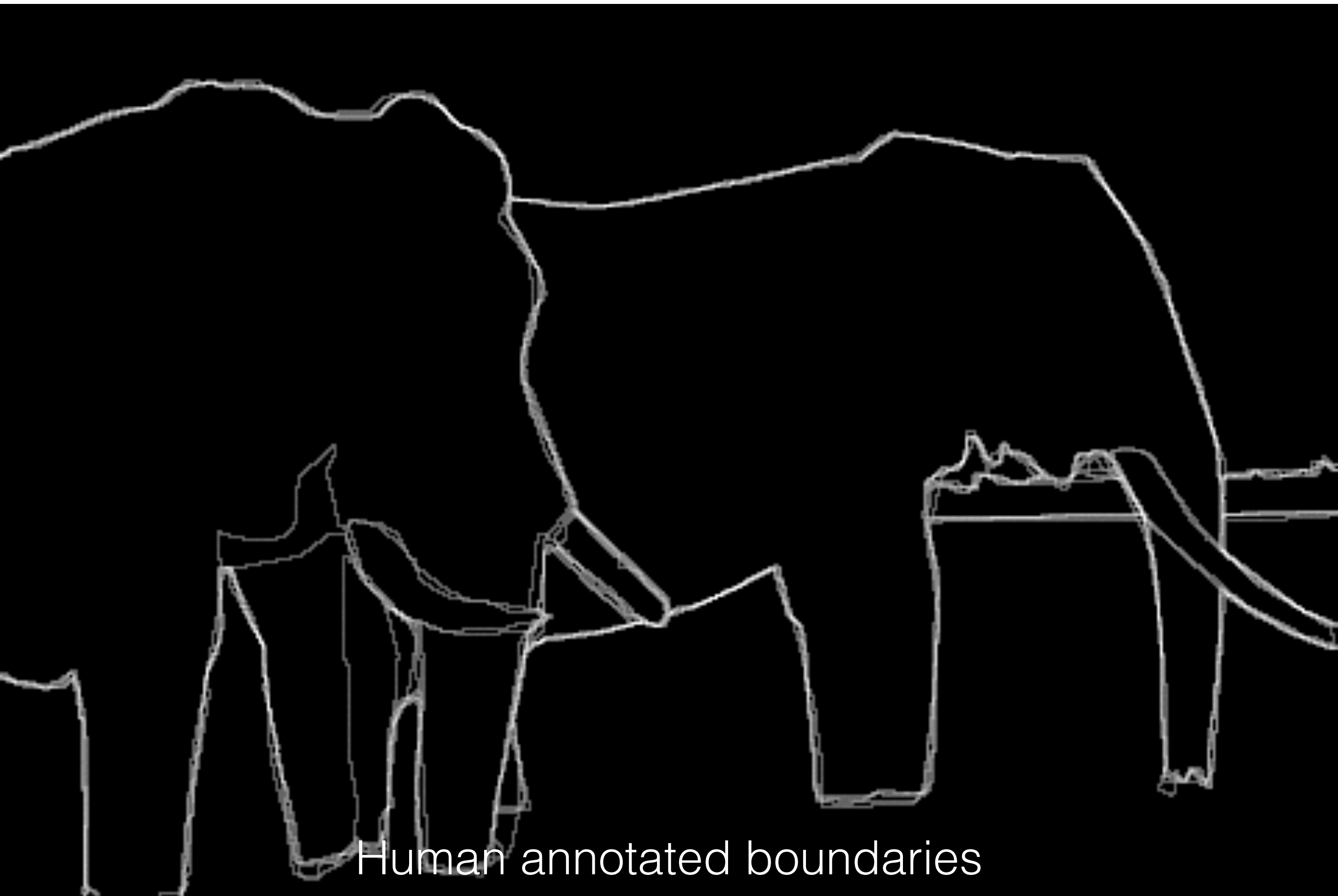


Defining boundaries

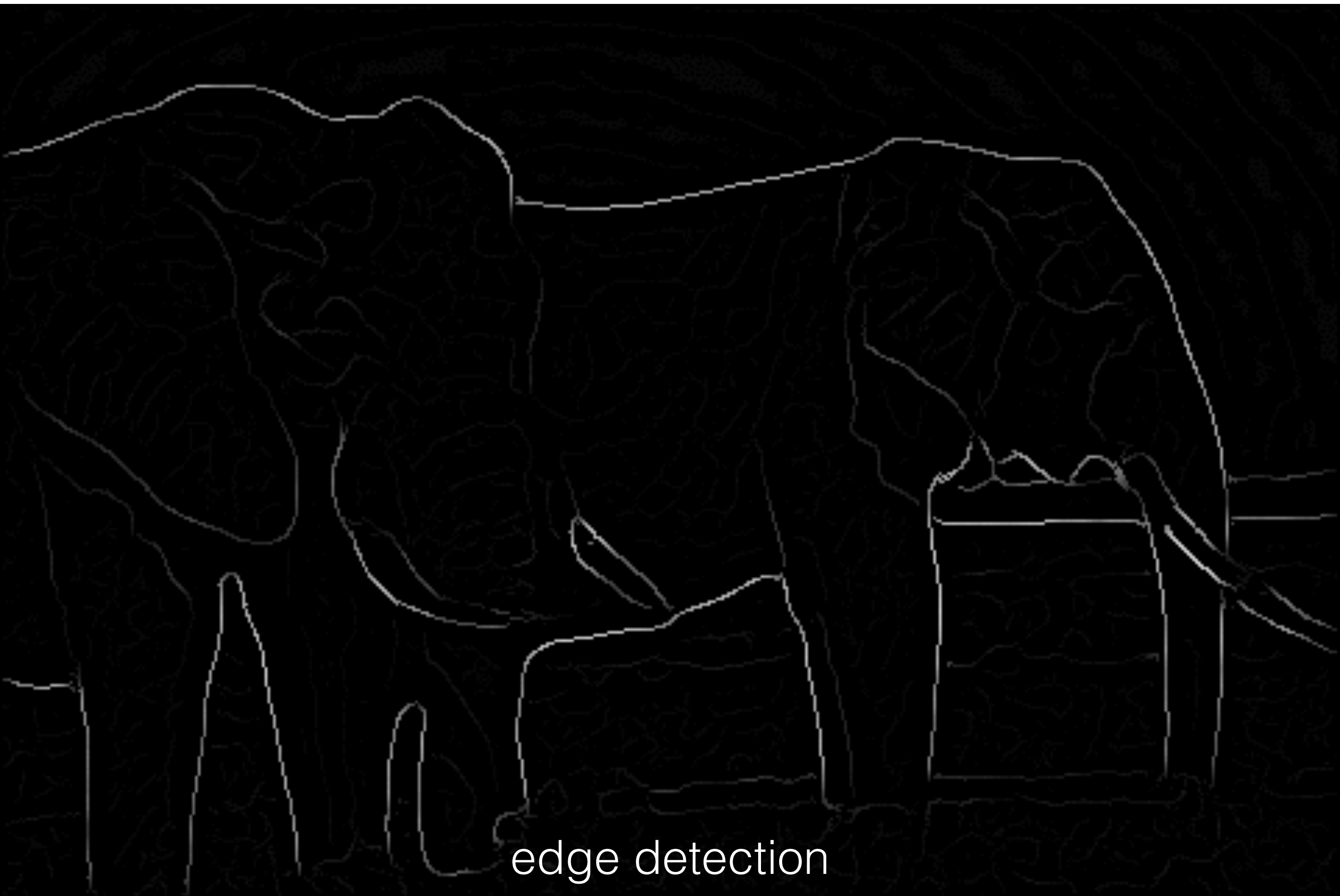
16-385 Computer Vision

Where are the object boundaries?

