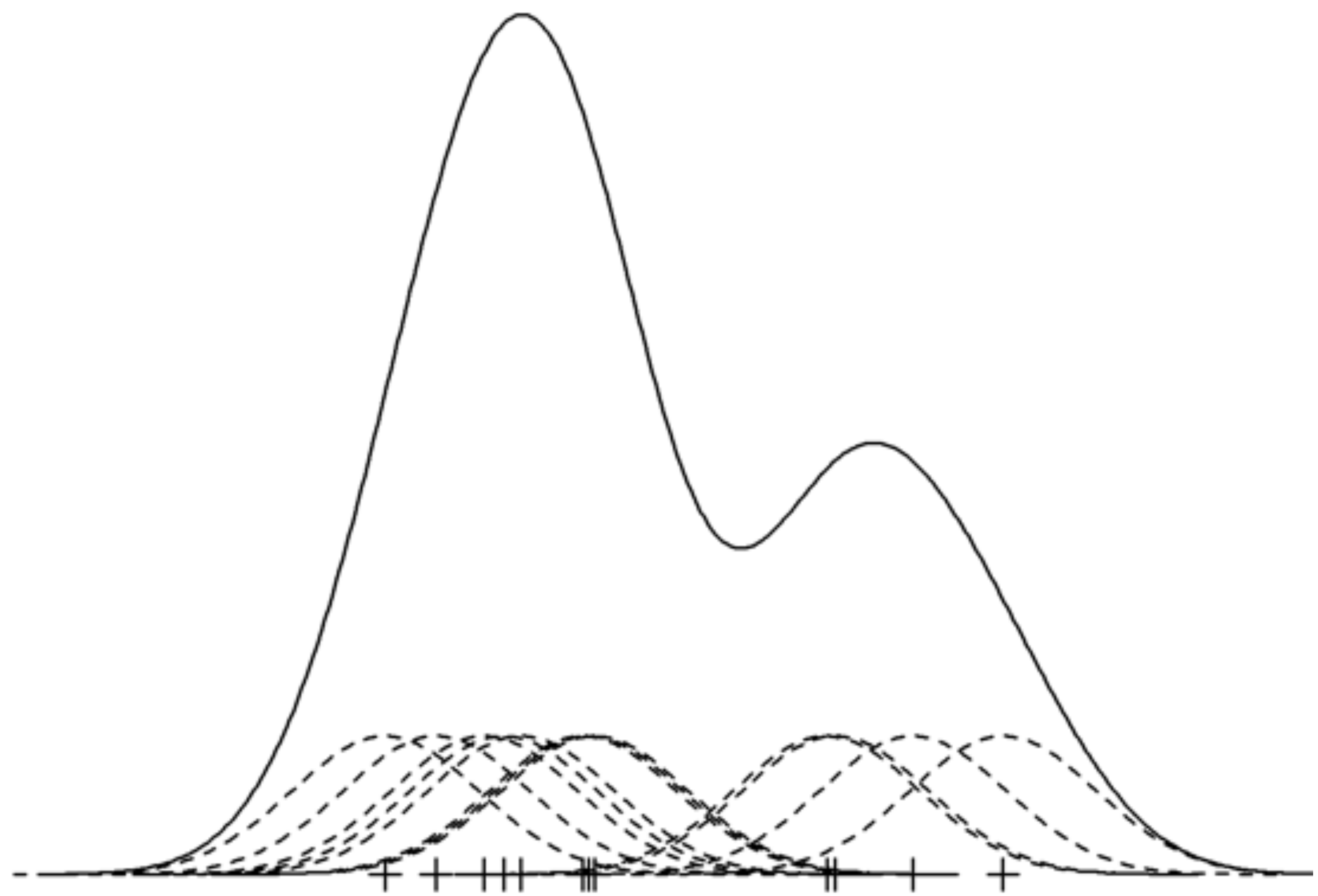


Mean-Shift Tracker

16-385 Computer Vision

Mean Shift Algorithm

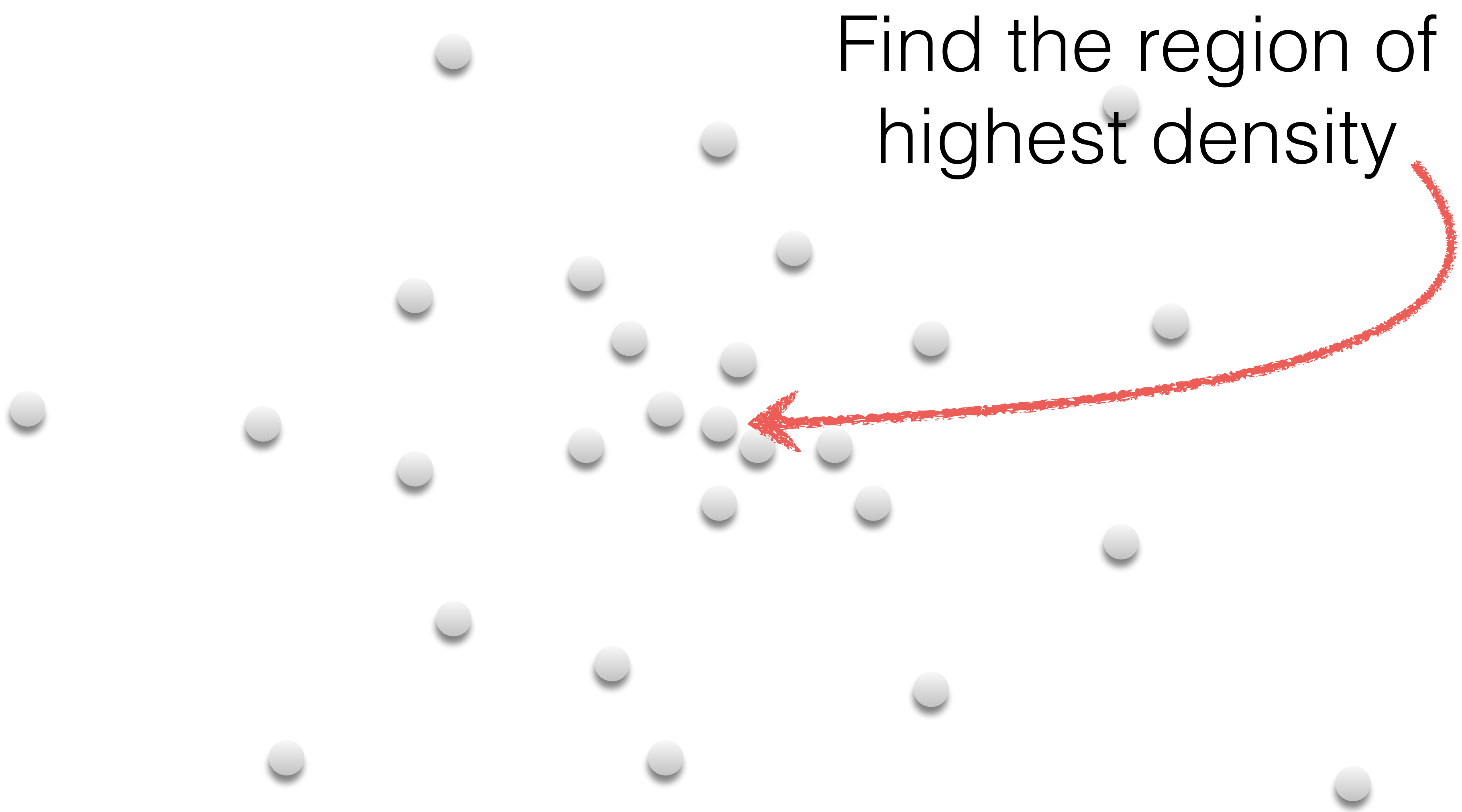
A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)

Mean Shift Algorithm

A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)



Mean Shift Algorithm

A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)

