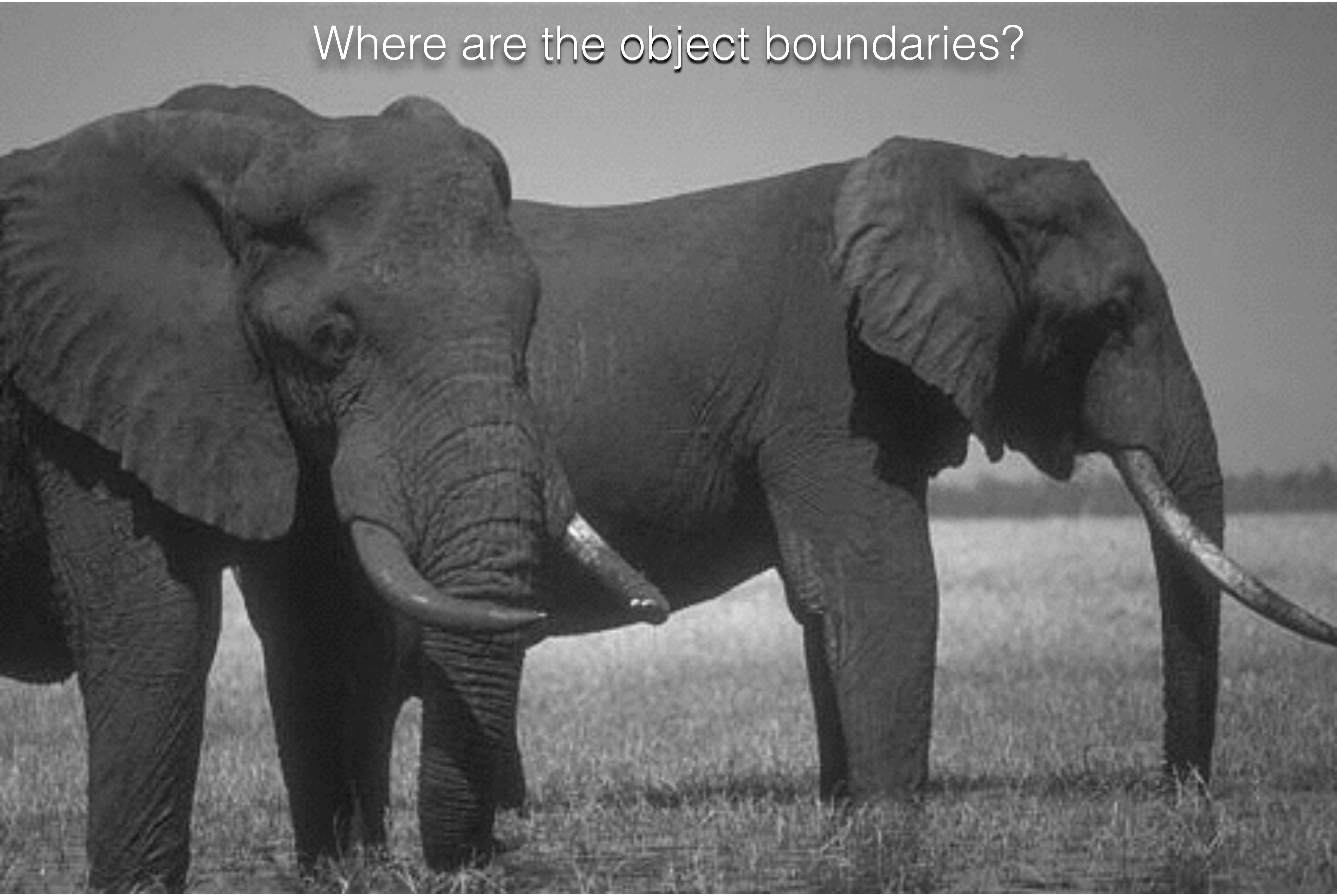
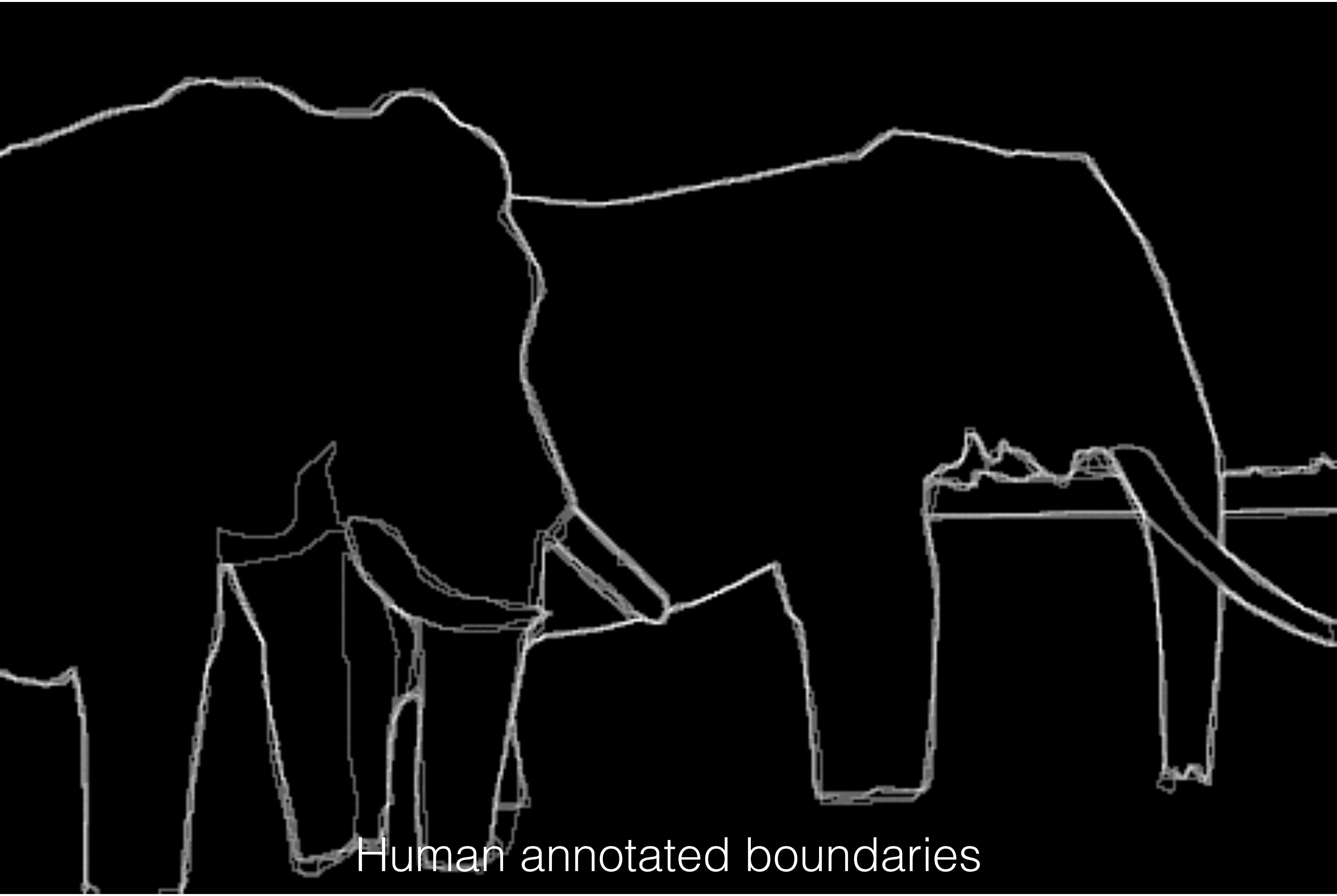
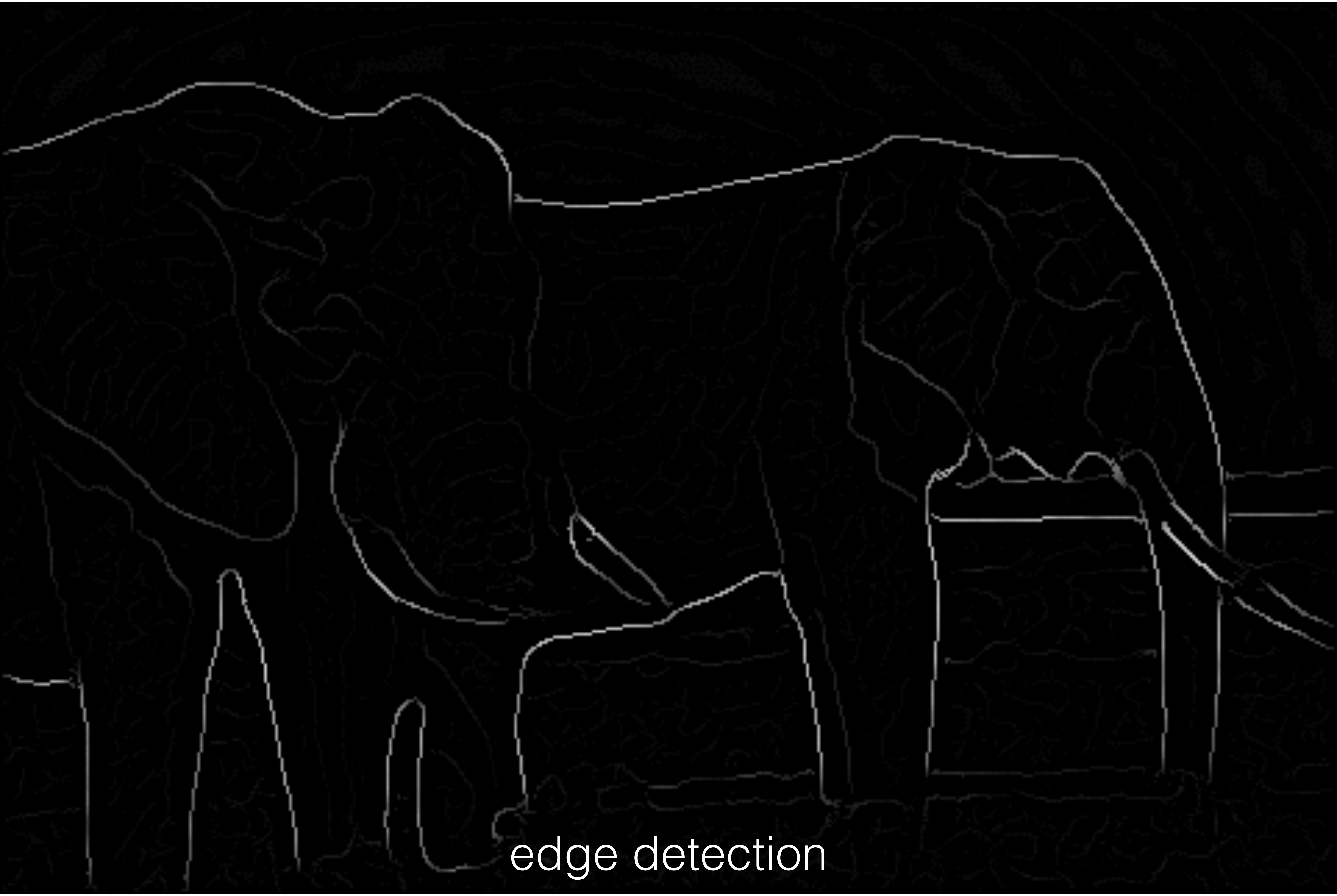
Defining boundaries