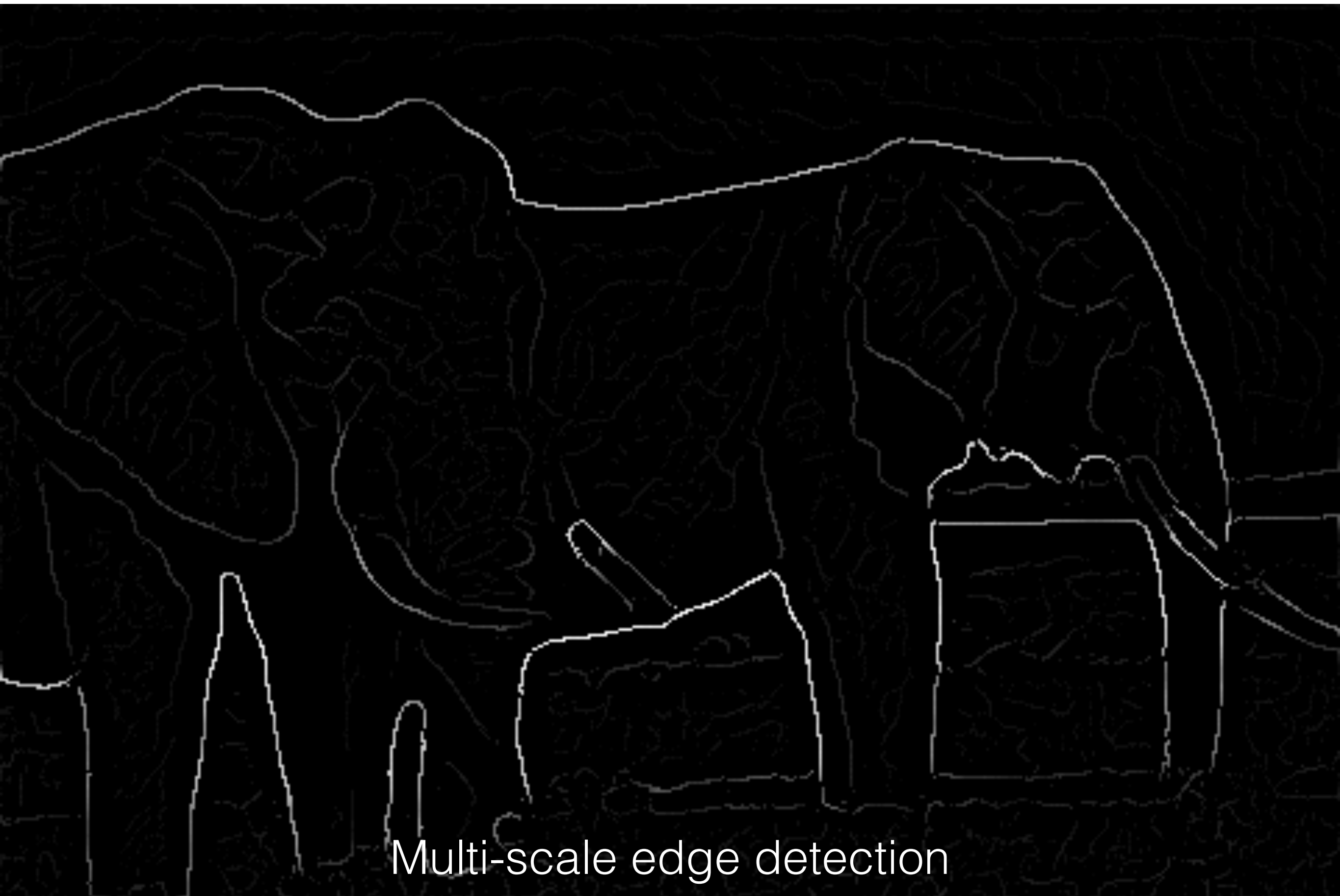




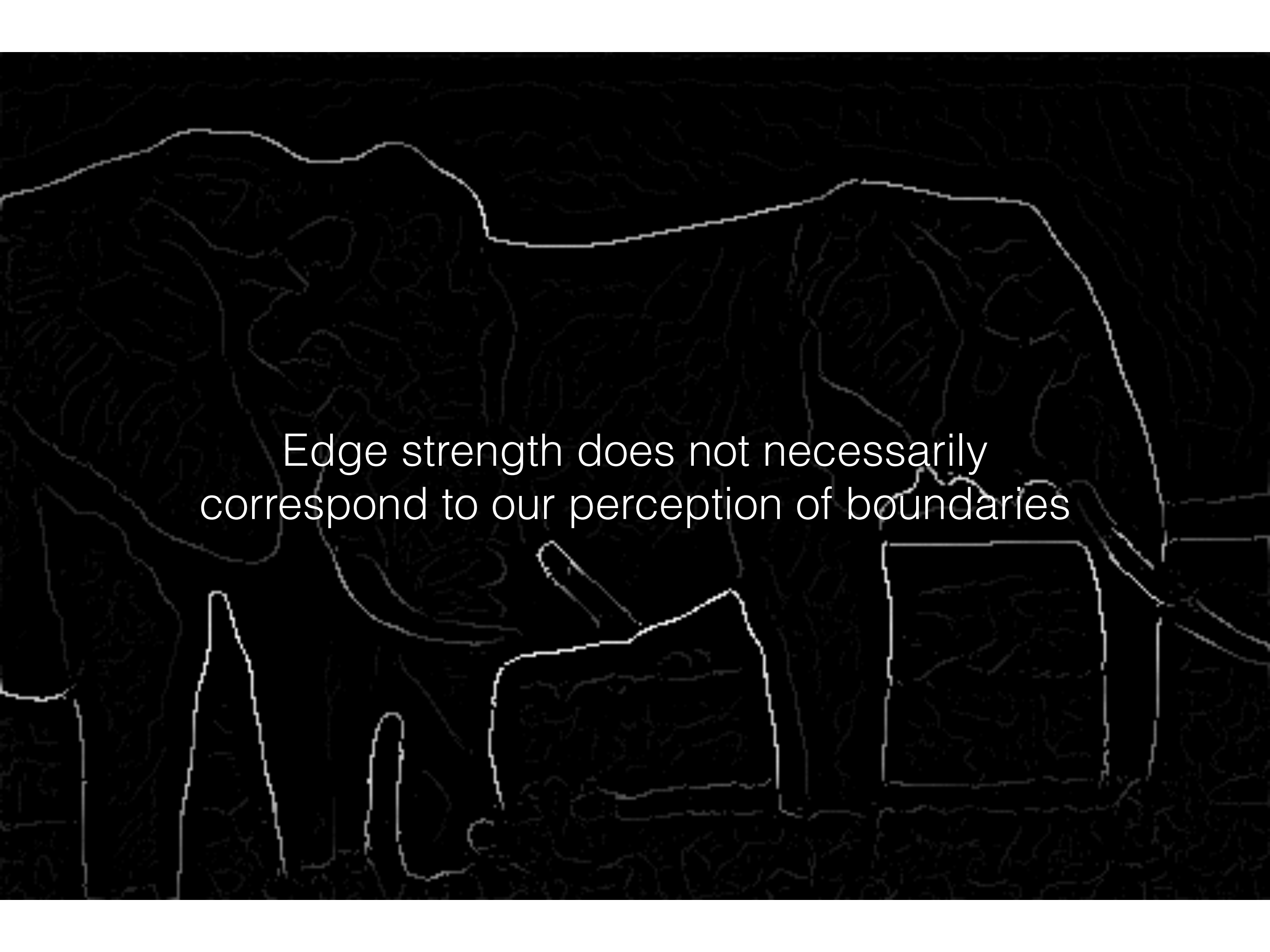
Human annotated boundaries



edge detection



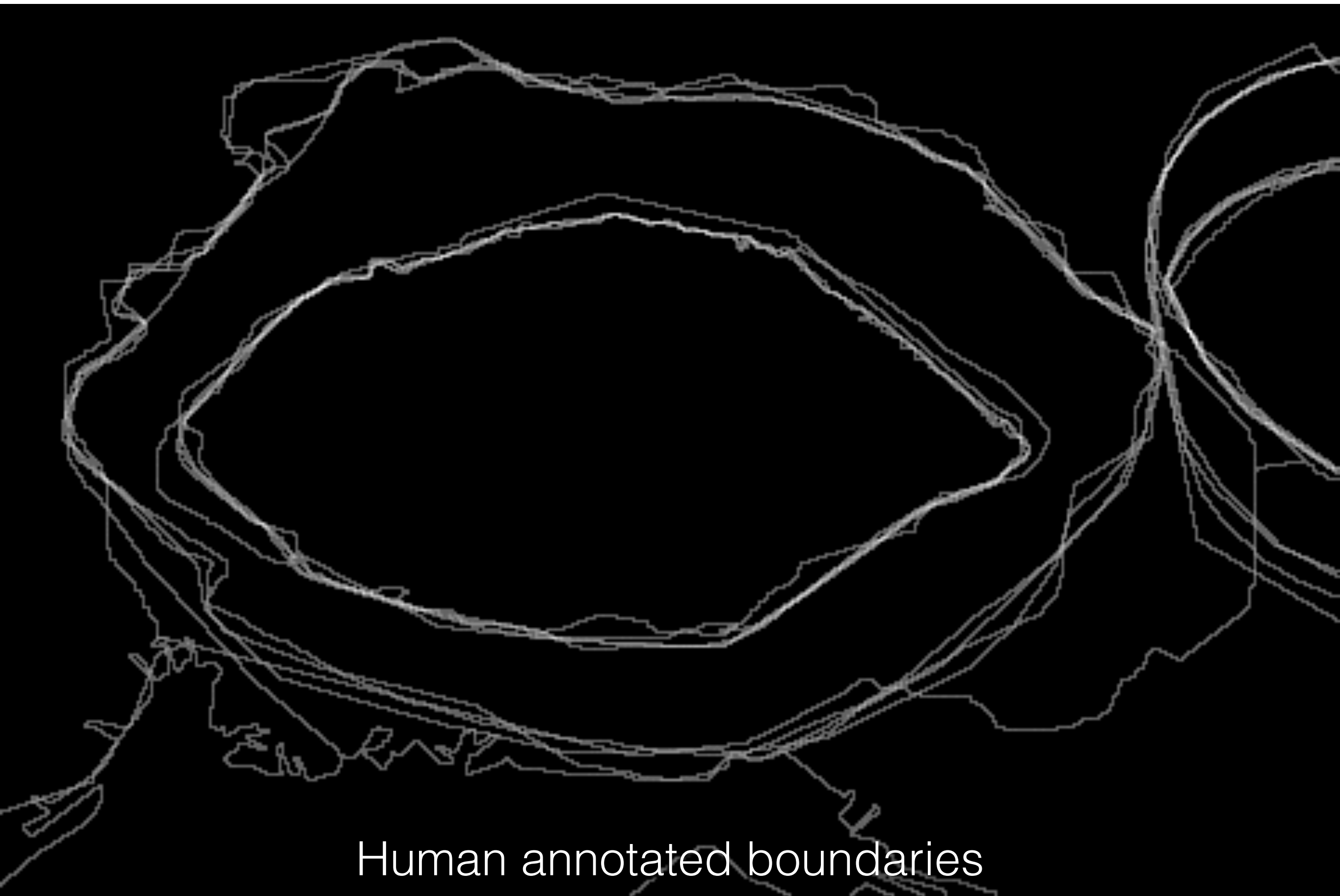
Multi-scale edge detection

The image shows a grayscale edge detection of a scene. The scene includes a large, dark, irregular shape on the left, a central area with some horizontal lines, and a large, light-colored rectangular shape on the right. The edges are highlighted in white against a black background. The text "Edge strength does not necessarily correspond to our perception of boundaries" is overlaid in the center of the image.

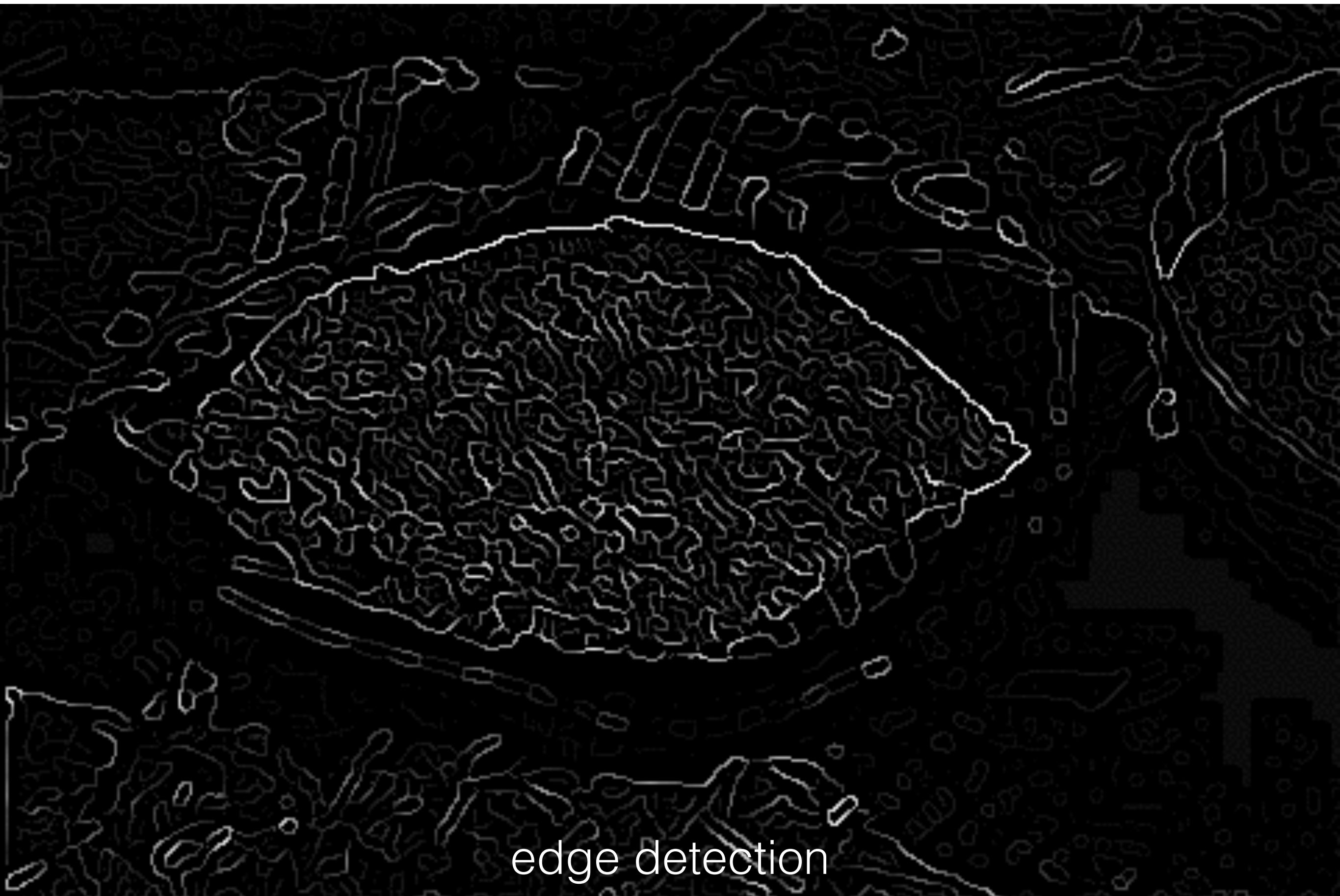
Edge strength does not necessarily
correspond to our perception of boundaries

Where are the object boundaries?