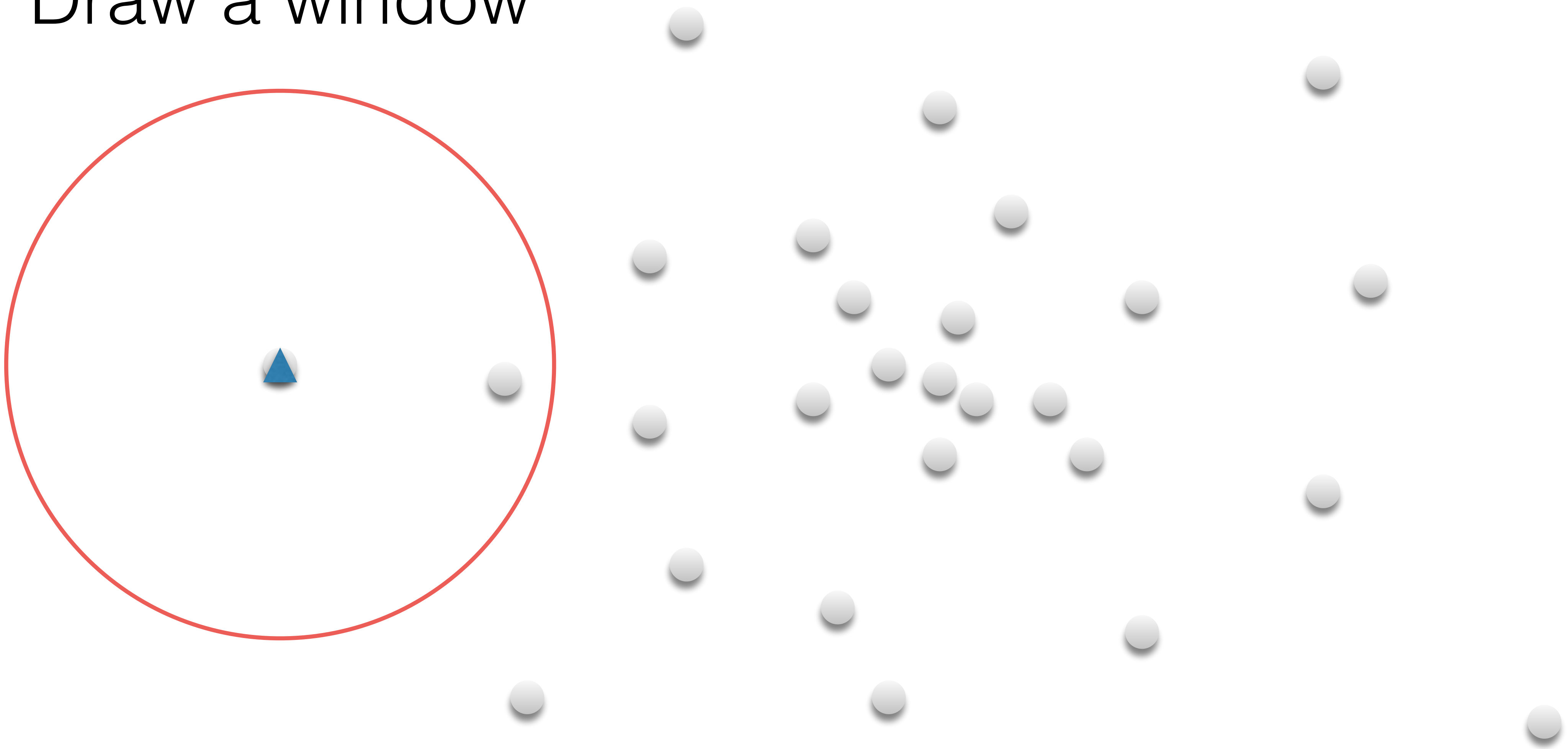
Pick a point

Mean Shift Algorithm

A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)