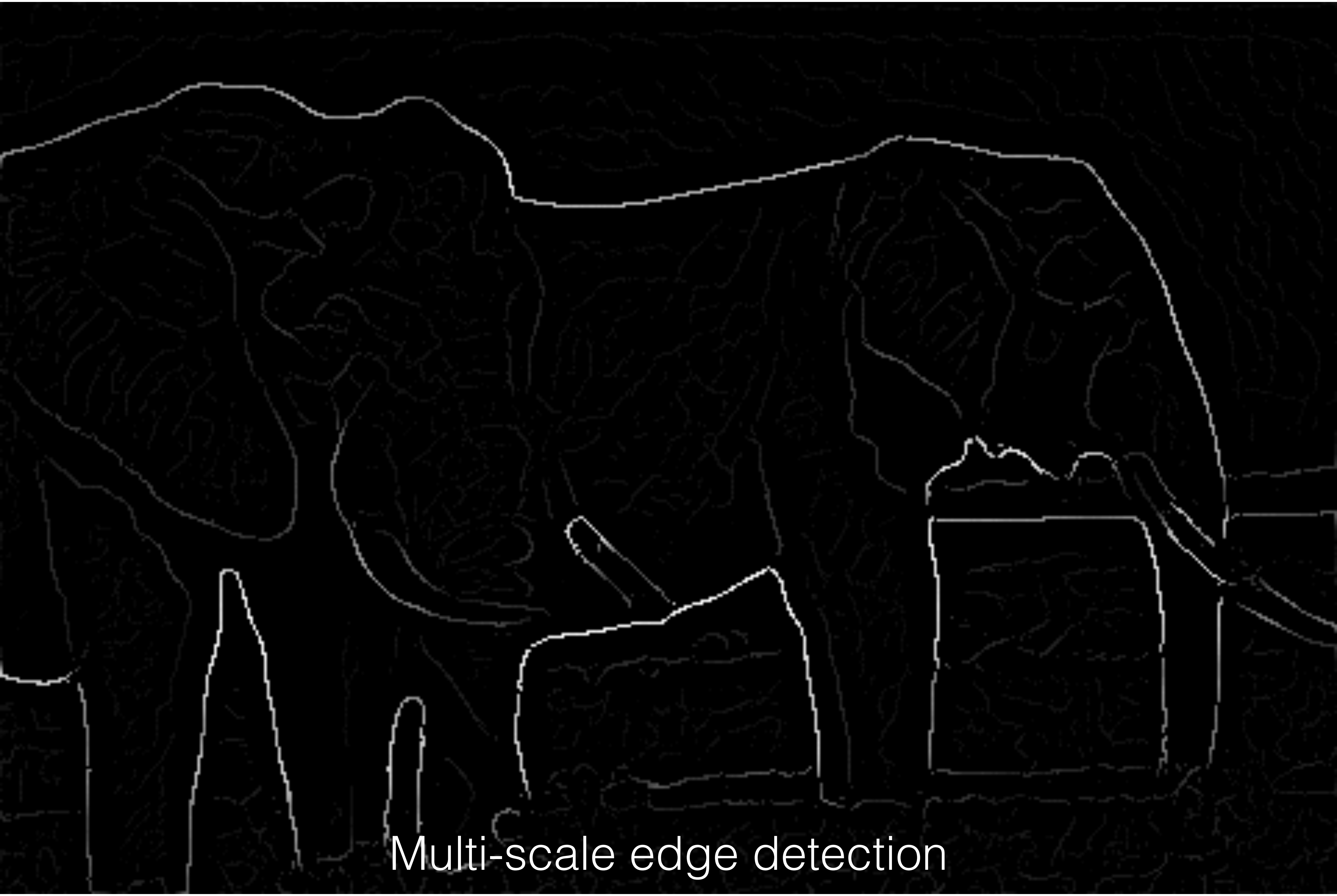
16-385 Computer Vision


Where are the object boundaries?

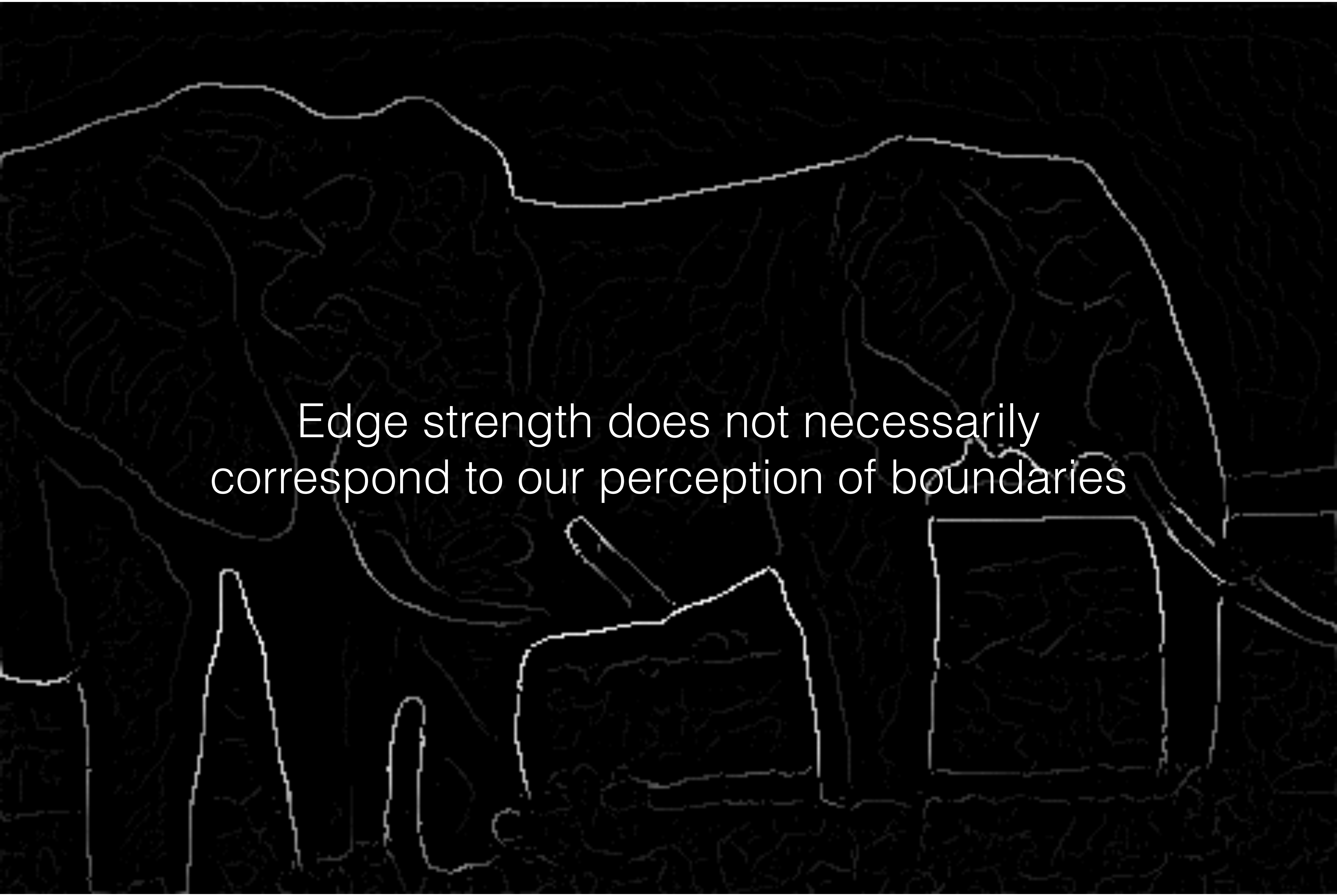


Human annotated boundaries

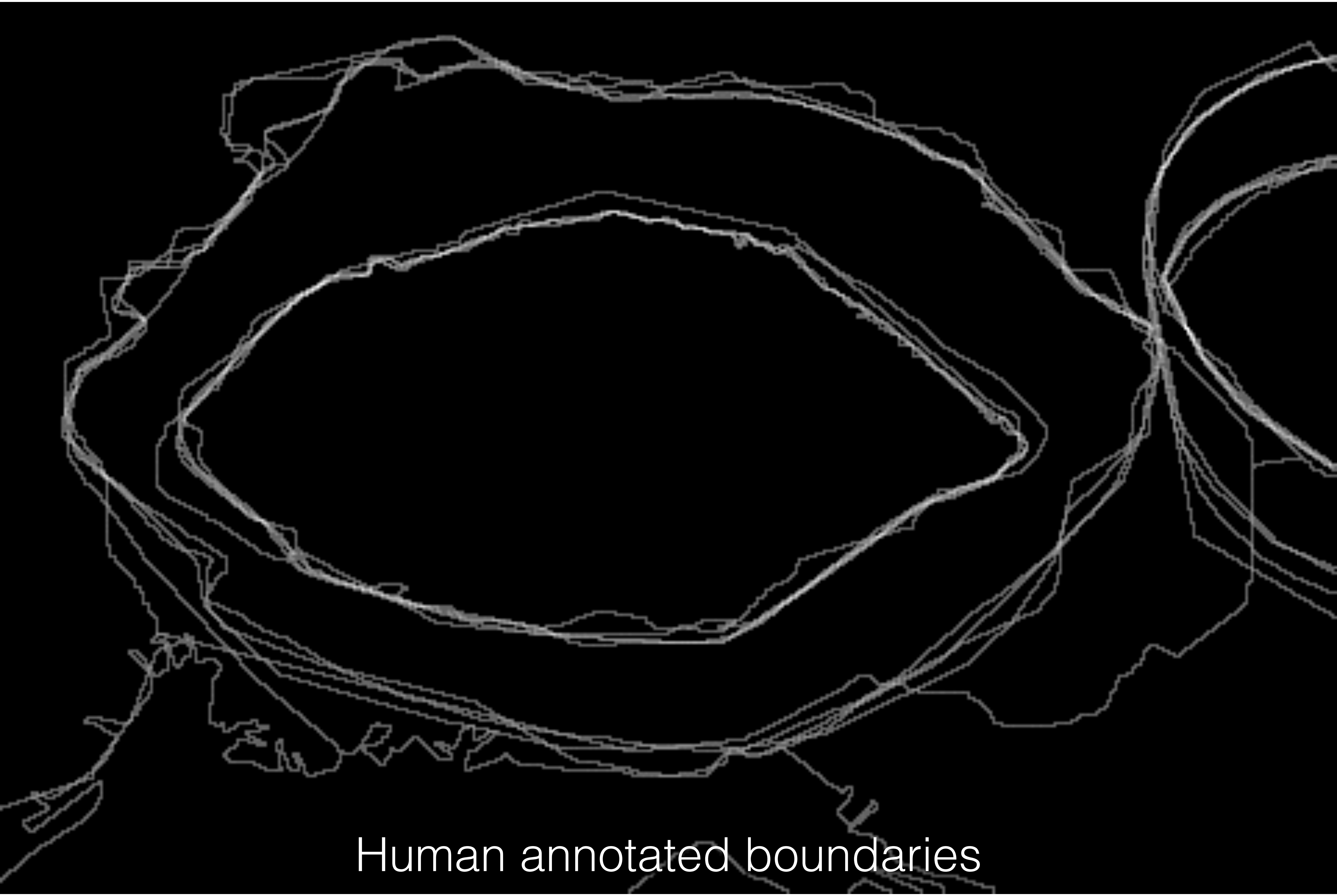

edge detection



Multi-scale edge detection


Edge strength does not necessarily
correspond to our perception of boundaries


Where are the object boundaries?