Human annotated boundaries



edge detection

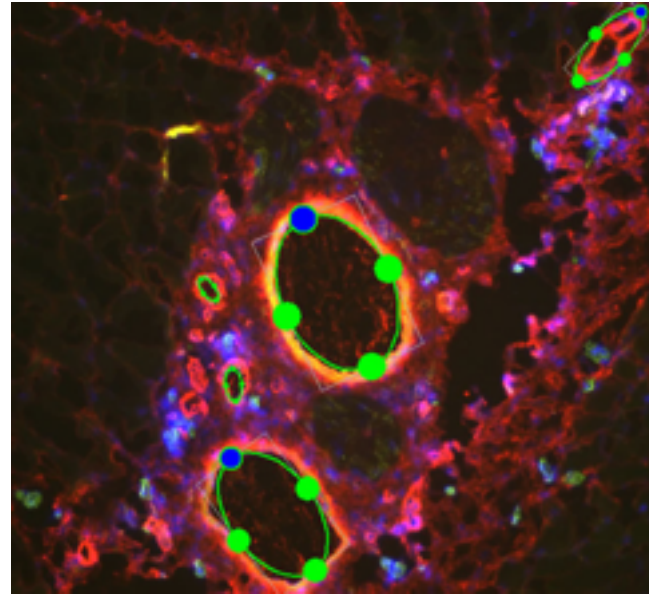
The image shows two sea anemones with long, thin, pinkish-white tentacles, positioned on a vibrant red surface. The red surface is densely populated with numerous small, dark, circular openings, likely the mouths of other marine organisms. The background is a dark, almost black, textured area. The text "Defining boundaries are hard for us too" is overlaid in white, sans-serif font across the middle of the image, centered between the two anemones.

Defining boundaries are hard for us too

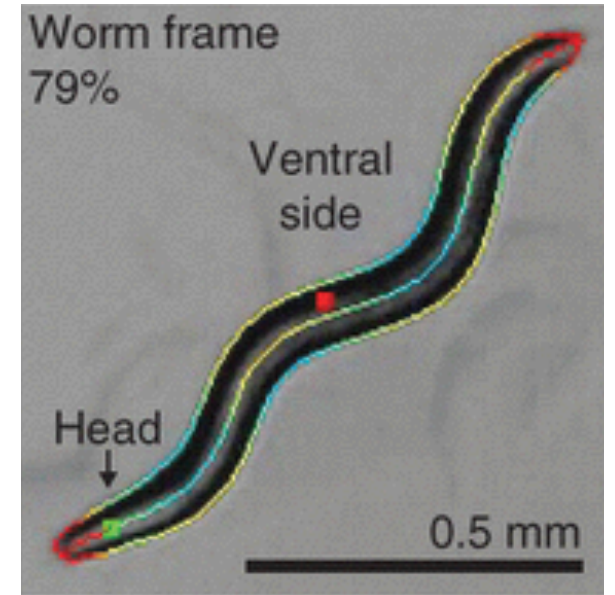
Applications



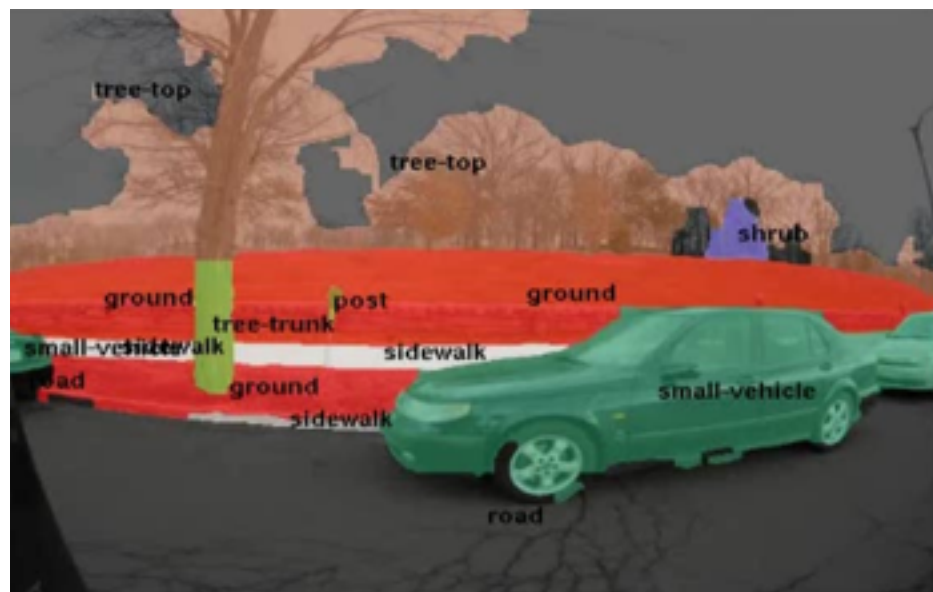
Autonomous Vehicles
(lane line detection)



tissue engineering
(blood vessel counting)



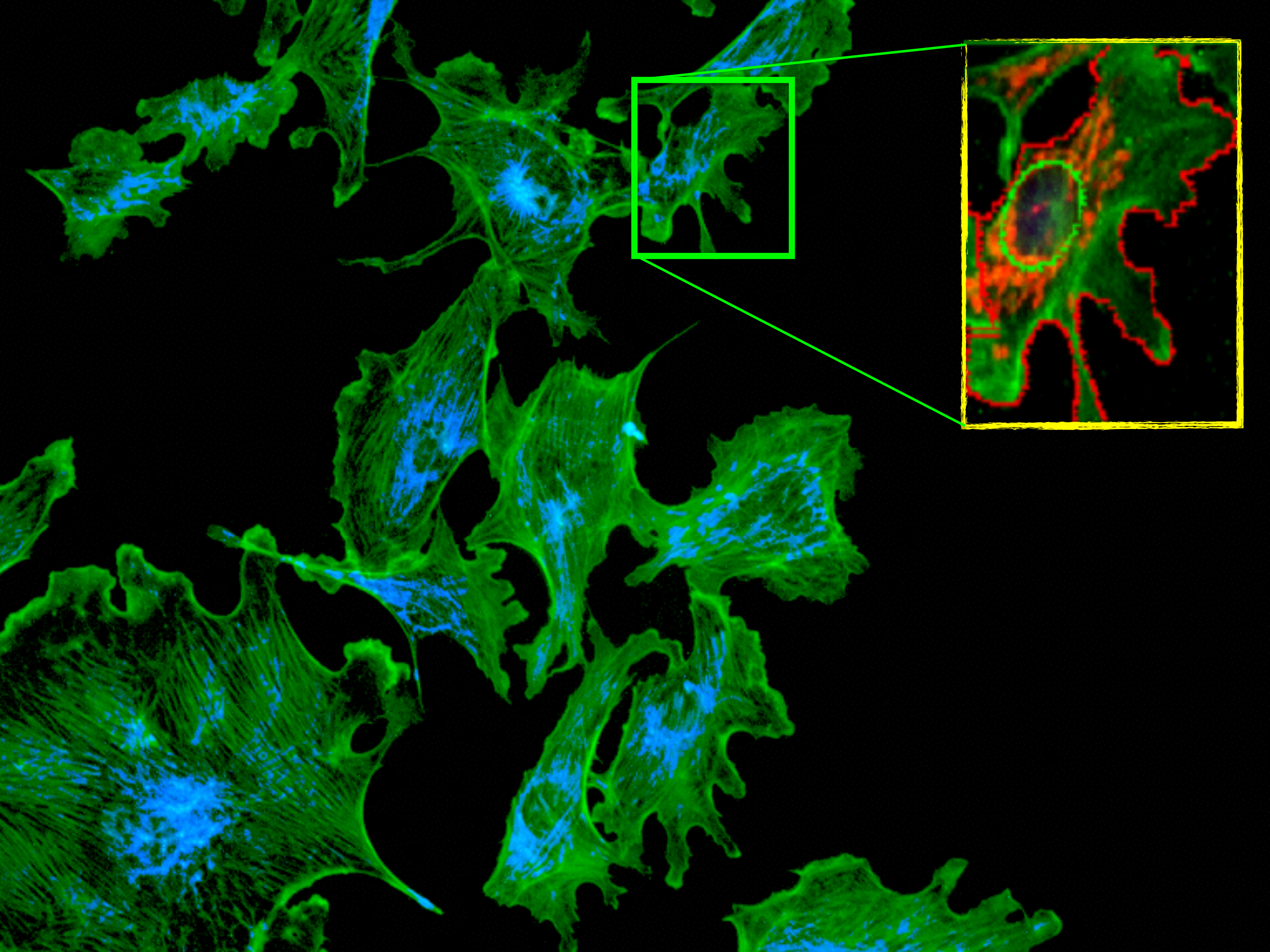
behavioral genetics
(earthworm contours)



Autonomous Vehicles
(semantic scene segmentation)



Computational Photography
(image inpainting)