Draw a window

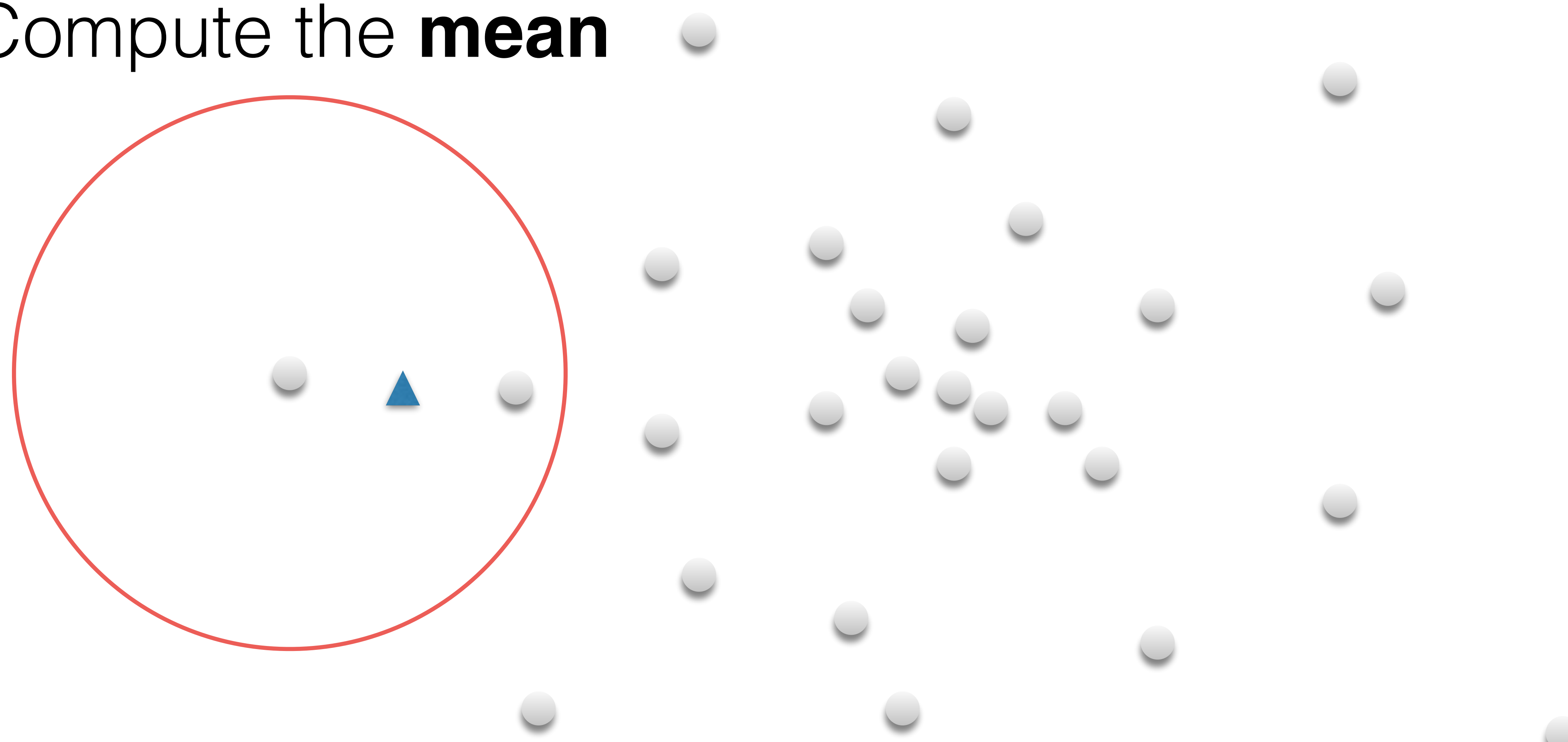


Mean Shift Algorithm

A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)