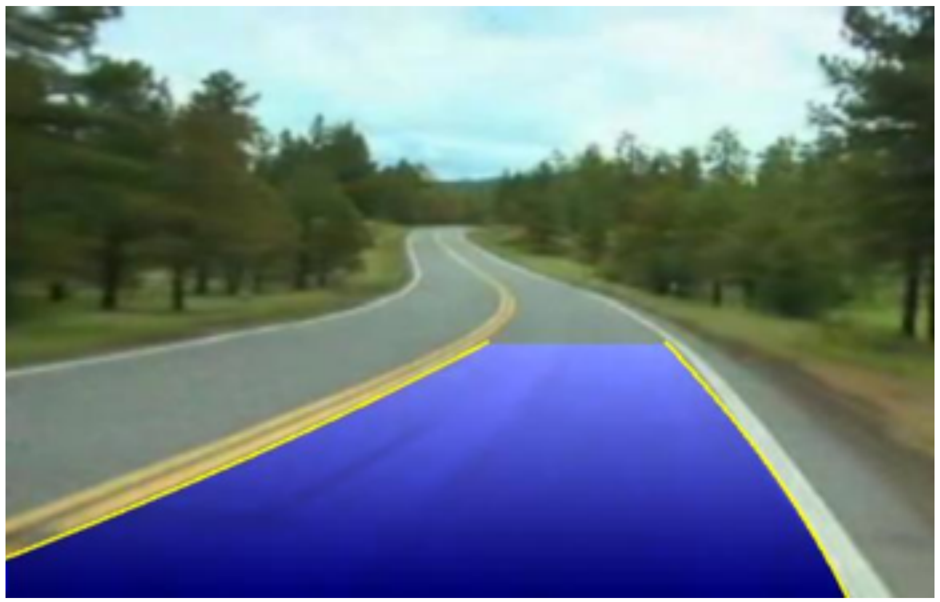


Human annotated boundaries

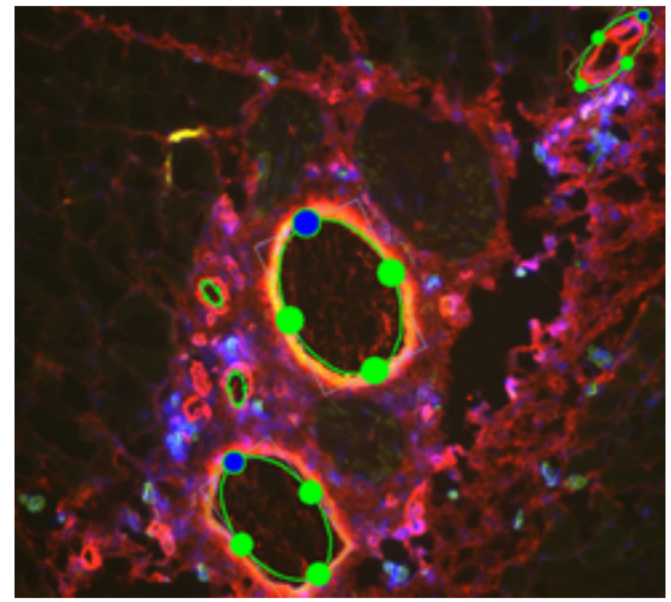

edge detection


Defining boundaries are hard for us too

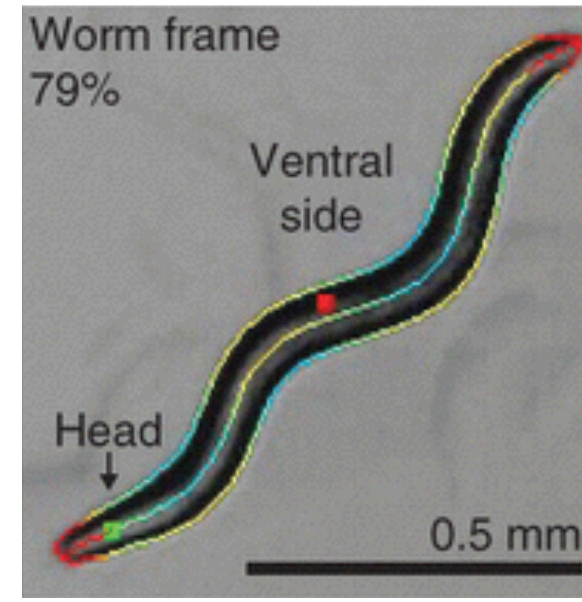
Applications



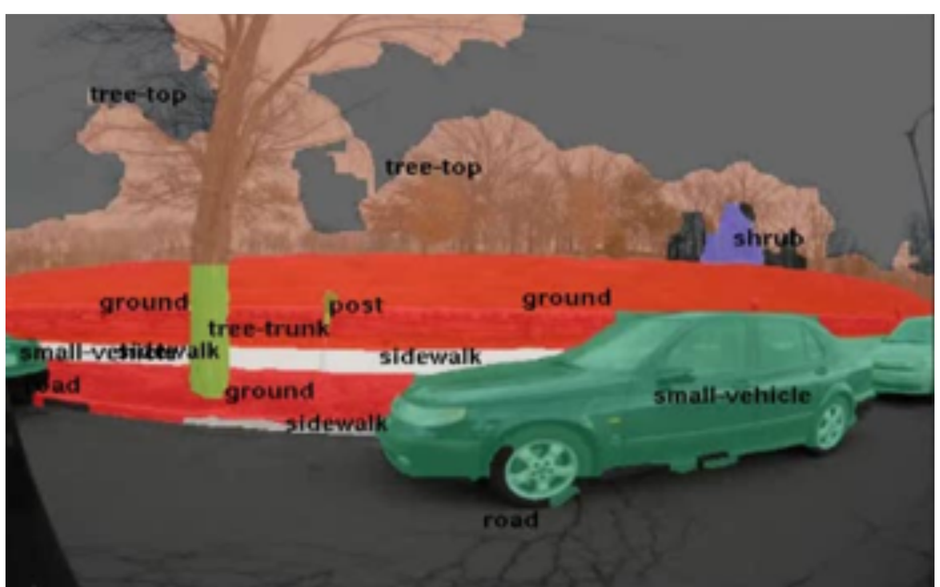
Autonomous Vehicles
(lane line detection)



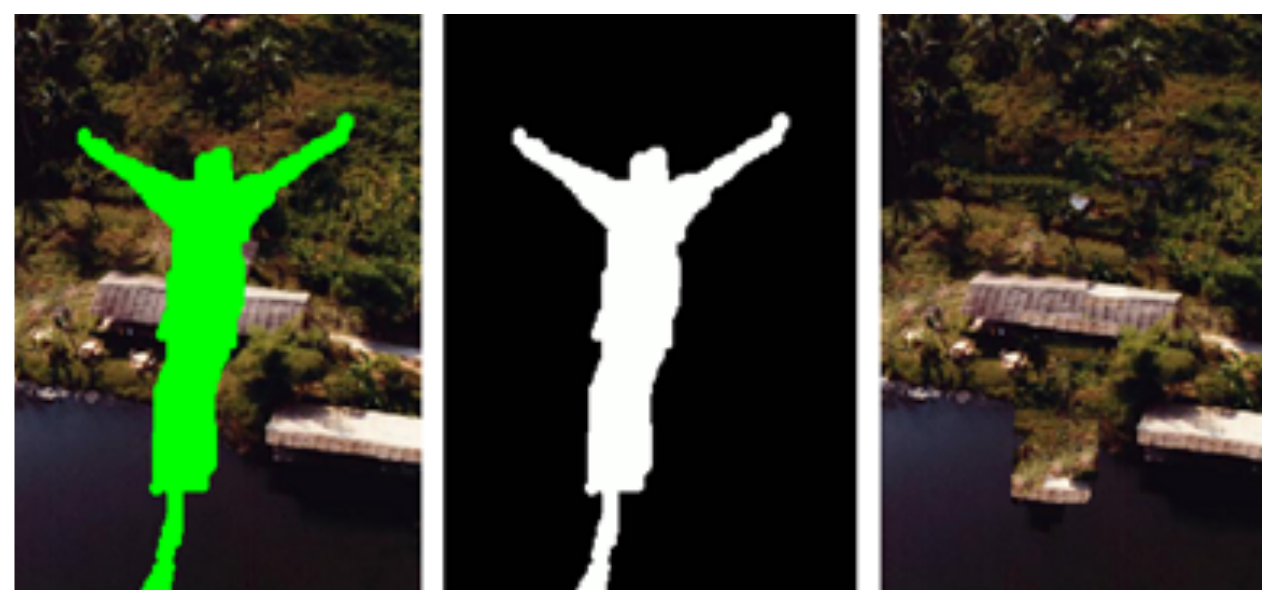
tissue engineering
(blood vessel counting)



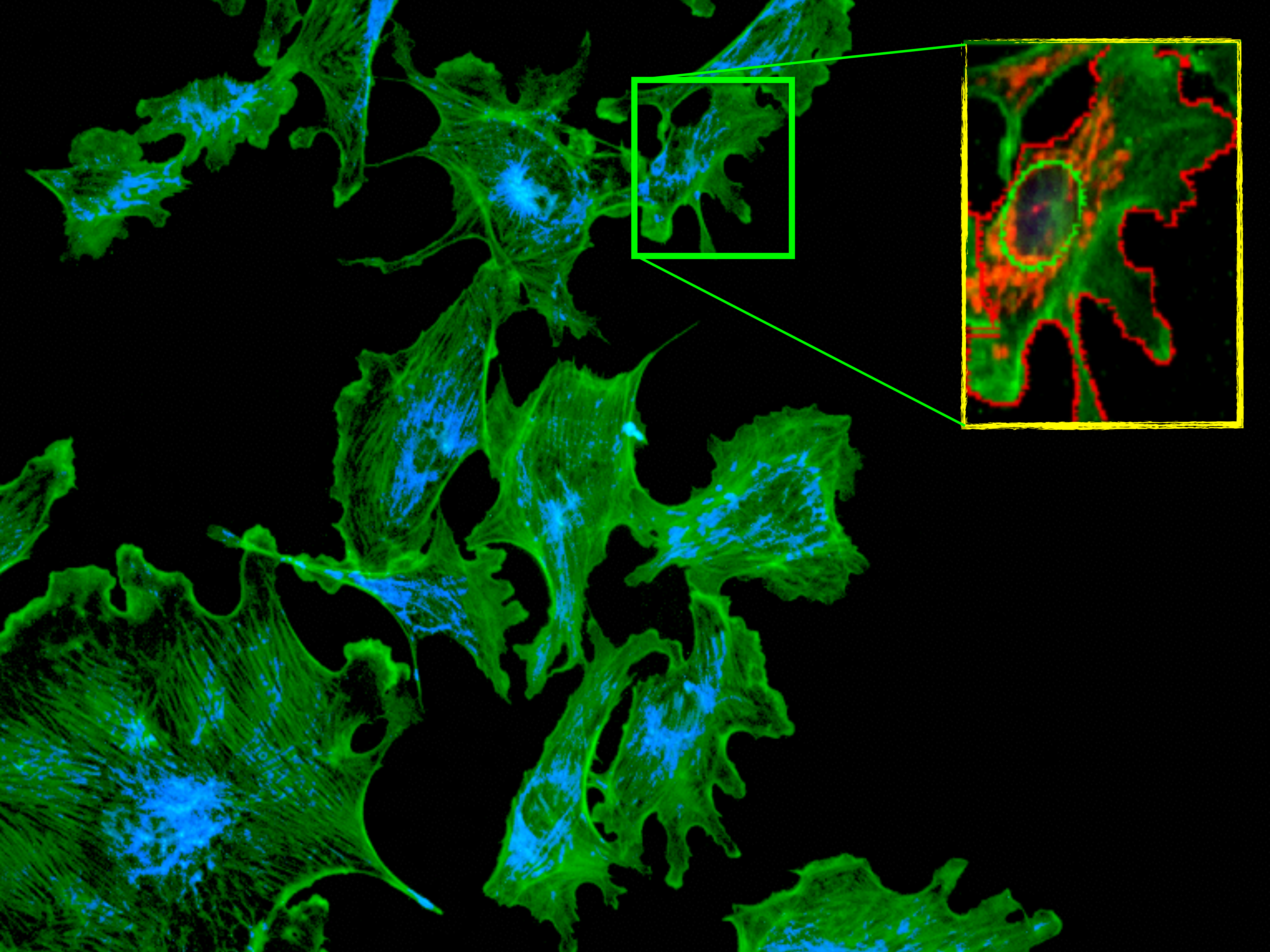
behavioral genetics
(earthworm contours)



Autonomous Vehicles
(semantic scene segmentation)

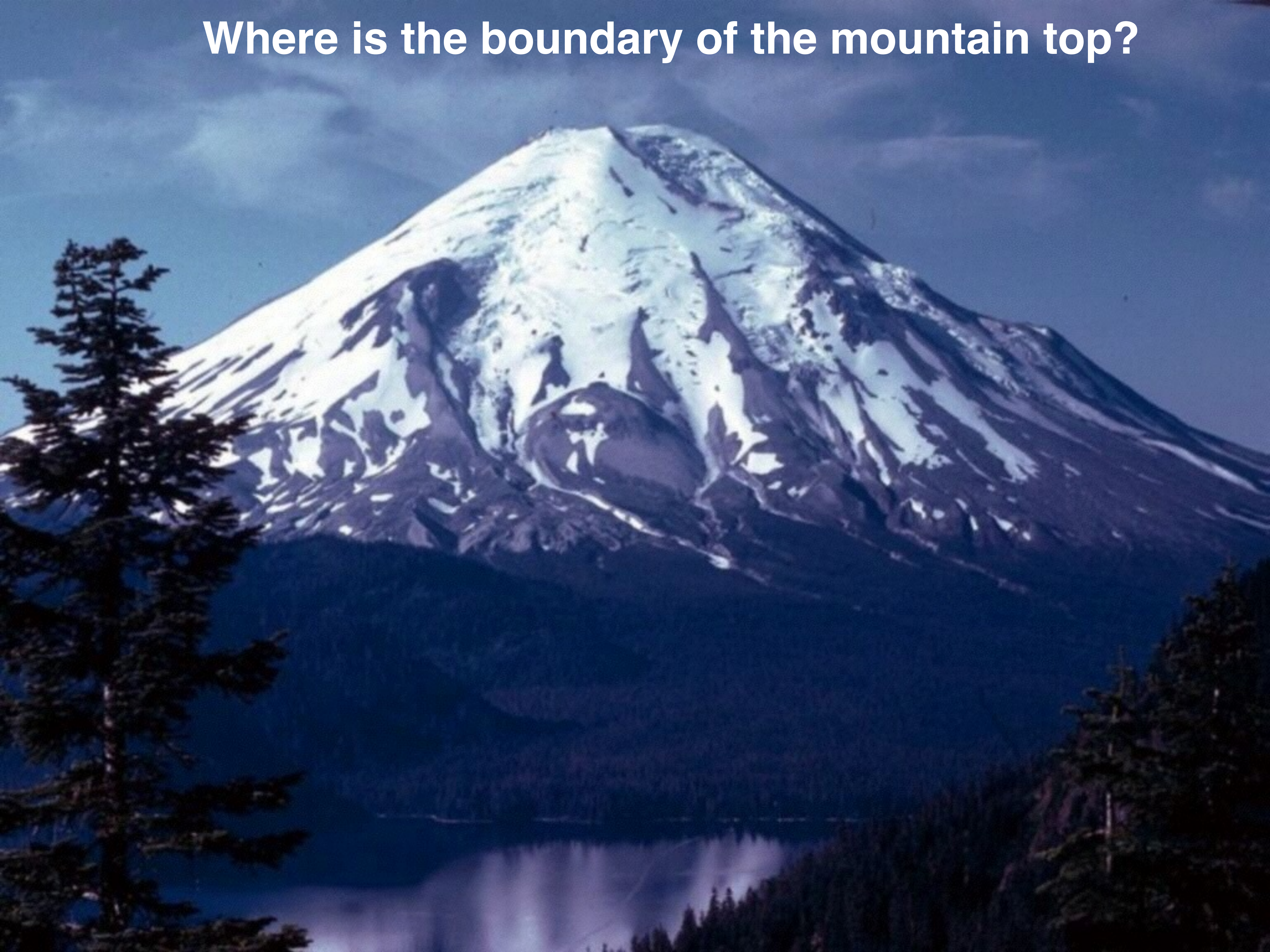


Computational Photography
(image inpainting)