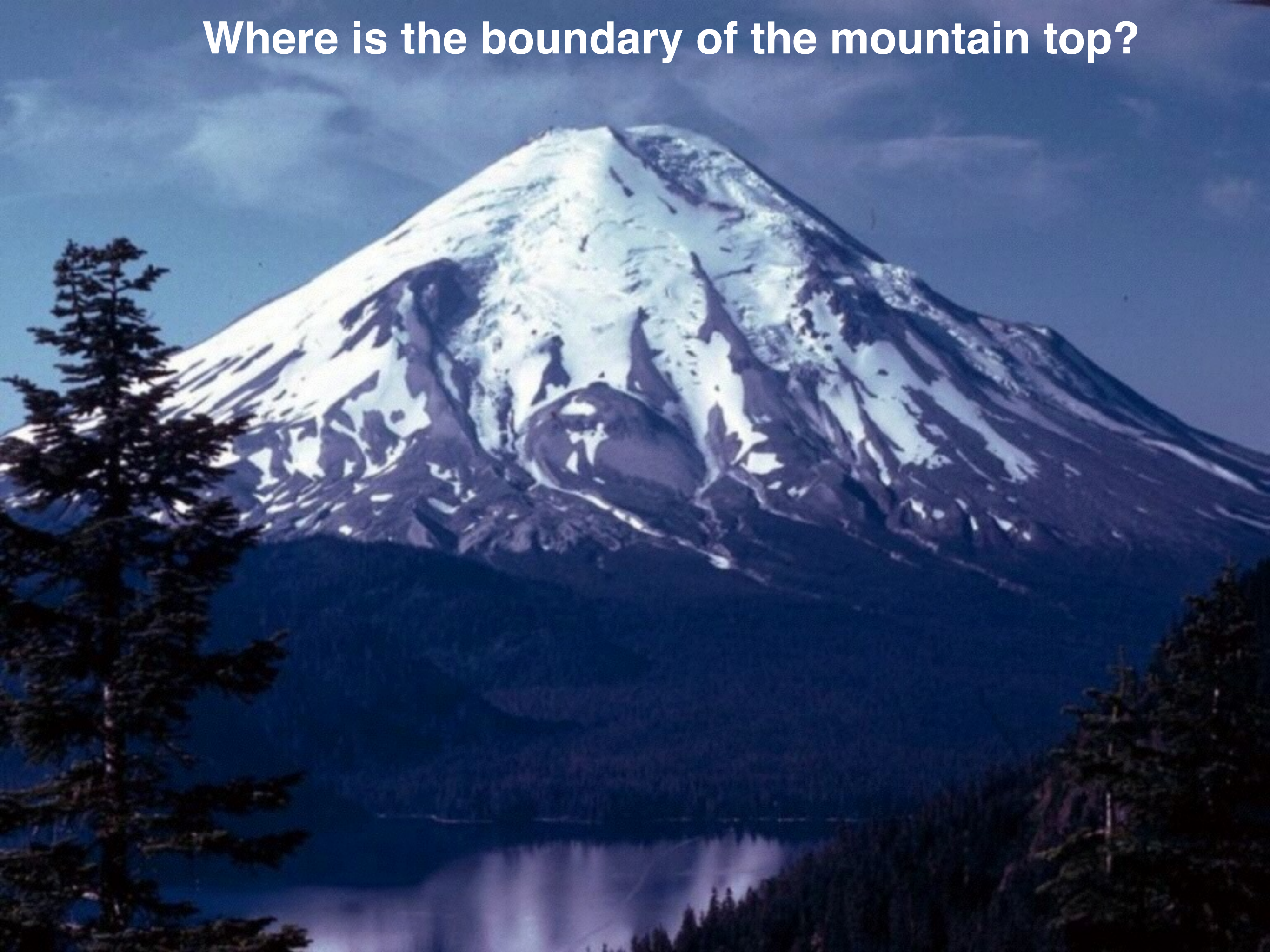




Extracting Lines

16-385 Computer Vision

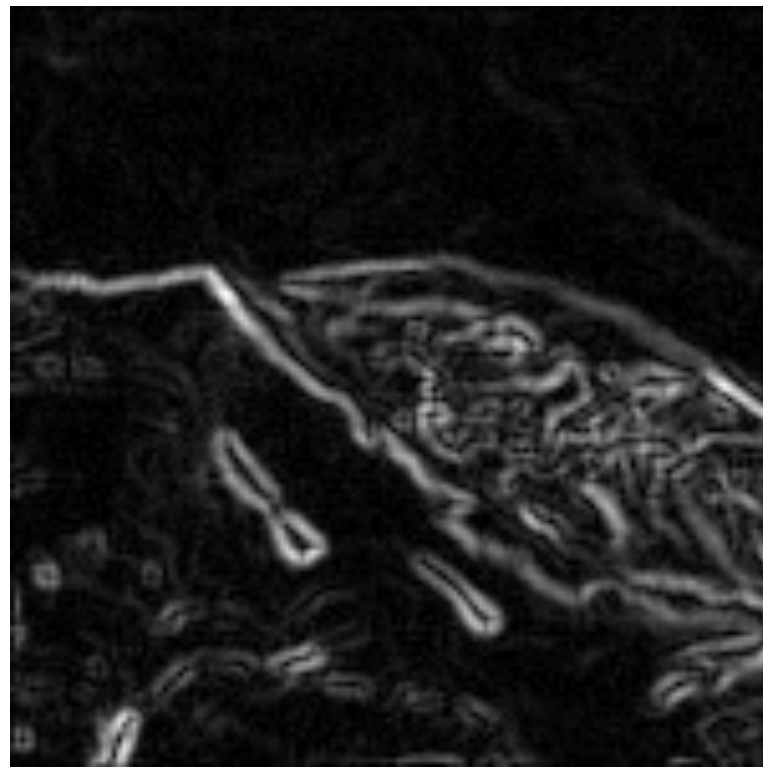
Where is the boundary of the mountain top?



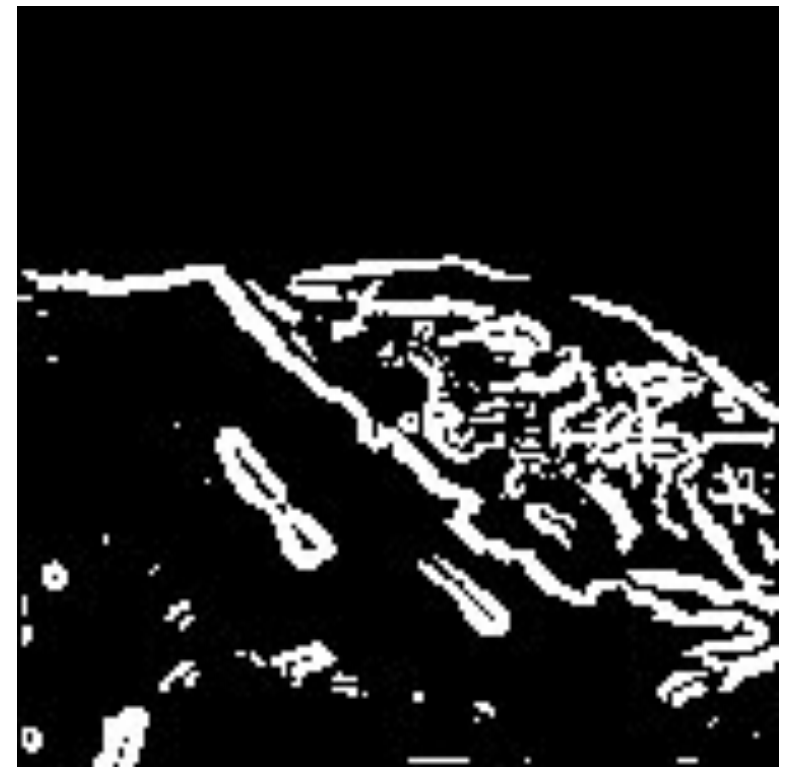
Lines are hard to find



Original image



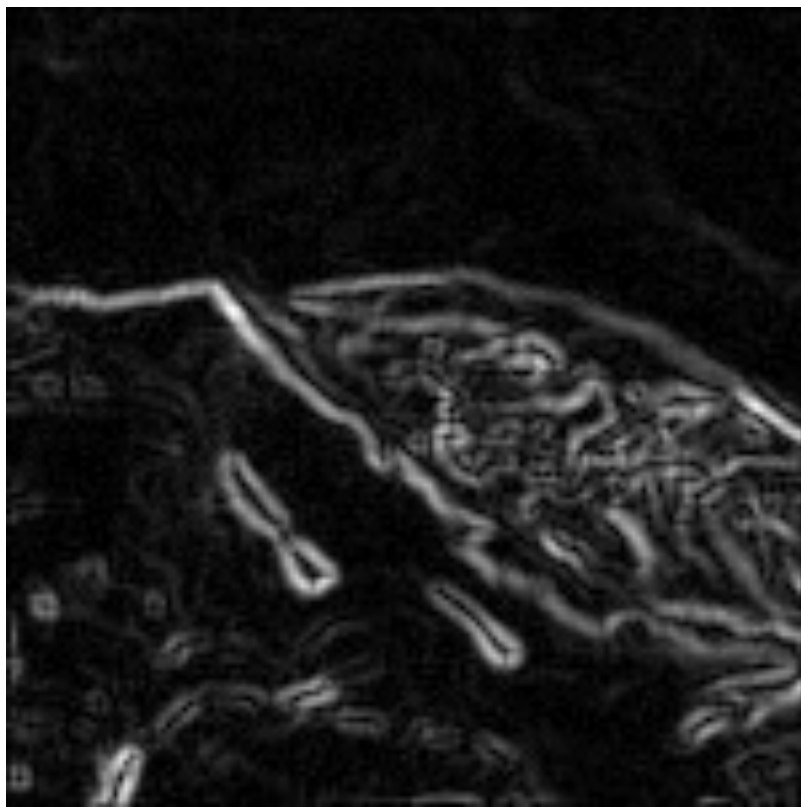
Edge detection



Thresholding

Noisy edge image
Incomplete boundaries

idea #1: morphology



Sobel

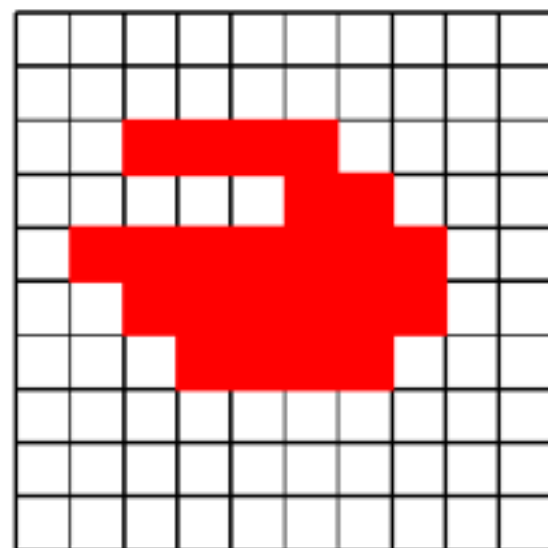


Threshold



Shrink, Expand, Shrink

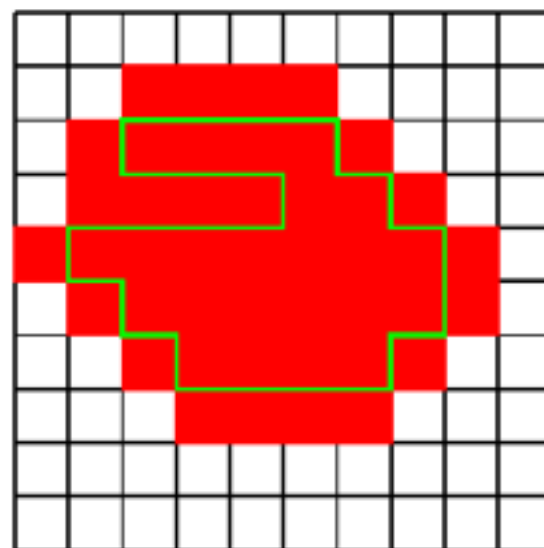
What are some problems with the approach?



A



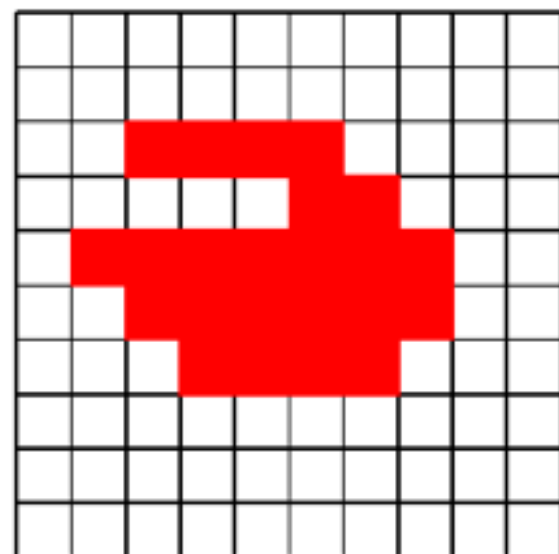
B



$A \oplus B$

Dilation

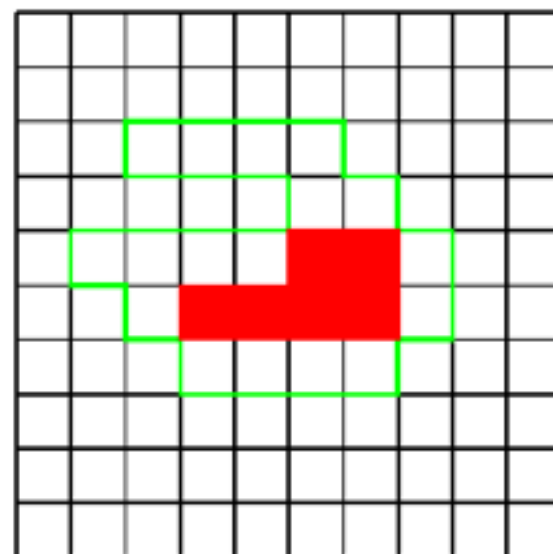
If filter response > 0 , set to 1



A



B



$A \ominus B$

Erosion

If filter response is MAX, set to 1

idea #2: breaking lines

Divide and Conquer:

Given: Boundary lies between points A and B

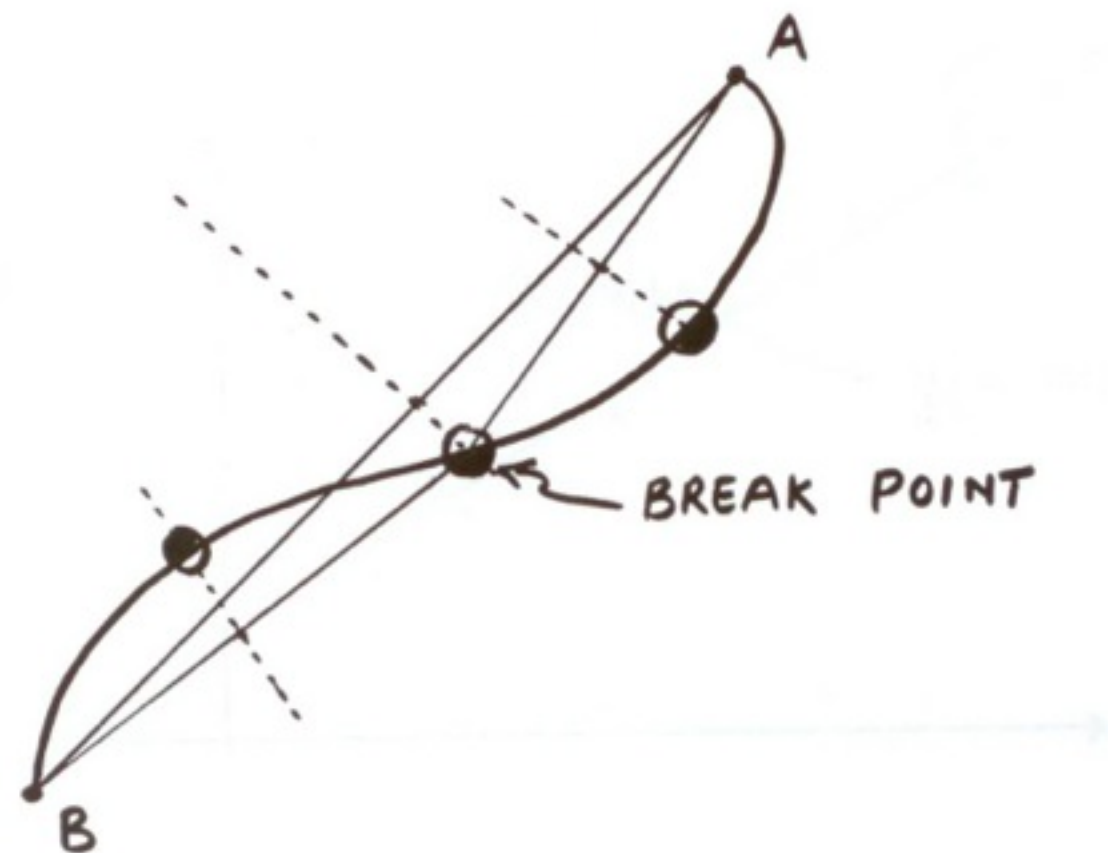
Task: Find boundary

Connect A and B with Line

Find strongest edge along line bisector

Use edge point as break point

Repeat



What are some problems with the approach?

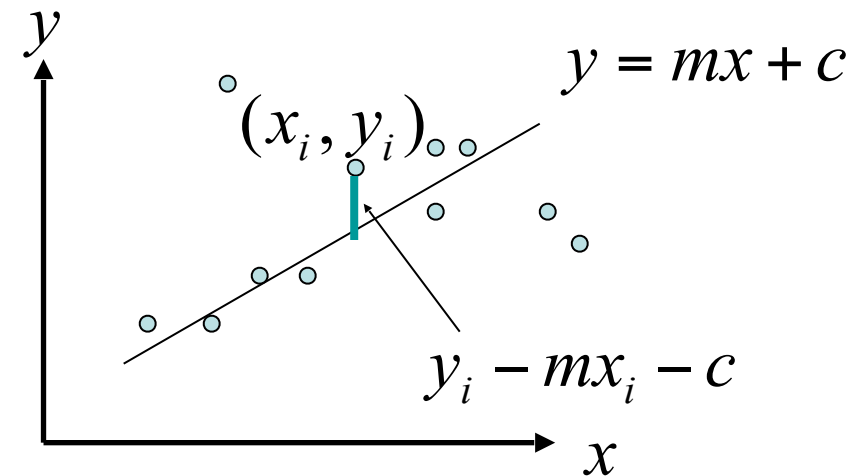
idea #3: line fitting

Given: Many (x_i, y_i) pairs

Find: Parameters (m, c)

Minimize: Average square distance:

$$E = \sum_i \frac{(y_i - mx_i - c)^2}{N}$$



What are some problems with the approach?

idea #3: line fitting

Given: Many (x_i, y_i) pairs

Find: Parameters (m, c)

Minimize: Average square distance:

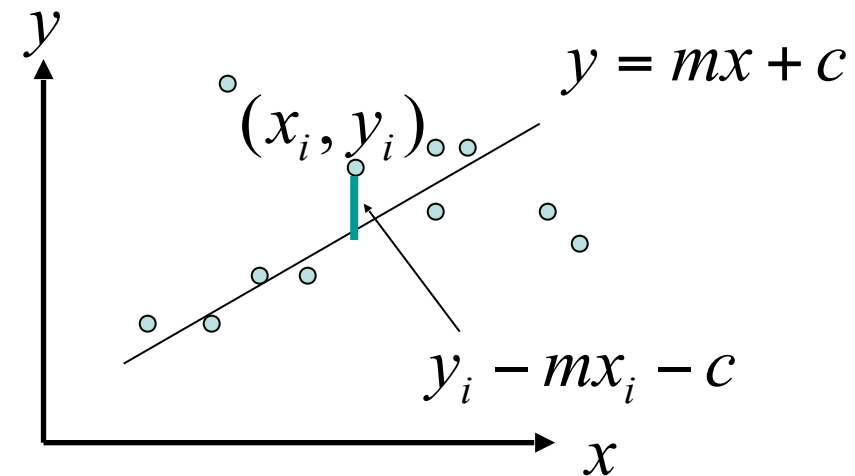
$$E = \sum_i \frac{(y_i - mx_i - c)^2}{N}$$

Using:

$$\frac{\partial E}{\partial m} = 0 \quad \& \quad \frac{\partial E}{\partial c} = 0$$

Note:

$$\bar{y} = \frac{\sum_i y_i}{N} \quad \bar{x} = \frac{\sum_i x_i}{N}$$

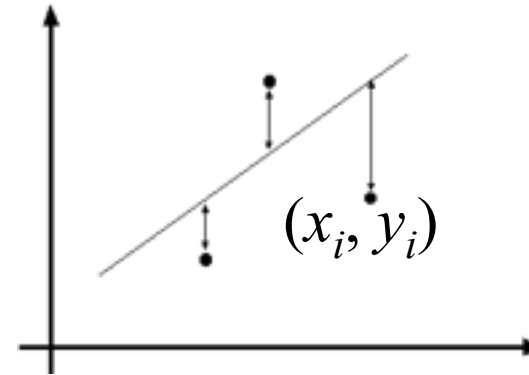


$$c = \bar{y} - m \bar{x}$$
$$m = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

What are some problems with the approach?

Data: $(x_1, y_1), \dots, (x_n, y_n)$

Line equation: $y_i = m x_i + b$



Find (m, b) to minimize

$$E = \sum_{i=1}^n (y_i - m x_i - b)^2$$

$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \quad X = \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \quad B = \begin{bmatrix} m \\ b \end{bmatrix}$$

$$E = \|Y - XB\|^2 = (Y - XB)^T (Y - XB) = Y^T Y - 2(XB)^T Y + (XB)^T (XB)$$

$$\frac{dE}{dB} = 2X^T XB - 2X^T Y = 0$$

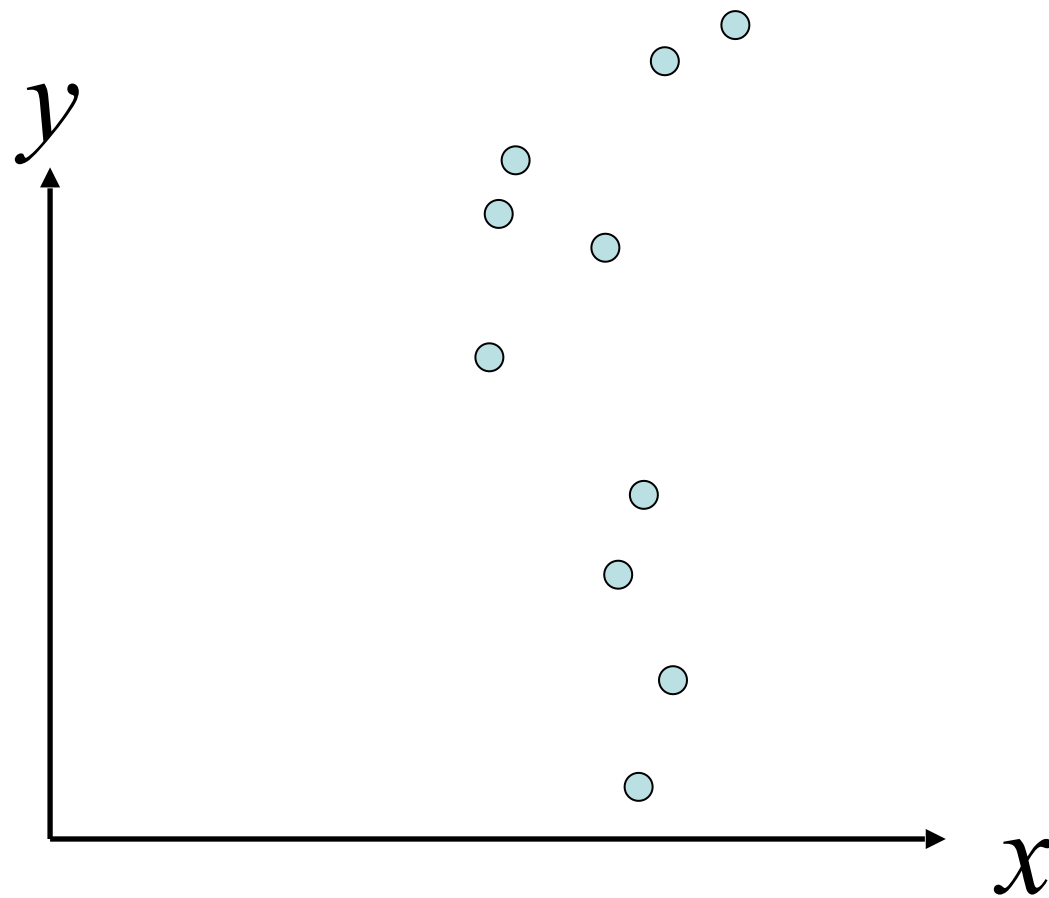
$$X^T XB = X^T Y$$

Normal equations: least squares solution to $XB=Y$

Problems with parameterizations

Where is the line that minimizes E ?