Compute the **mean**

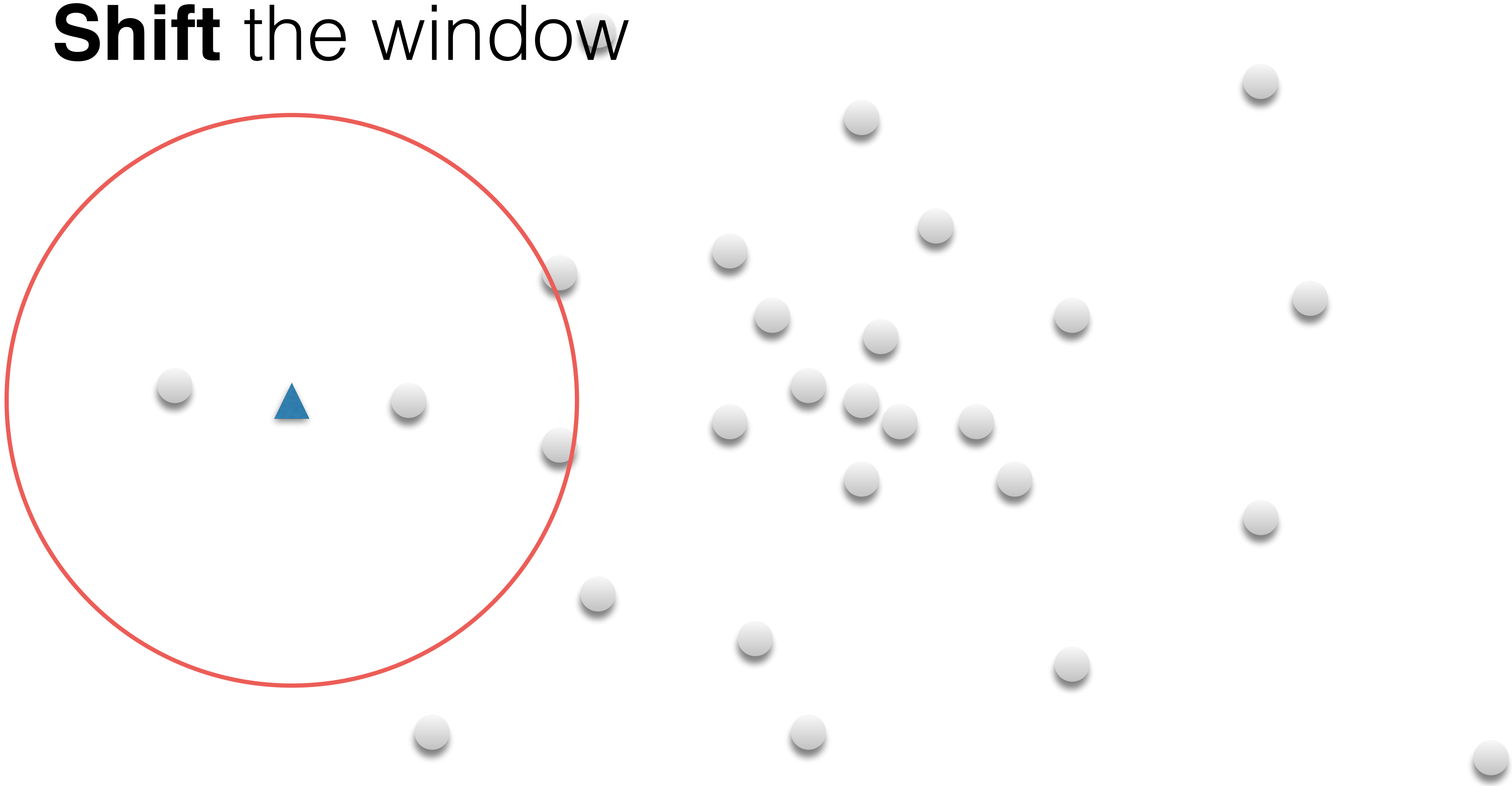


Mean Shift Algorithm

A ‘mode seeking’ algorithm

Fukunaga & Hostetler (1975)