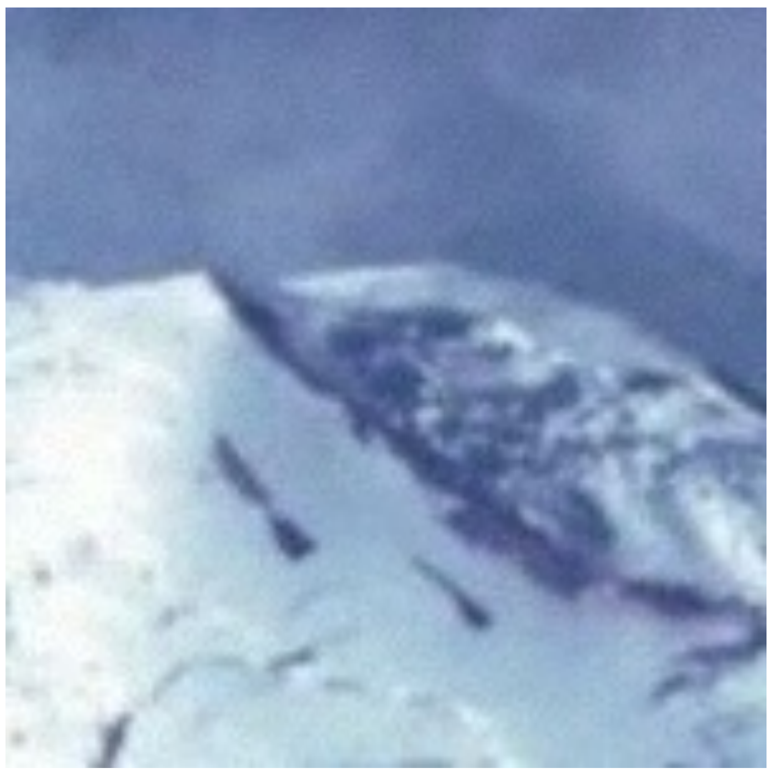


Extracting Lines

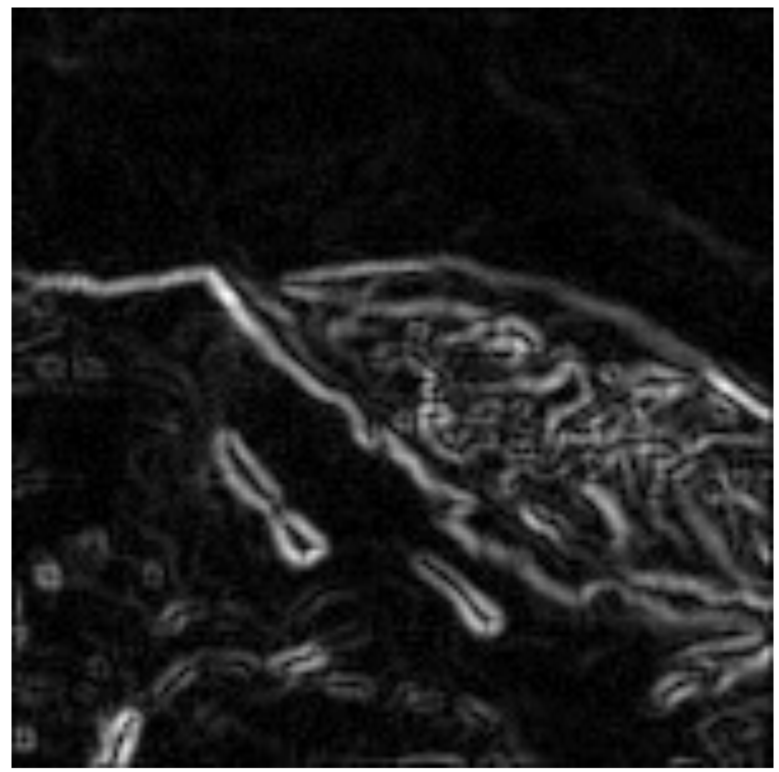
16-385 Computer Vision


Where is the boundary of the mountain top?