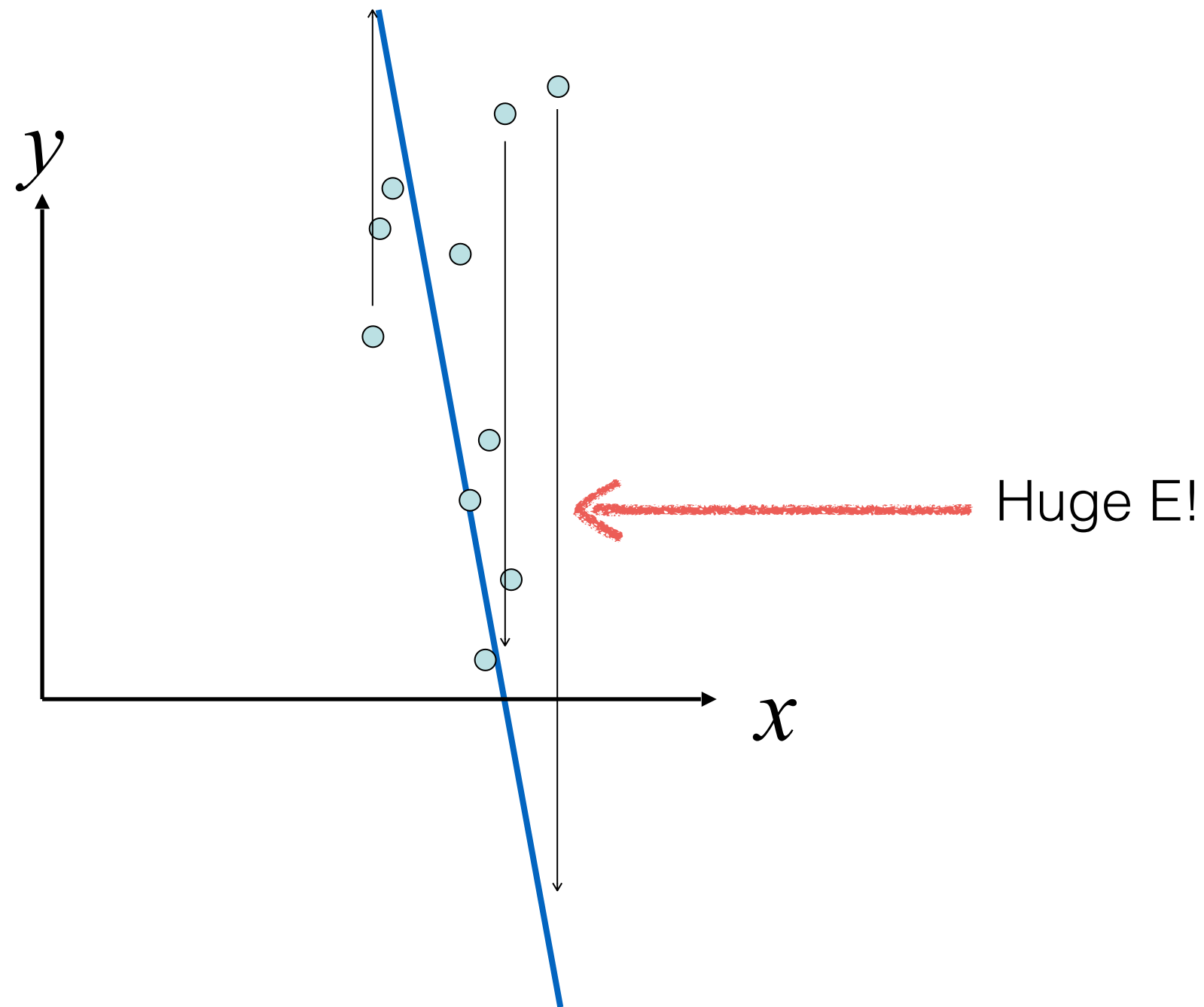
$$E = \sum_{i=1}^n (y_i - mx_i - b)^2$$



Problems with parameterizations

Where is the line that minimizes E ?

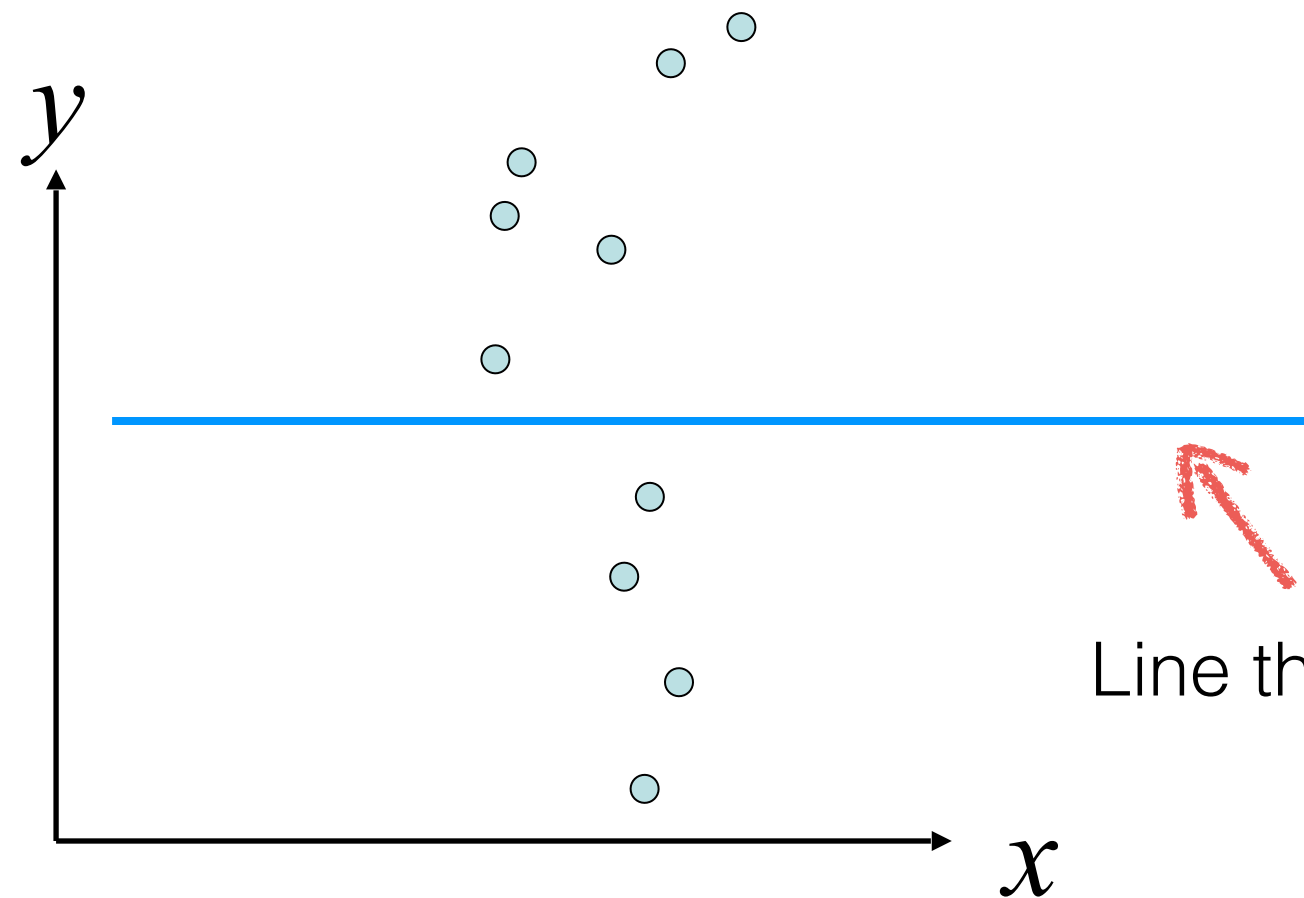
$$E = \sum_{i=1}^n (y_i - mx_i - b)^2$$



Problems with parameterizations

Where is the line that minimizes E ?

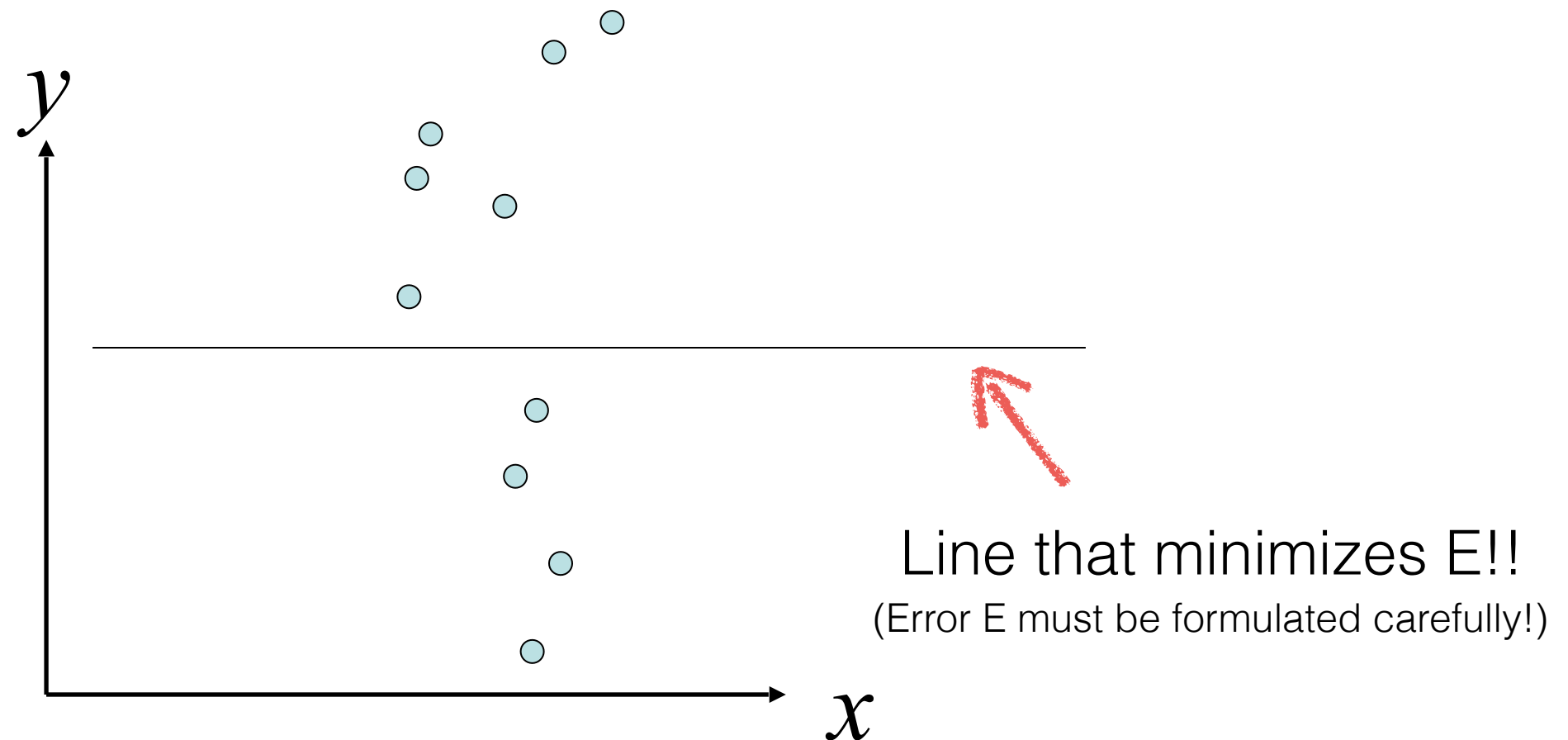
$$E = \sum_{i=1}^n (y_i - mx_i - b)^2$$



Line that minimizes E !!

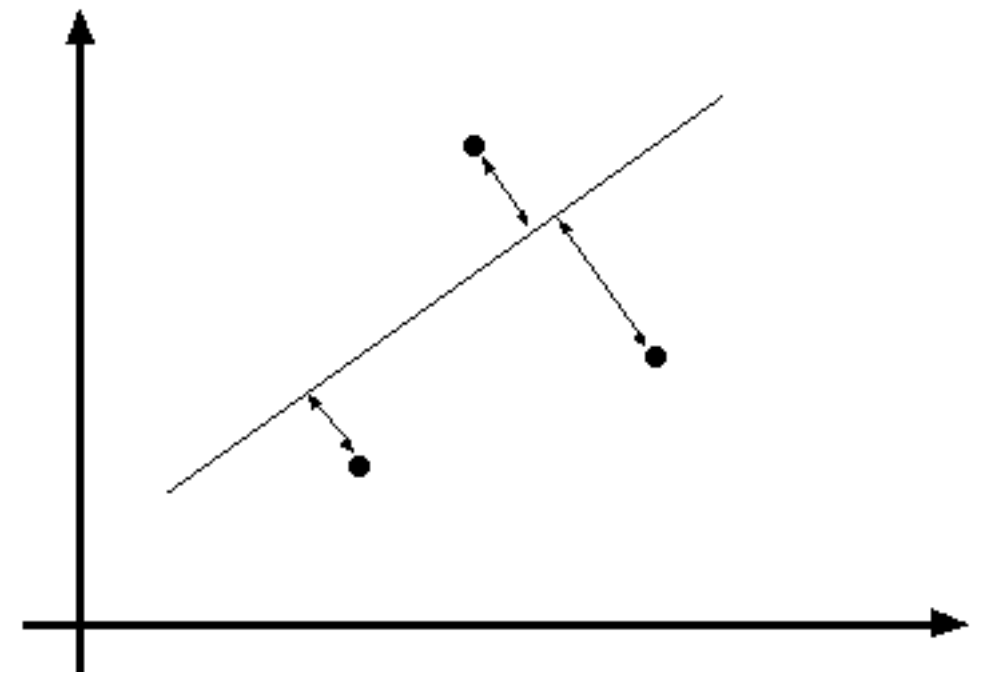
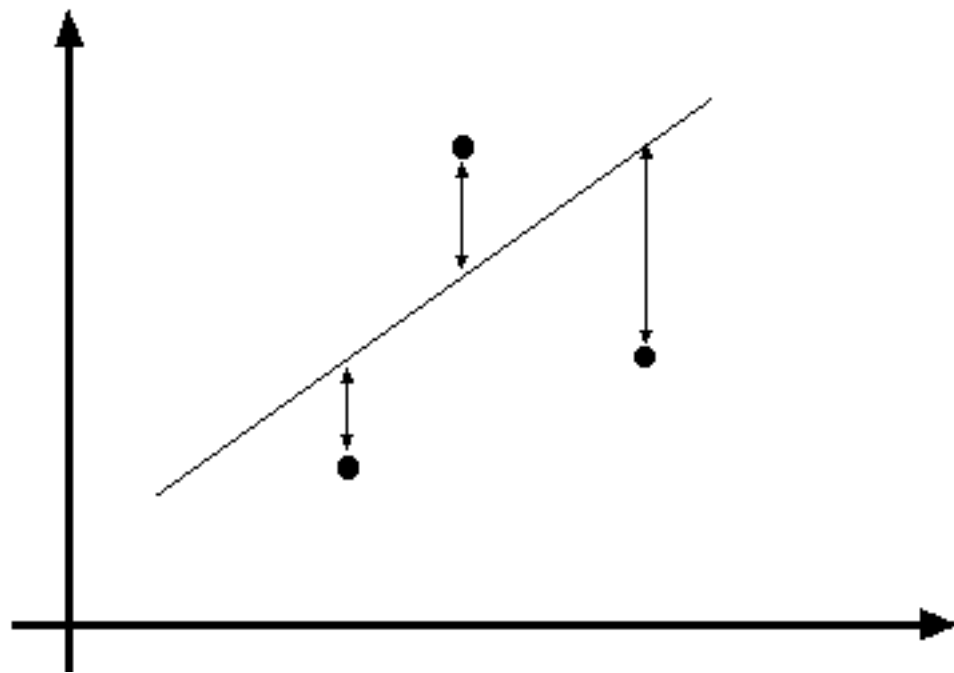
Problems with parameterizations