Shift the window



Mean Shift Algorithm

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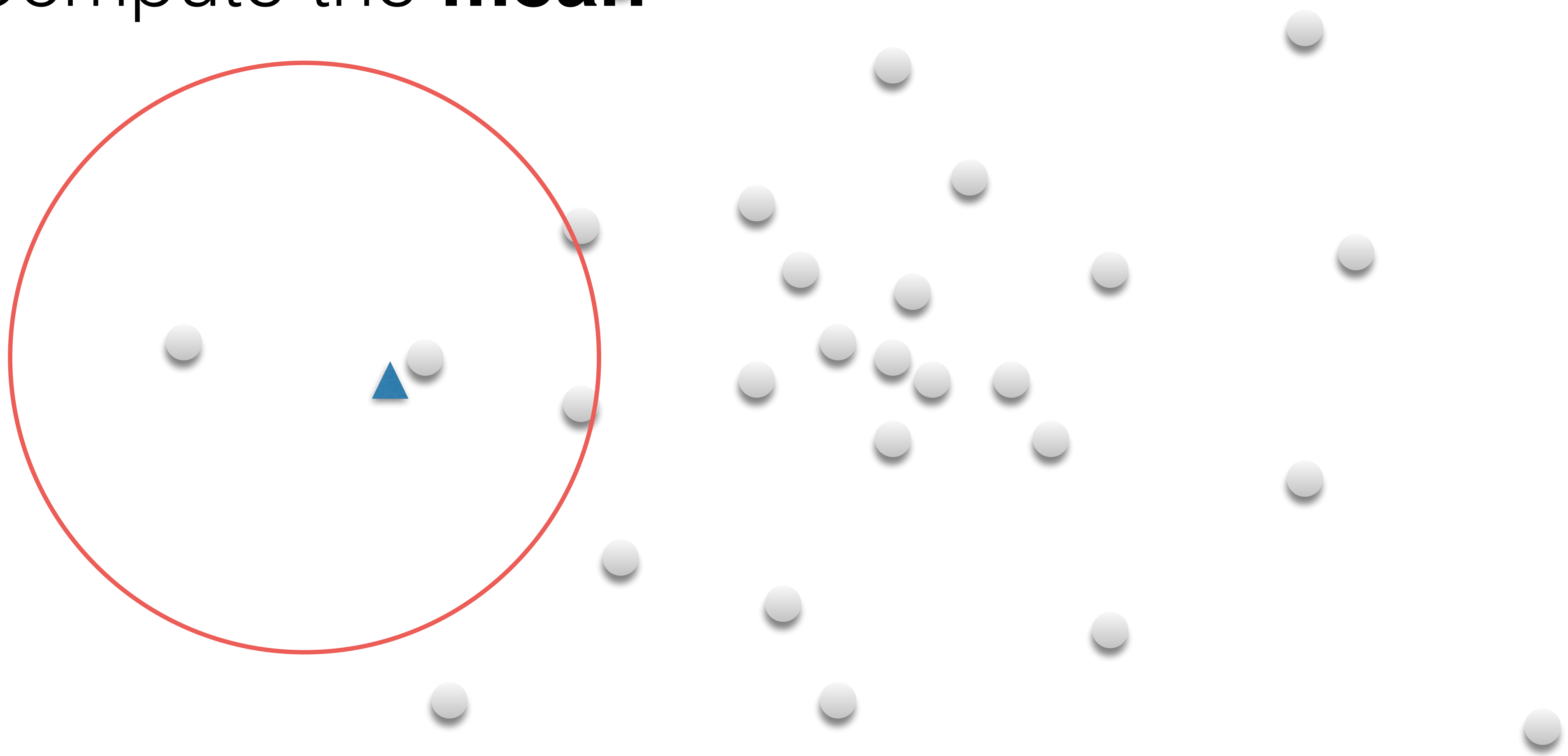
Fukunaga & Hostetler (1975)