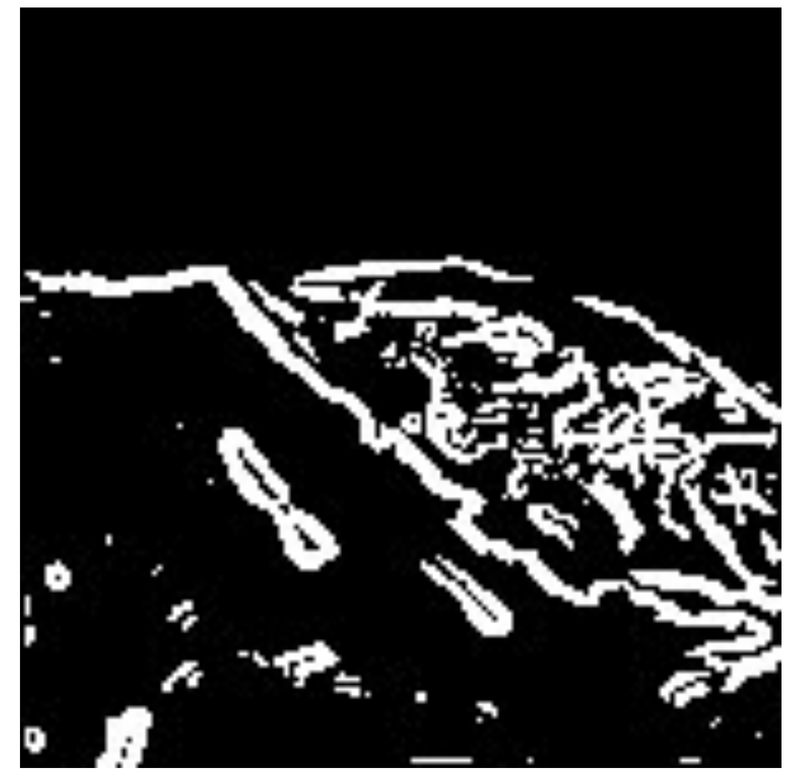
Lines are hard to find



Original image



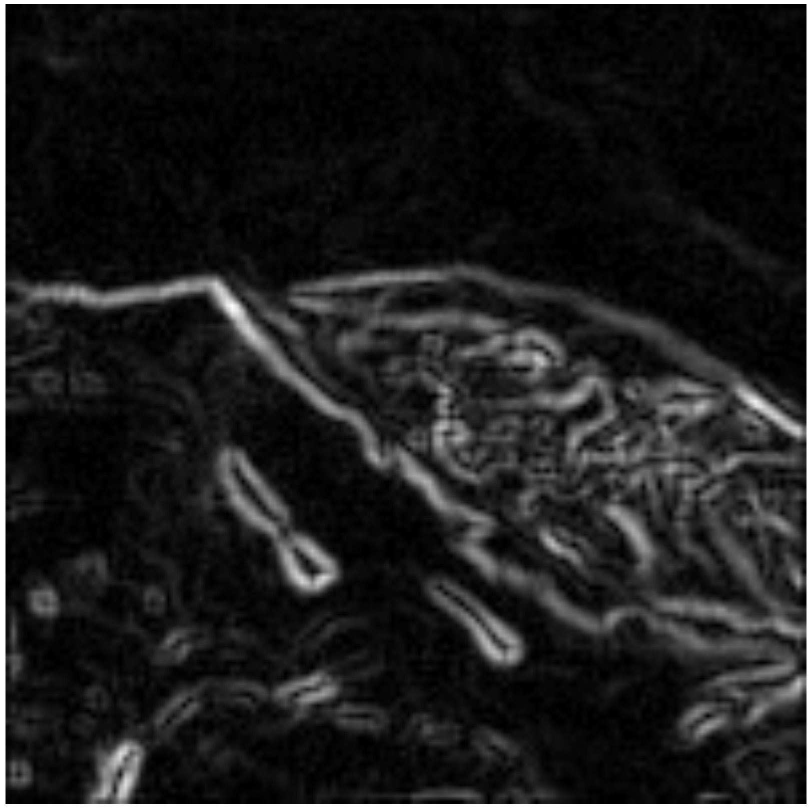
Edge detection



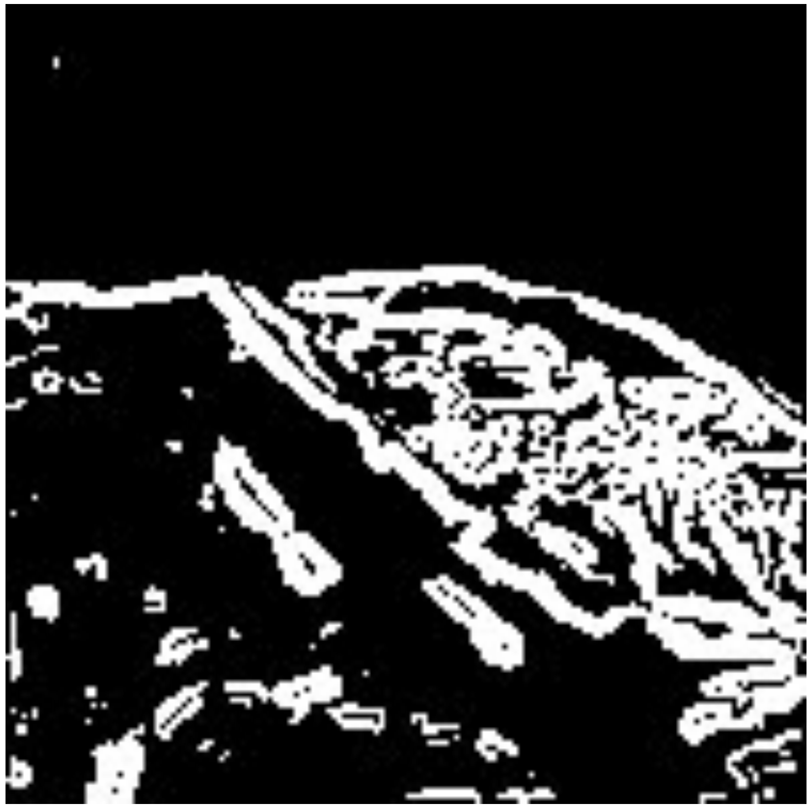
Thresholding

Noisy edge image
Incomplete boundaries

idea #1: morphology



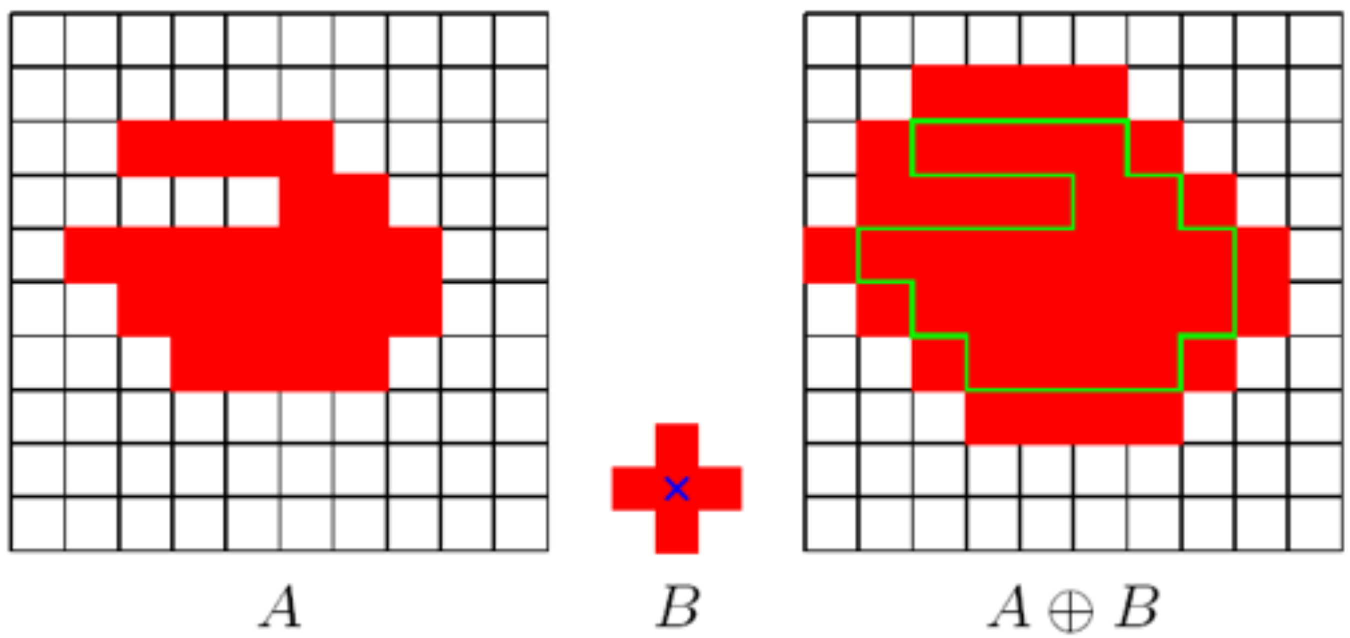
Sobel



Threshold

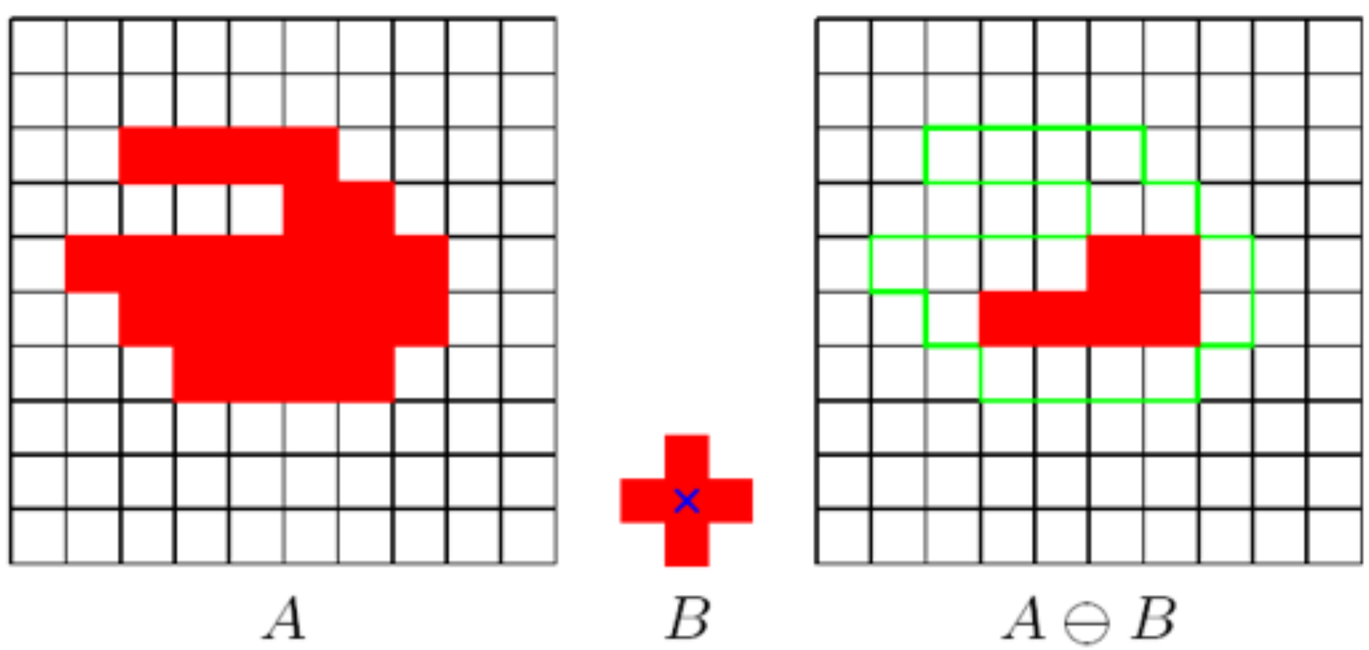
Shrink, Expand, Shrink

What are some problems with the approach?



Dilation

If filter response > 0, set to 1



Erosion

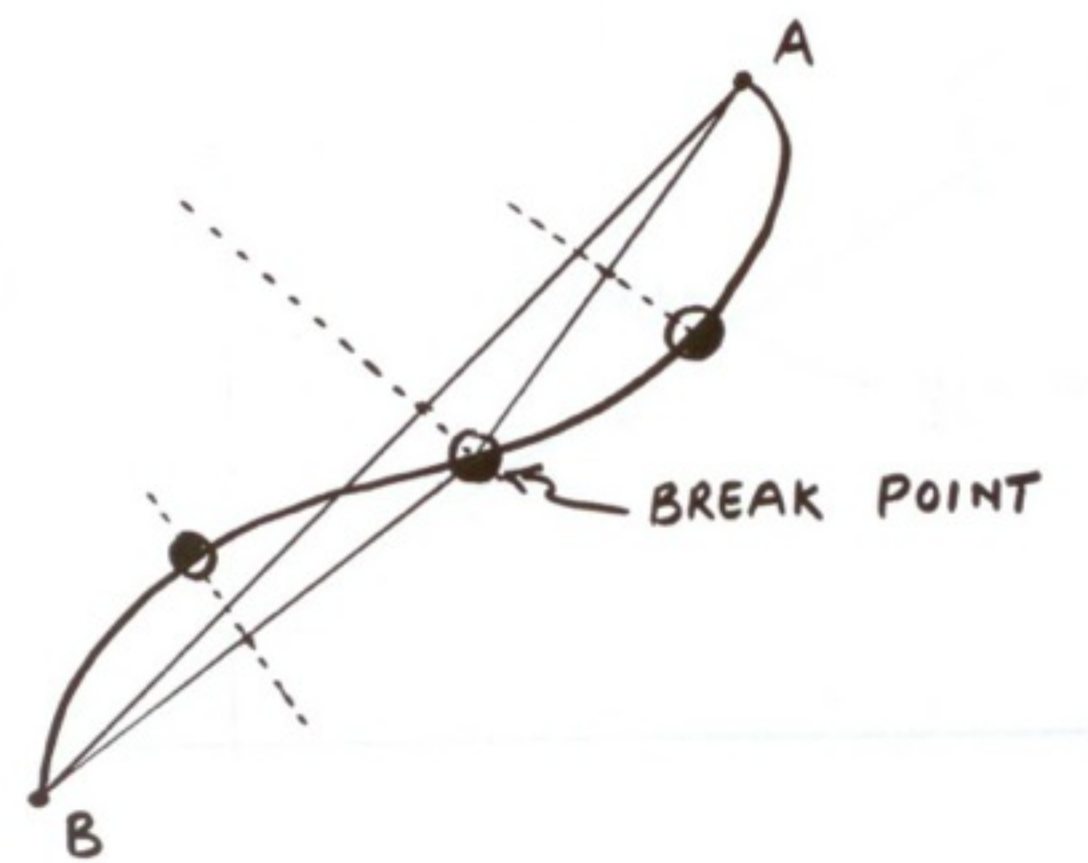
If filter response is MAX, set to 1

idea #2: breaking lines

Divide and Conquer:

Given: Boundary lies between points A and B

Task: Find boundary




```
Connect A and B with Line
  Find strongest edge along line bisector
  Use edge point as break point
Repeat
```

What are some problems with the approach?

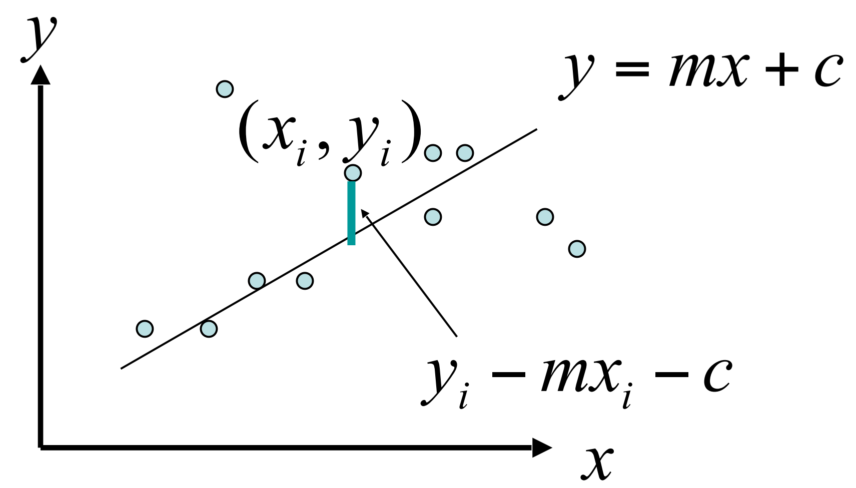
idea #3: line fitting

Given: Many (x_i, y_i) pairs

Find: Parameters (m, c)

Minimize: Average square distance:

$$E = \sum_i \frac{(y_i - mx_i - c)^2}{N}$$

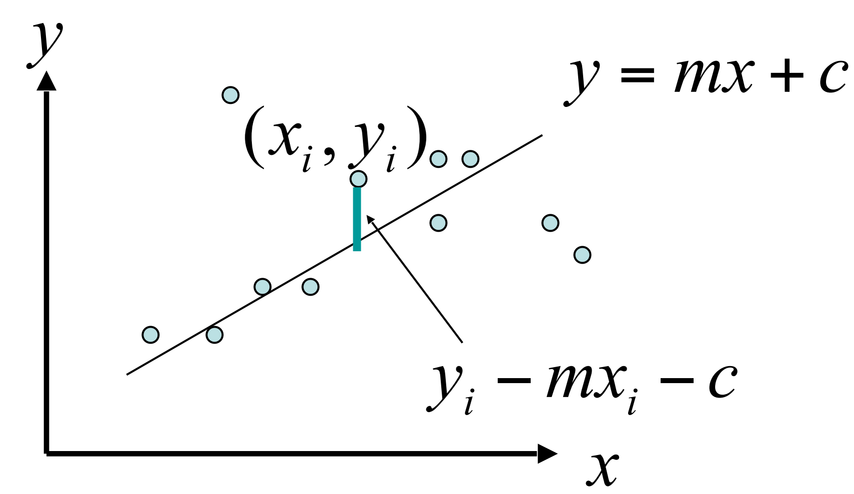


What are some problems with the approach?

idea #3: line fitting

Given: Many (x_i, y_i) pairs

Find: Parameters (m, c)



Minimize: Average square distance:

$$E = \sum_i \frac{(y_i - mx_i - c)^2}{N}$$

Using:

$$\frac{\partial E}{\partial m} = 0 \quad \& \quad \frac{\partial E}{\partial c} = 0$$

Note:

$$\bar{y} = \frac{\sum_i y_i}{N} \qquad \bar{x} = \frac{\sum_i x_i}{N}$$

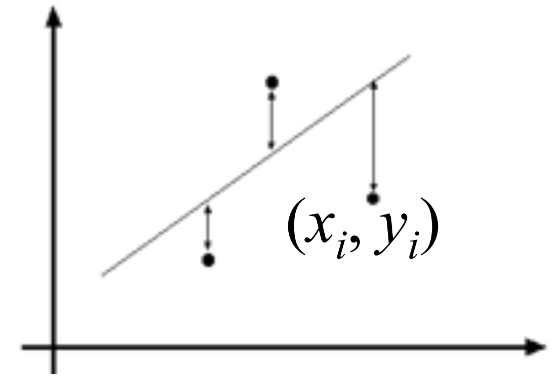
$$c = \bar{y} - m\bar{x}$$

$$m = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

What are some problems with the approach?