Where is the line that minimizes E ?

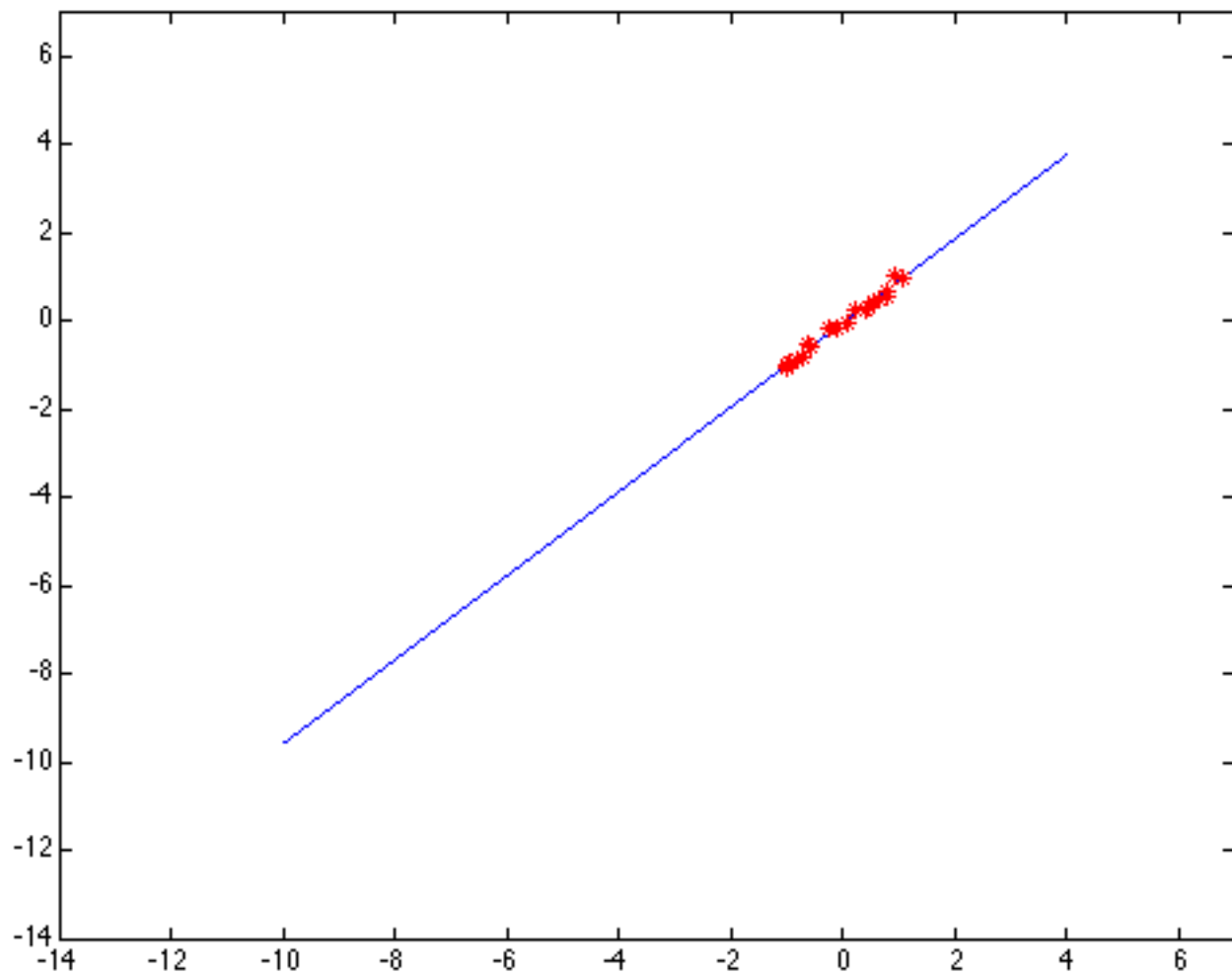


Use this instead:
$$E = \frac{1}{N} \sum_i (\rho - x_i \cos\theta + y_i \sin\theta)^2$$
 I'll explain this later ...

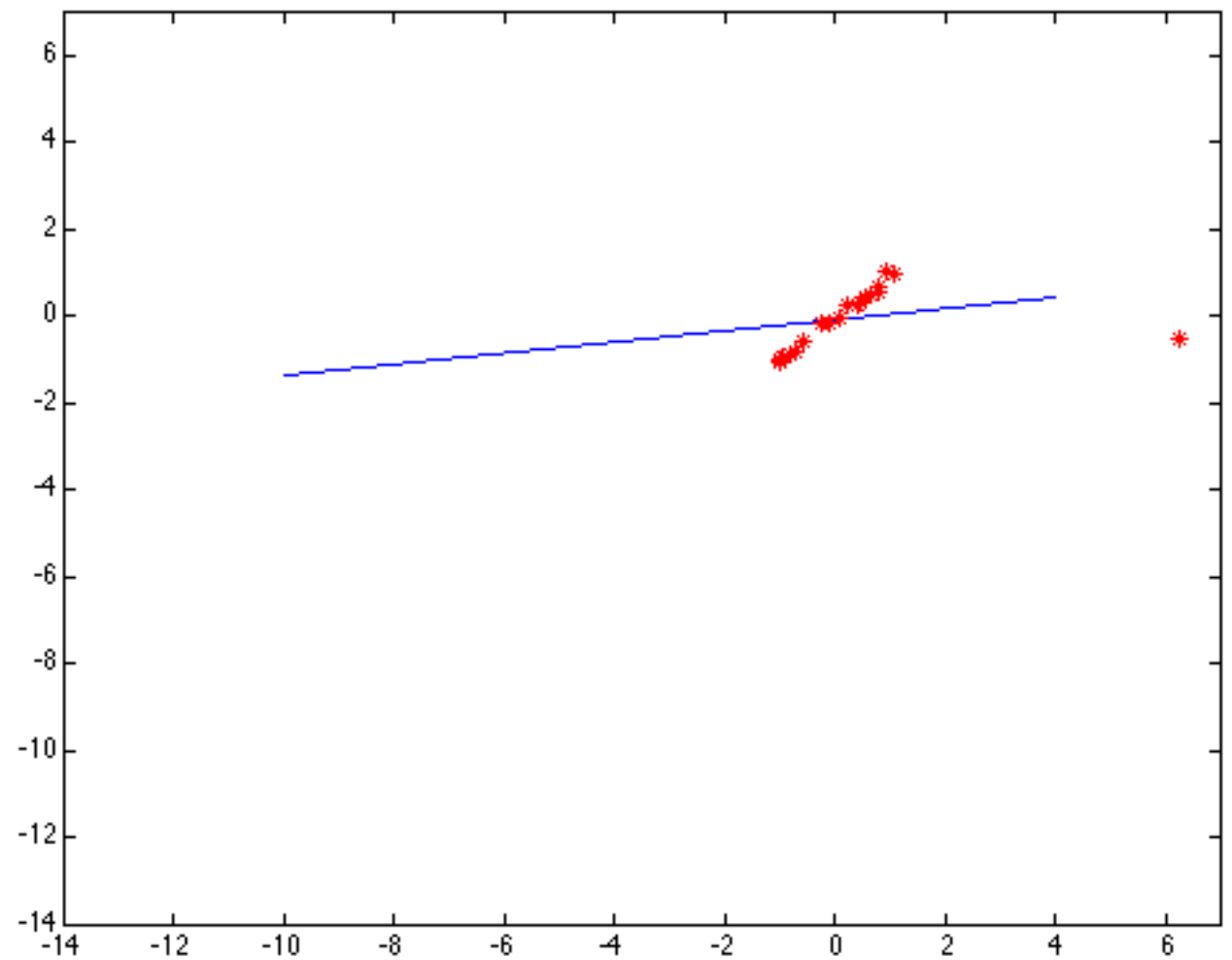
Line fitting can be maximum likelihood
- but choice of model is important