Data: $(x_1, y_1), \ldots, (x_n, y_n)$

Line equation: $y_i = m\, x_i + b$



Find (m, b) to minimize $\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$

$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \qquad X = \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \qquad B = \begin{bmatrix} m \\ b \end{bmatrix}$$

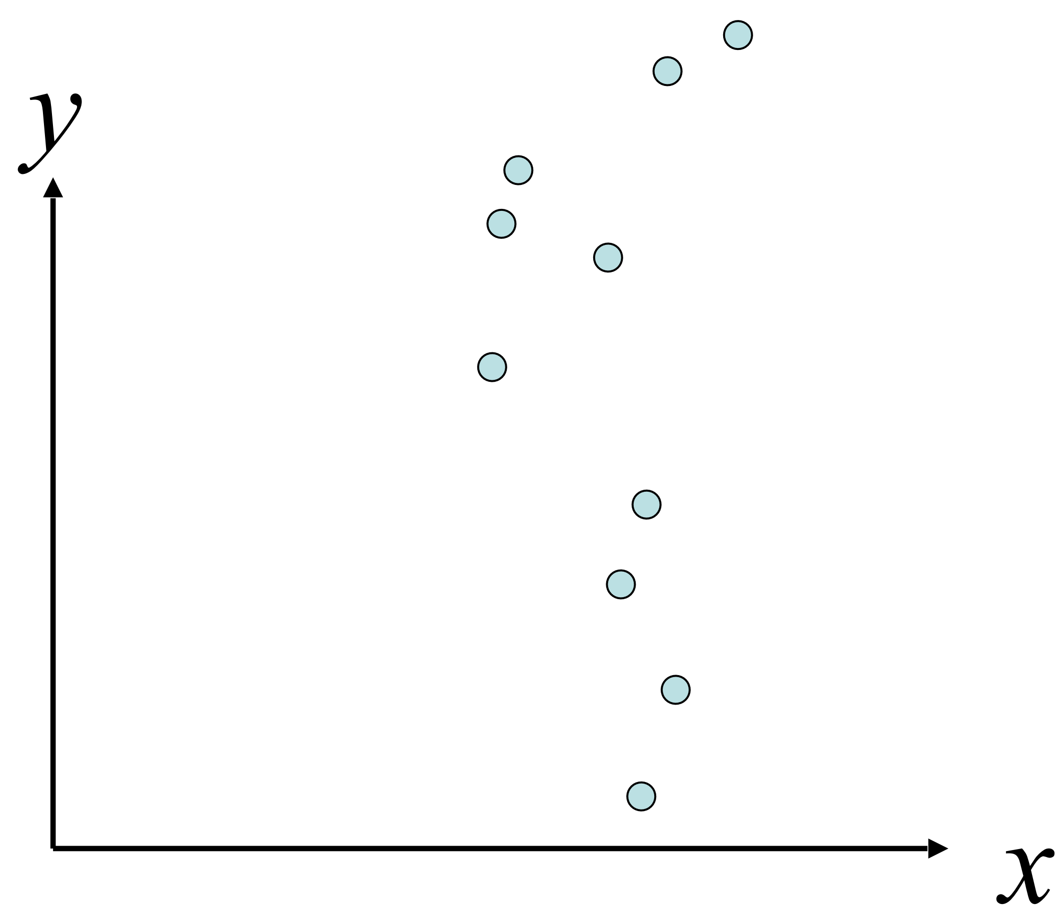
$$E = \|Y - XB\|^2 = (Y - XB)^T (Y - XB) = Y^T Y - 2(XB)^T Y + (XB)^T (XB)$$

$$\frac{dE}{dB} = 2X^T XB - 2X^T Y = 0 \qquad \boxed{X^T XB = X^T Y}$$

Normal equations: least squares solution to $XB=Y$

Problems with parameterizations

Where is the line that minimizes E? $E = \sum_{i=1}^{n}(y_i - mx_i - b)^2$



Problems with parameterizations

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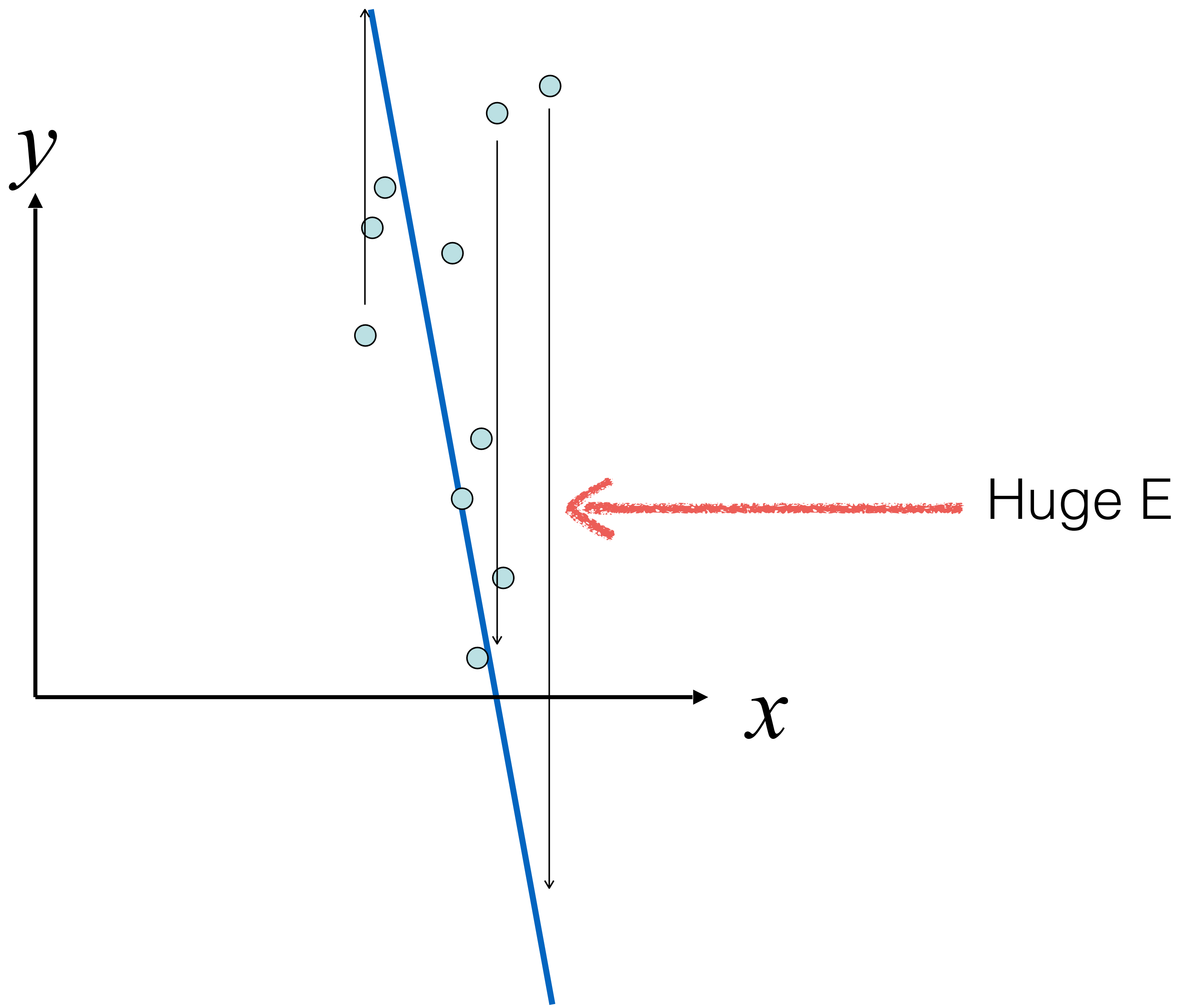
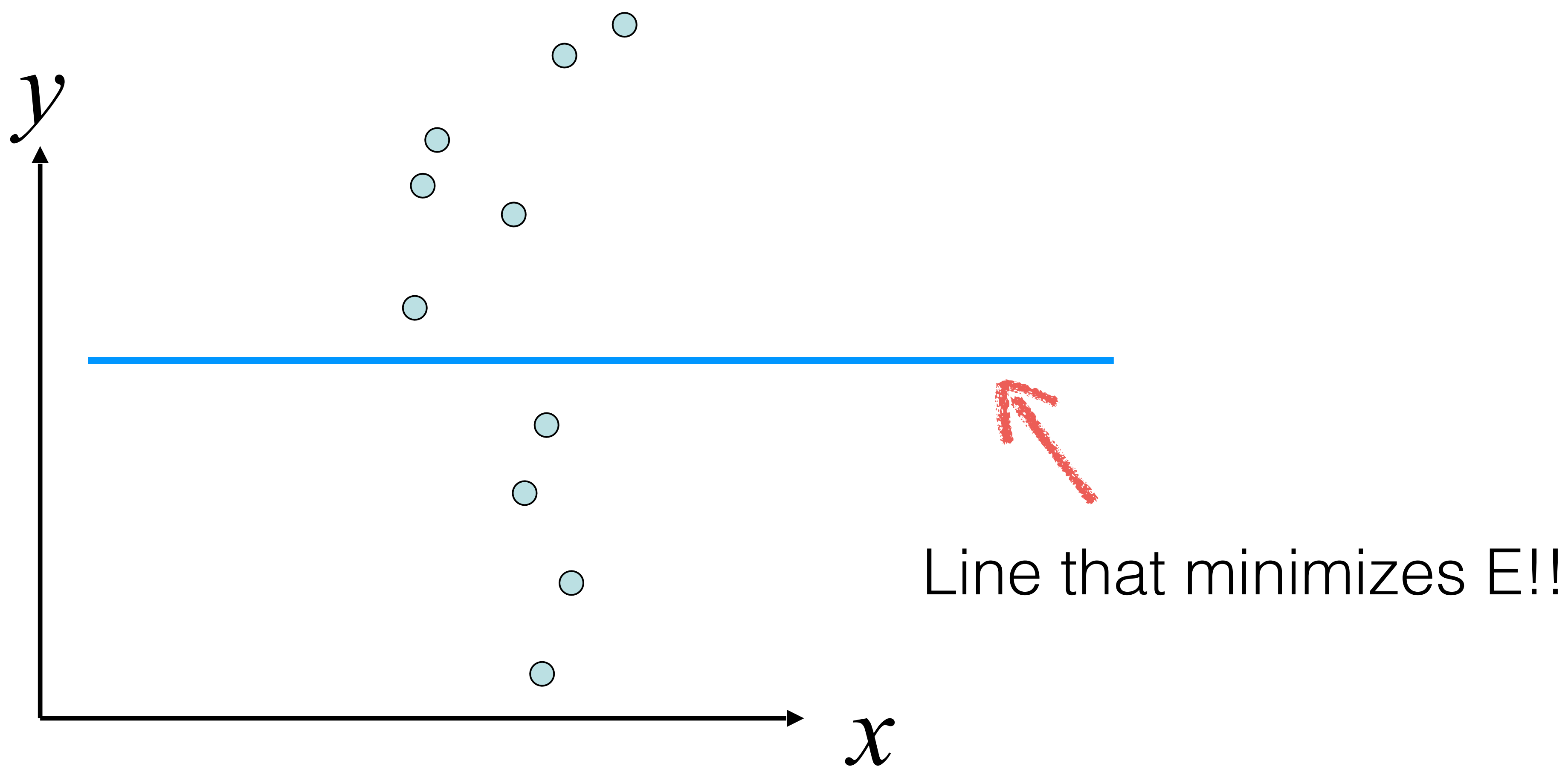


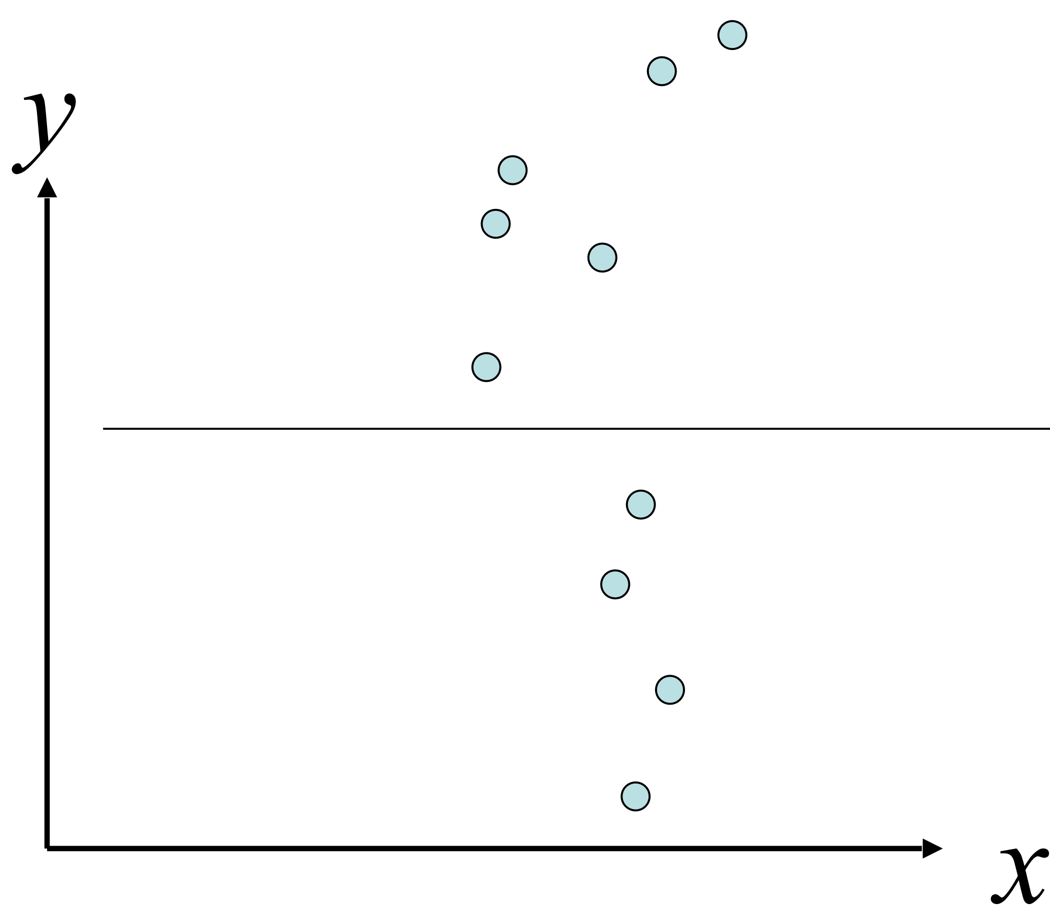
Problems with parameterizations

Where is the line that minimizes E? $\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$



Problems with parameterizations

Where is the line that minimizes E?

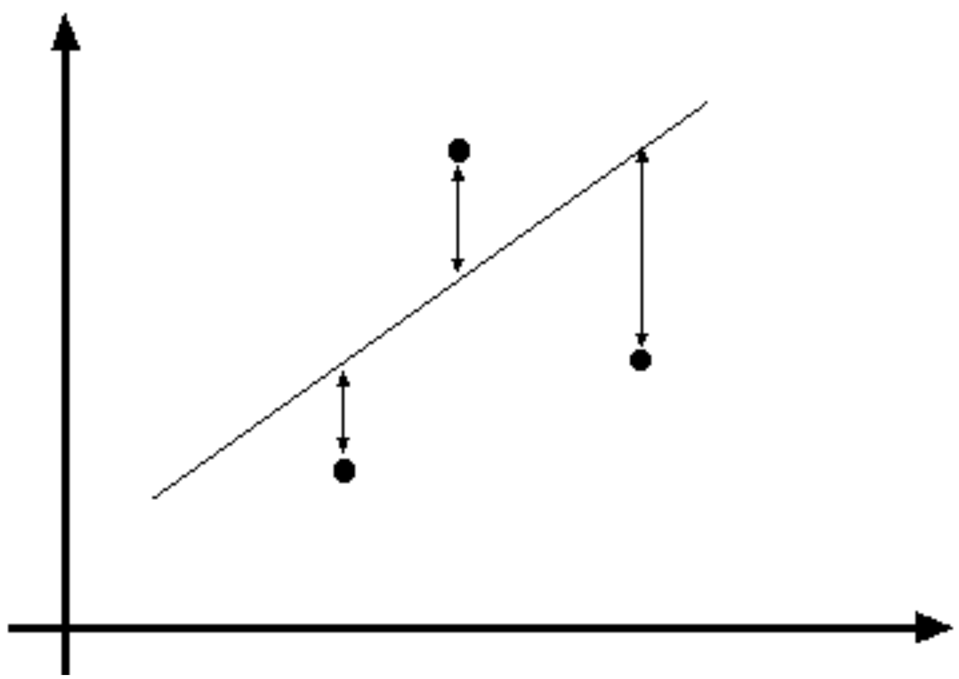


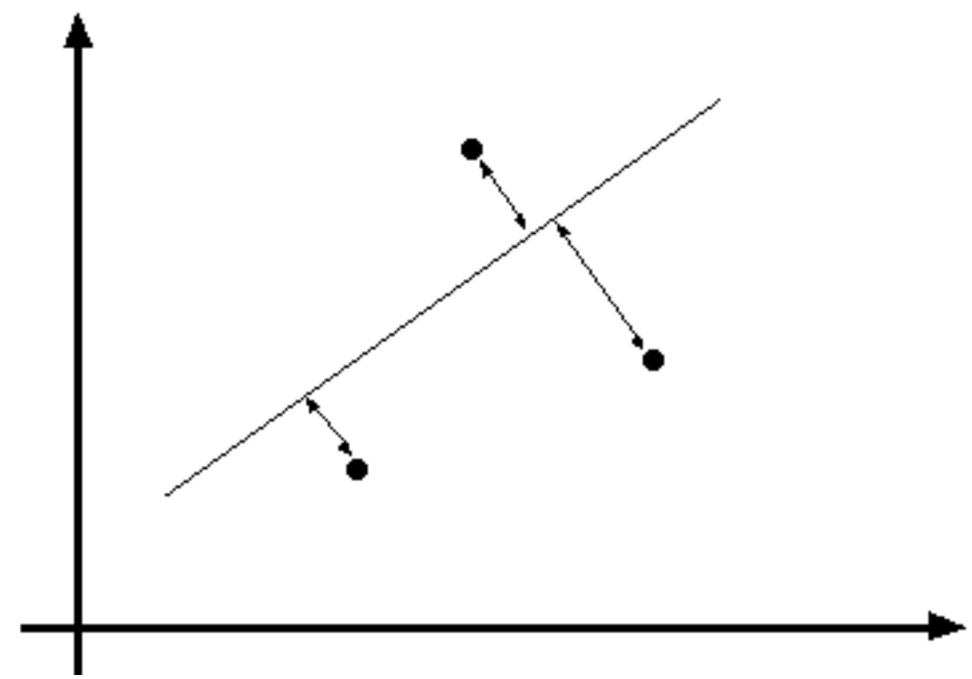
Line that minimizes E!!

(Error E must be formulated carefully!)

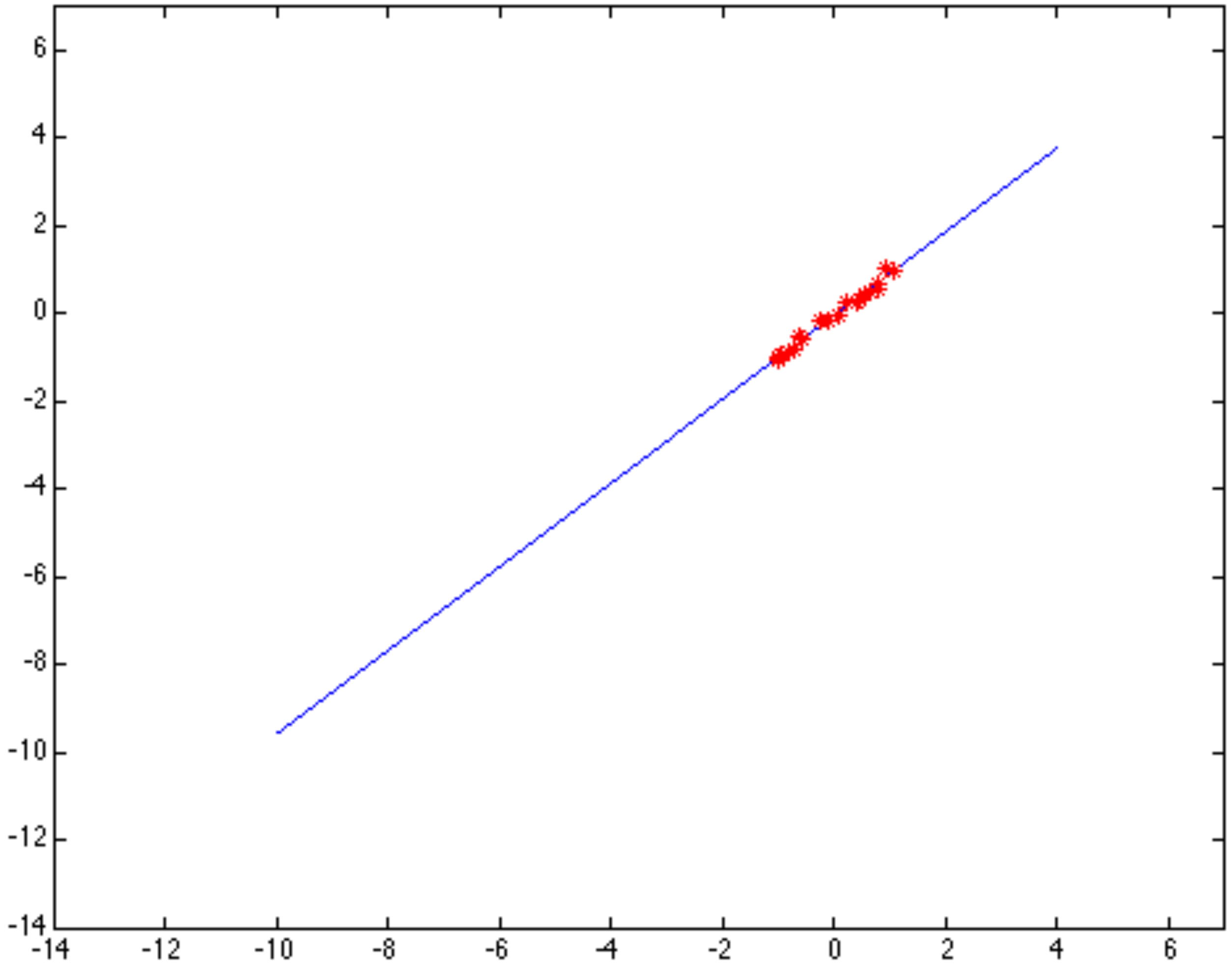
Use this instead: $$E = \frac{1}{N}\sum_i (\rho - x_i \cos\theta + y_i \sin\theta)^2$$ I'll explain this later …