Problems with noise



Least-squares error fit

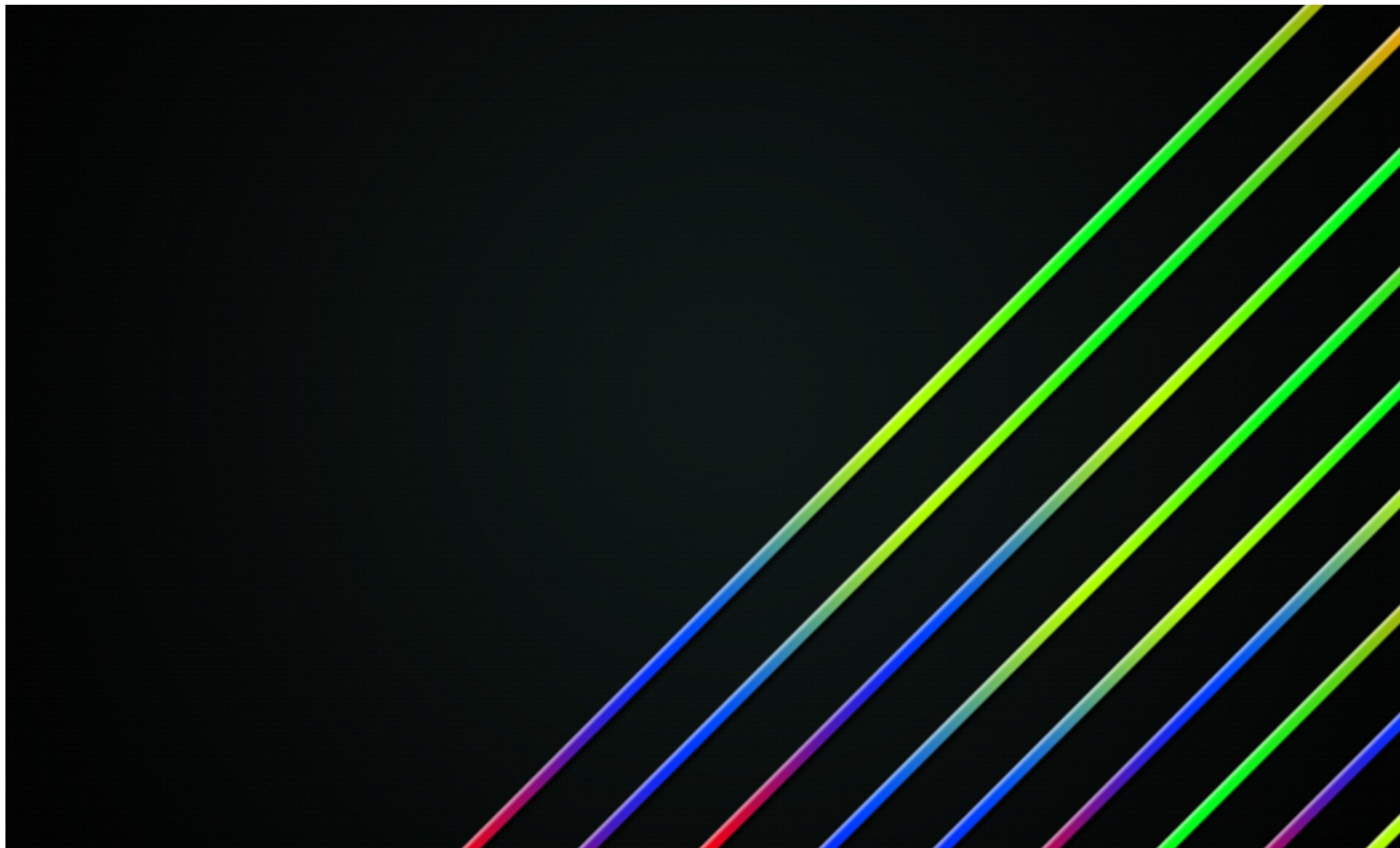


Squared error heavily penalizes outliers

Model fitting is difficult because...

- **Extraneous data:** clutter or multiple models
 - We do not know what is part of the model?
 - Can we pull out models with a few parts from much larger amounts of background clutter?
- **Missing data:** only some parts of model are present
- **Noise**
- **Cost:**
 - It is not feasible to check all combinations of features by fitting a model to each possible subset

So what can we do?





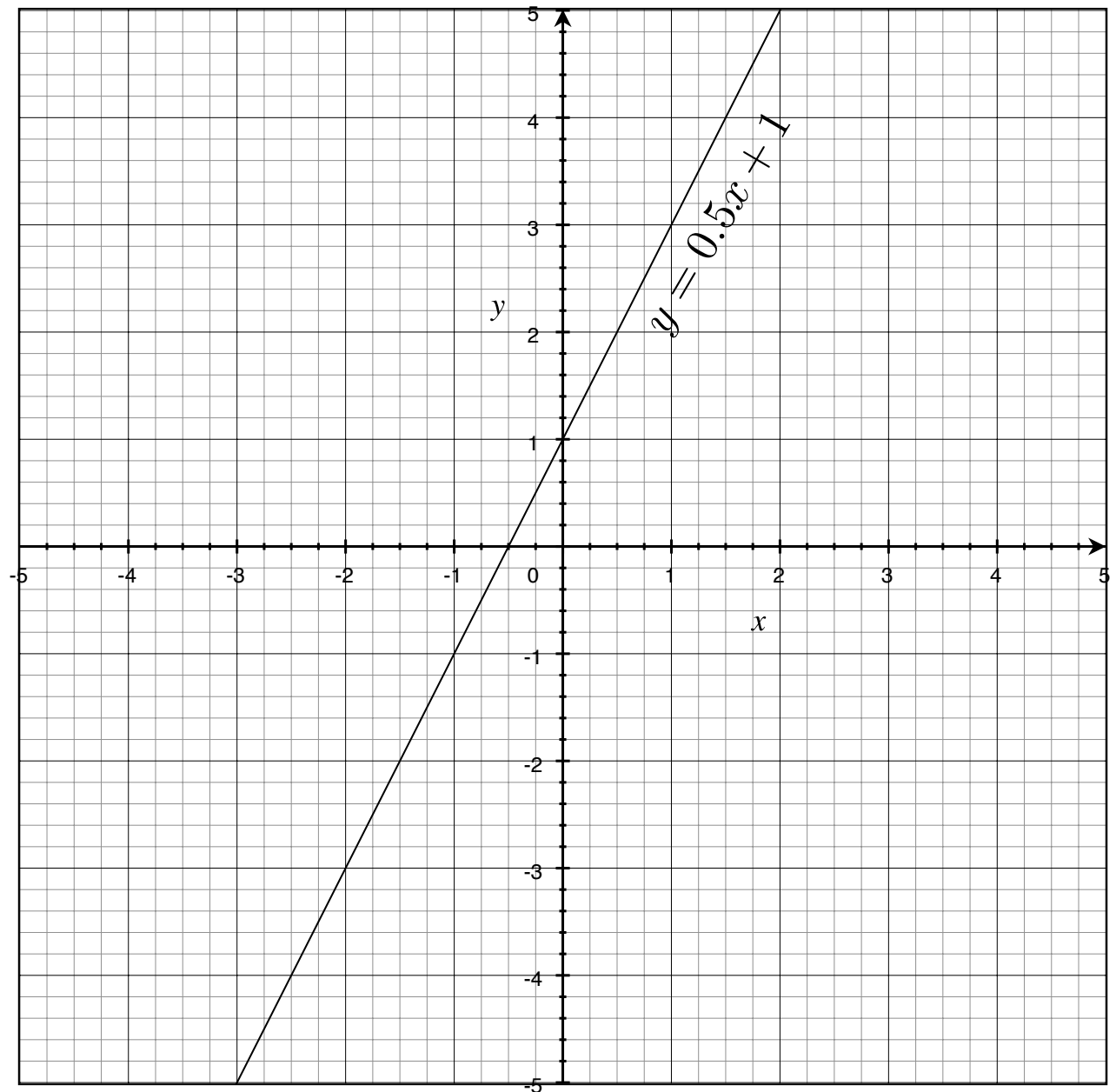
Lines Parameterization

16-385 Computer Vision

Slope intercept form

$$y = mx + b$$

 slope  y-intercept



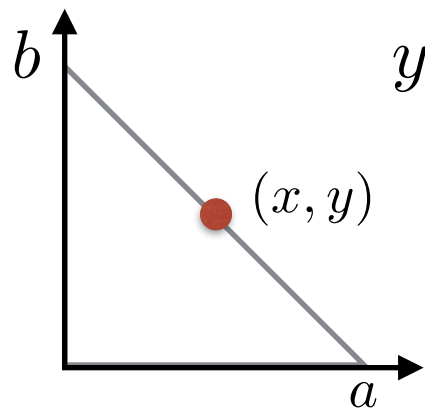
Double intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

x-intercept

y-intercept

Derivation:

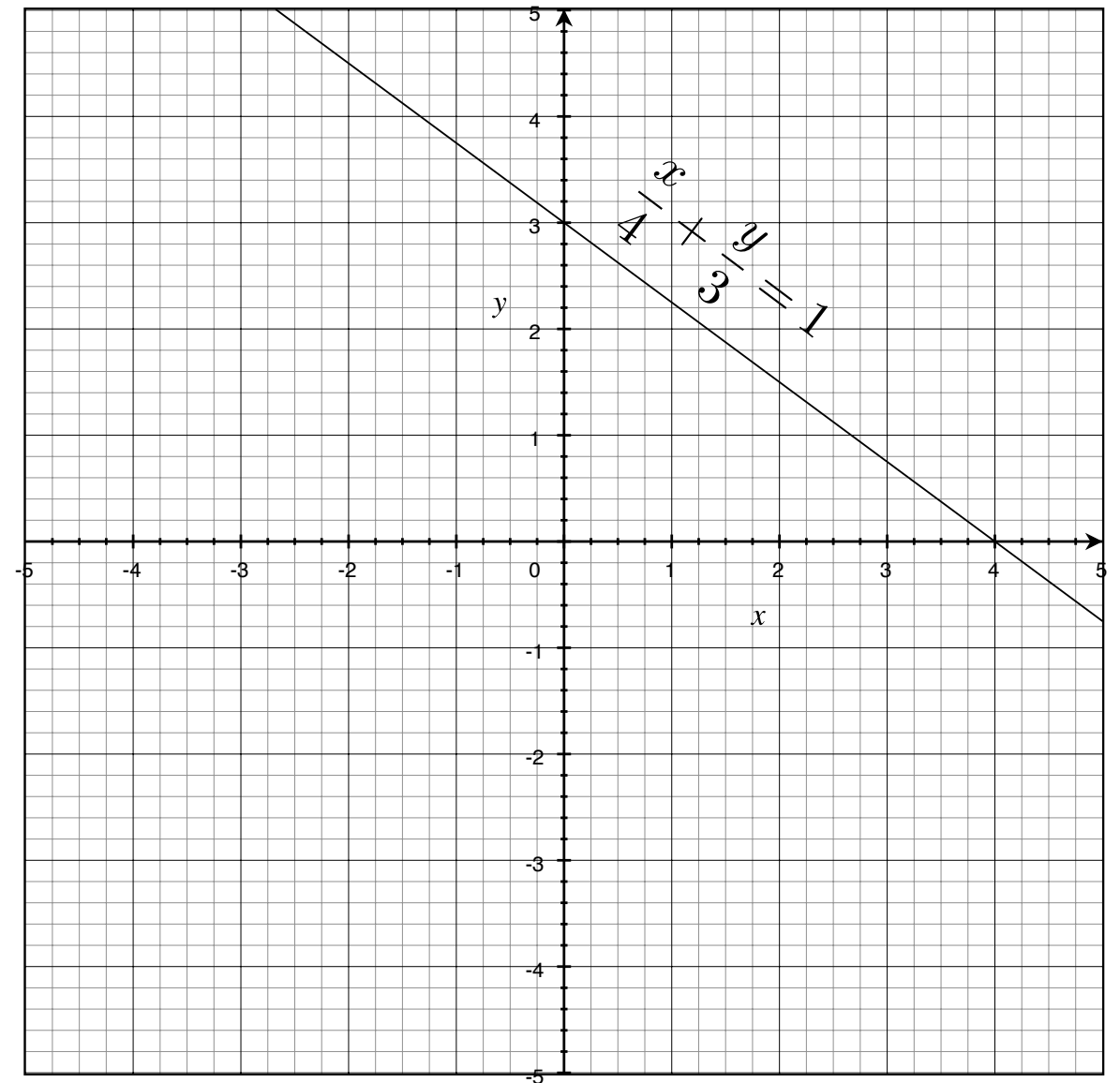


(Similar slope) $\frac{y - b}{x - 0} = \frac{0 - y}{a - x}$

$$ya + yx - ba + bx = -yx$$

$$ya + bx = ba$$

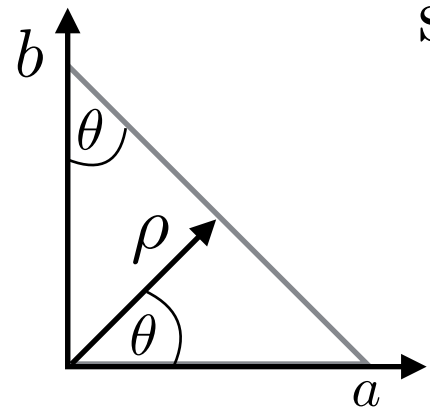
$$\frac{y}{b} + \frac{x}{a} = 1$$



Normal Form

$$x \cos \theta + y \sin \theta = \rho$$

Derivation:



$$\cos \theta = \frac{\rho}{a} \rightarrow a = \frac{\rho}{\cos \theta}$$

$$\sin \theta = \frac{\rho}{b} \rightarrow b = \frac{\rho}{\sin \theta}$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$x \cos \theta + y \sin \theta = \rho$$