Line fitting can be maximum likelihood
- but choice of model is important

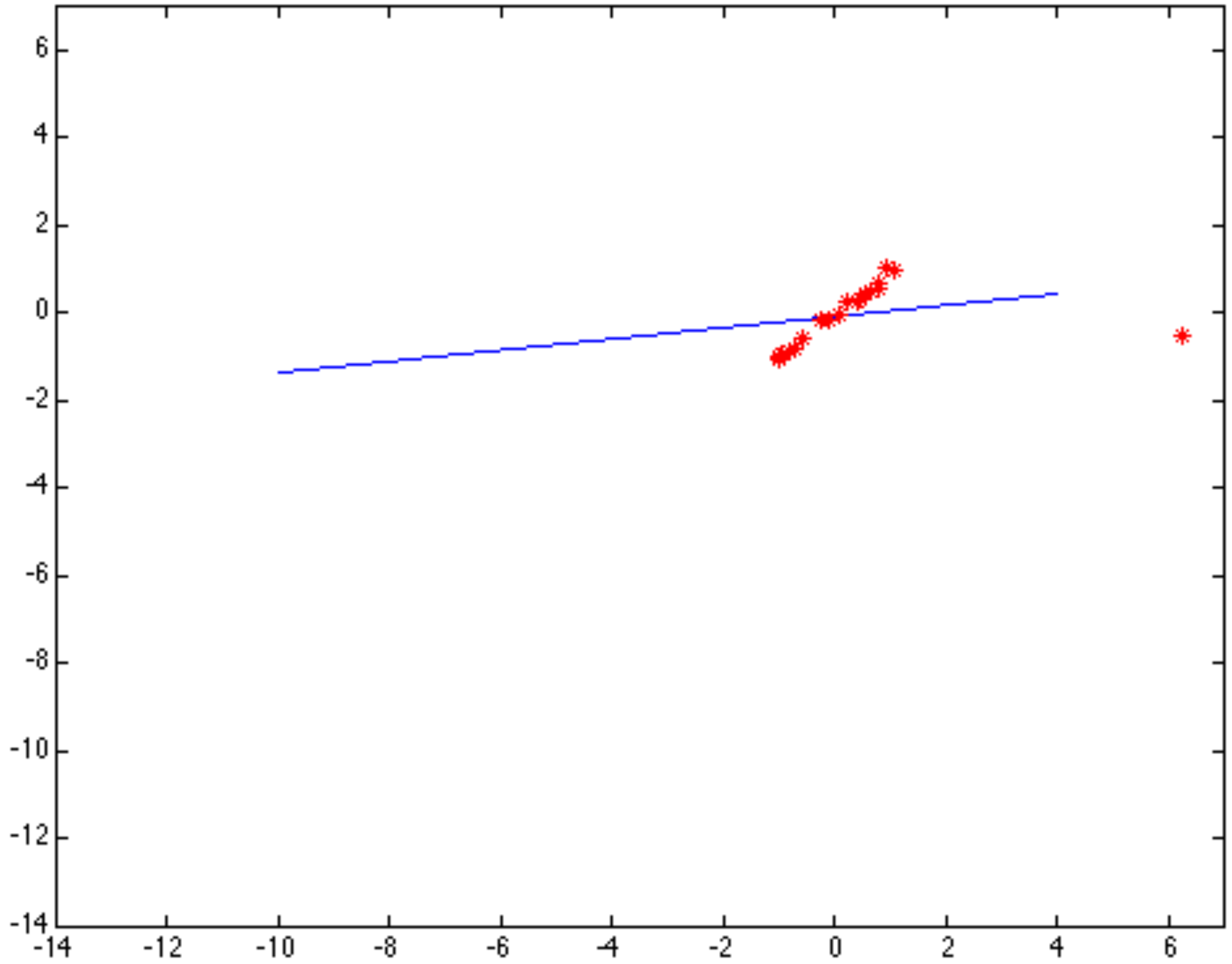




Problems with noise



Least-squares error fit



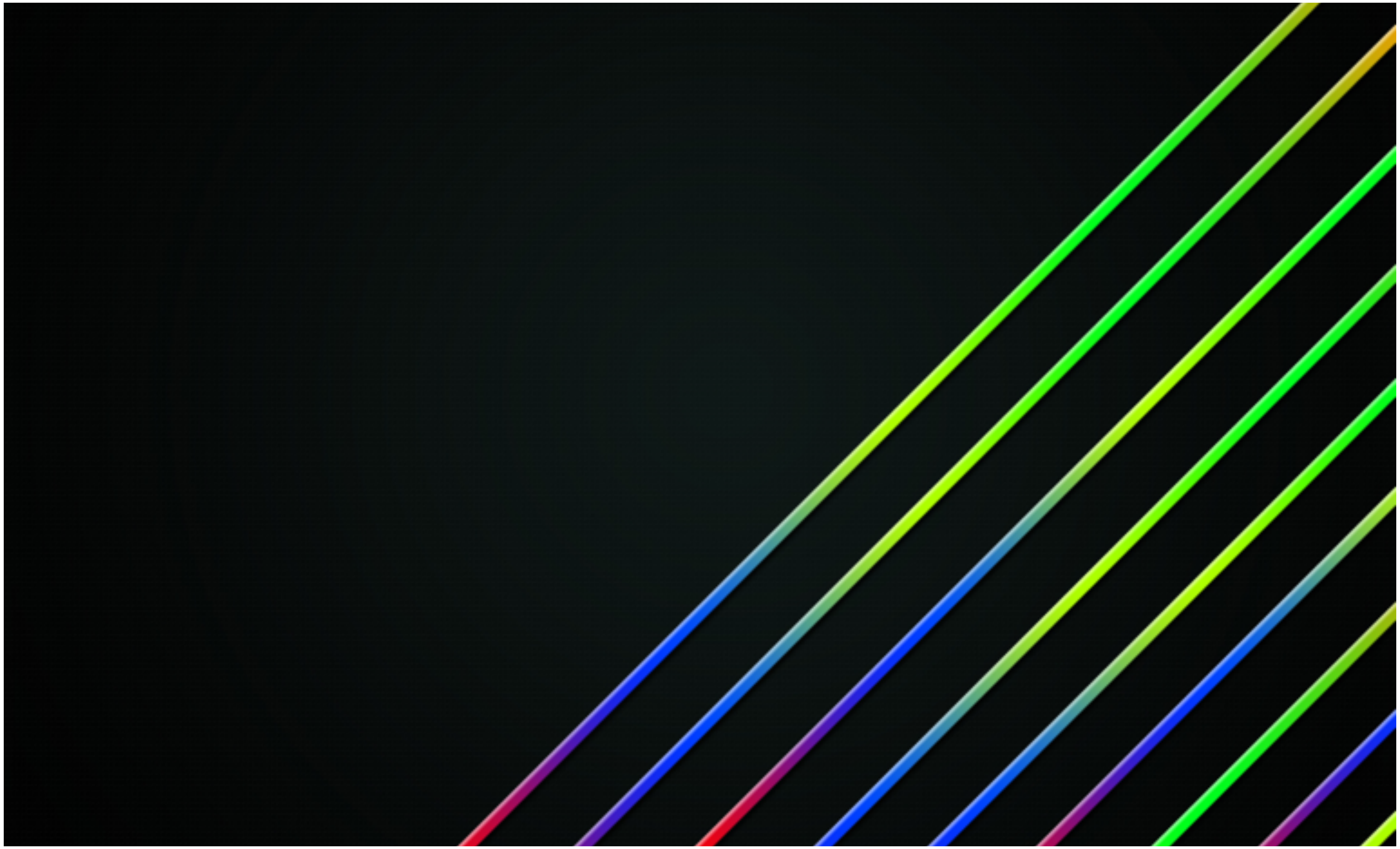
Squared error heavily penalizes outliers

Model fitting is difficult because…

- **Extraneous data:** clutter or multiple models
  - We do not know what is part of the model?
  - Can we pull out models with a few parts from much larger amounts of background clutter?
- **Missing data:** only some parts of model are present
- **Noise**

- Cost:
  - It is not feasible to check all combinations of features by fitting a model to each possible subset

So what can we do?



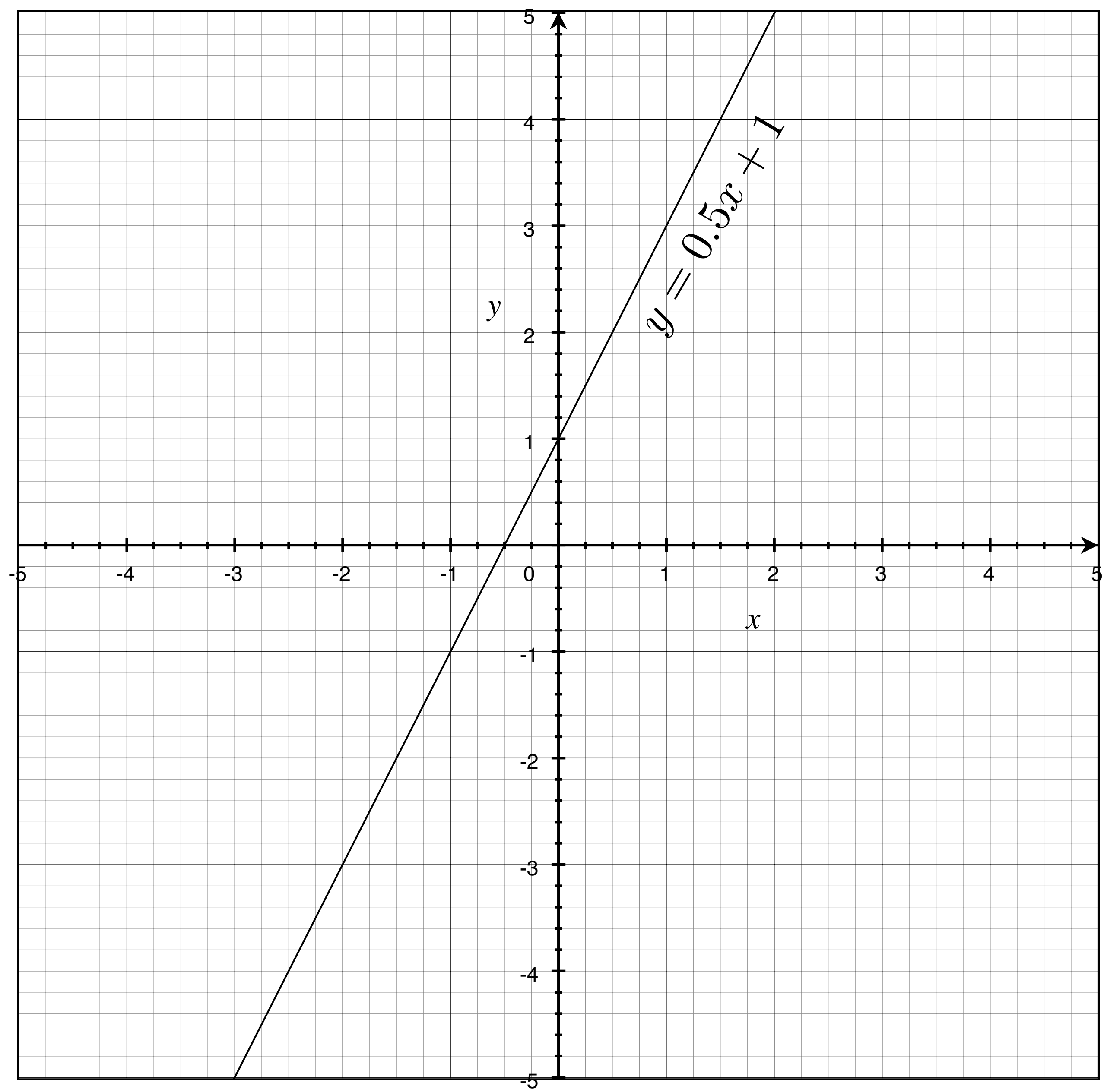
Lines Parameterization

16-385 Computer Vision

Slope intercept form

$$y = mx + b$$

slope y-intercept



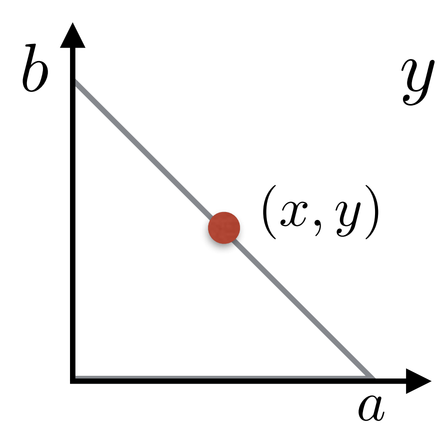
Double intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

x-intercept

y-intercept

Derivation:

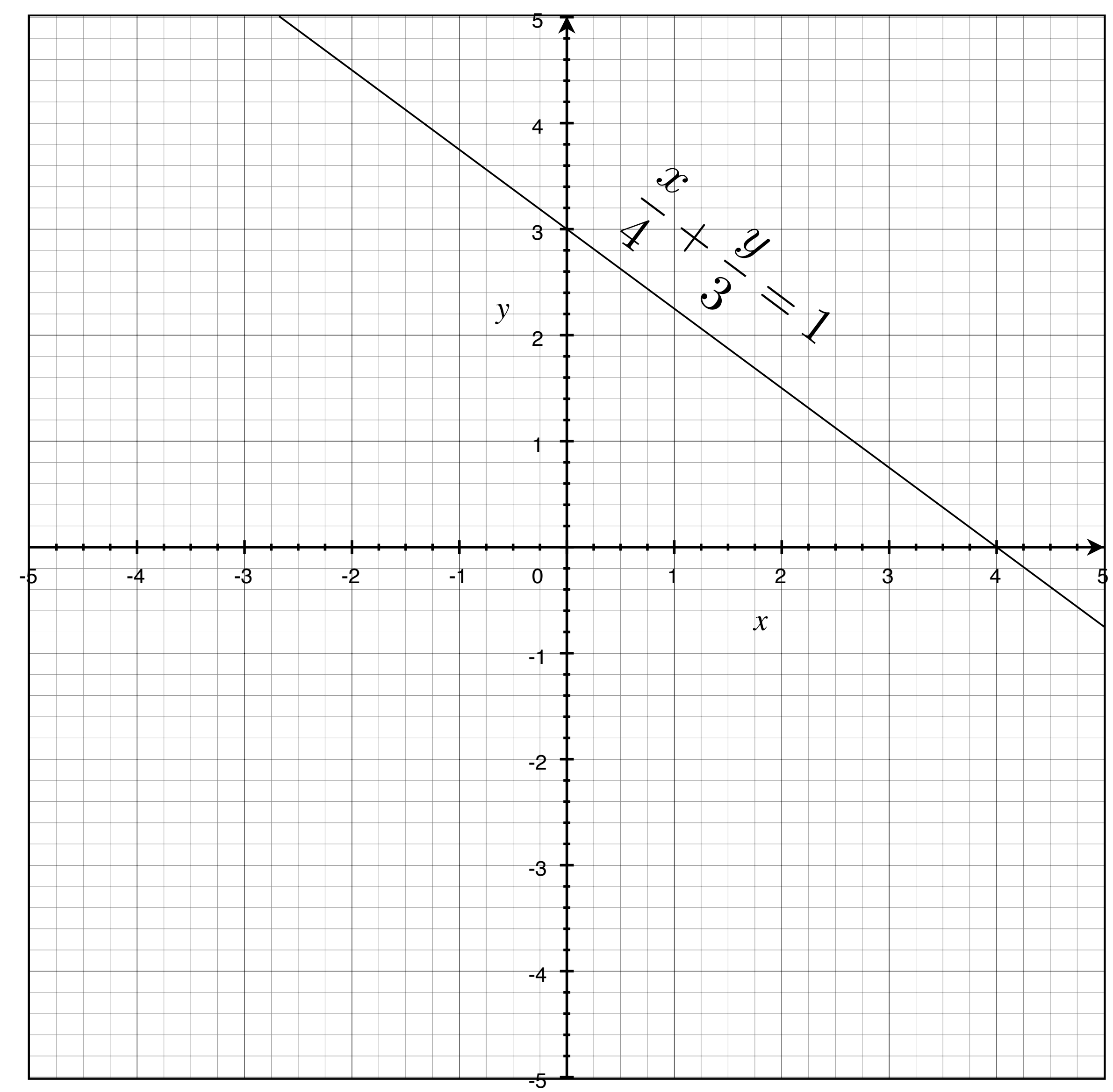


(Similar slope) $$\frac{y-b}{x-0} = \frac{0-y}{a-x}$$

$$ya + yx - ba + bx = -yx$$

$$ya + bx = ba$$

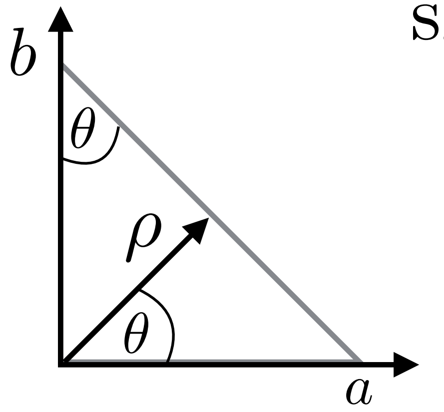
$$\frac{y}{b} + \frac{x}{a} = 1$$



Normal Form

$$x \cos\theta + y \sin\theta = \rho$$

Derivation:



$$\cos\theta = \frac{\rho}{a} \rightarrow a = \frac{\rho}{\cos\theta}$$

$$\sin\theta = \frac{\rho}{b} \rightarrow b = \frac{\rho}{\sin\theta}$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$x \cos\theta + y \sin\theta = \rho$$