Compute the **mean**



Mean Shift Algorithm

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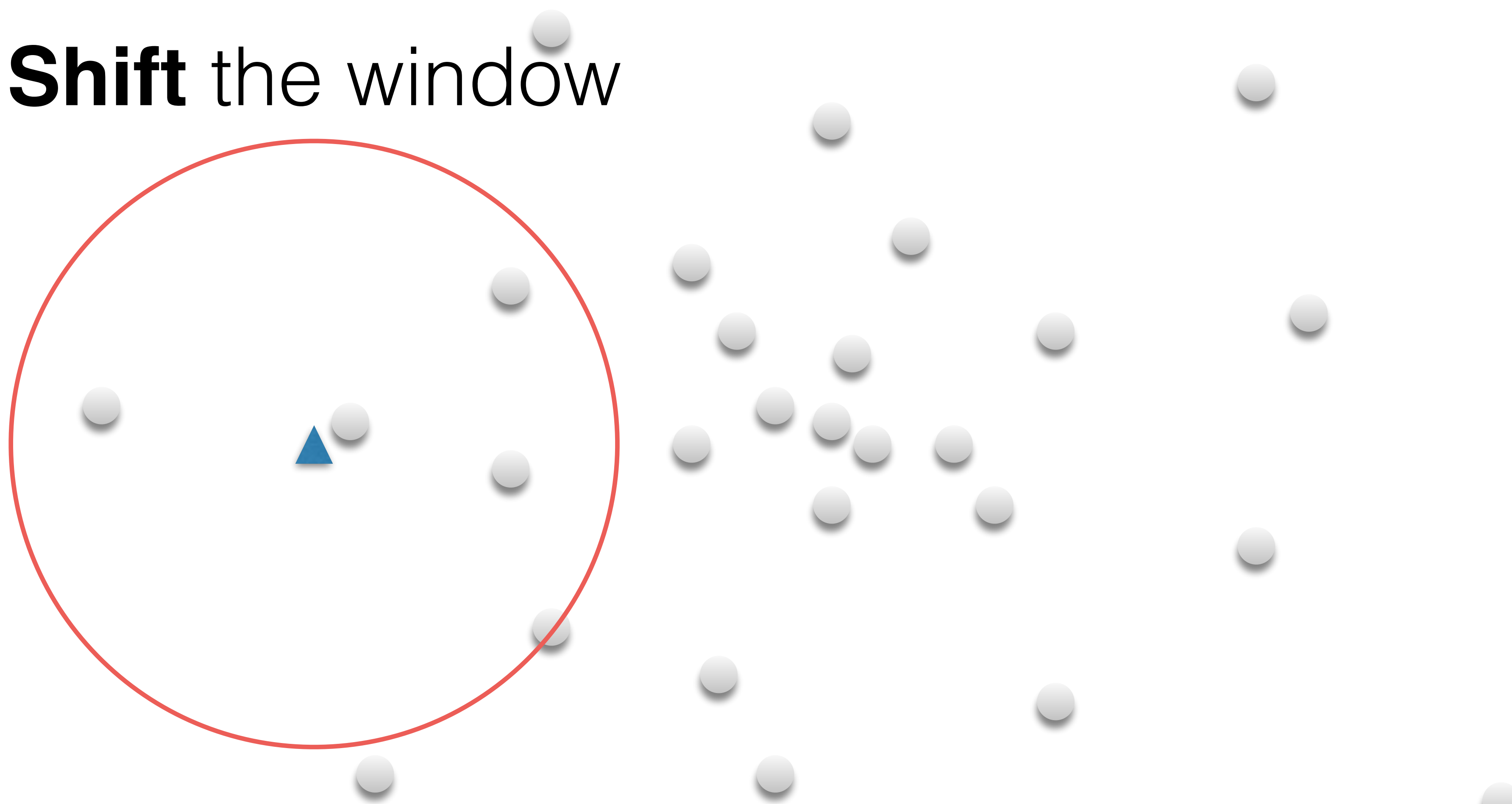
Shift the window



Mean Shift Algorithm

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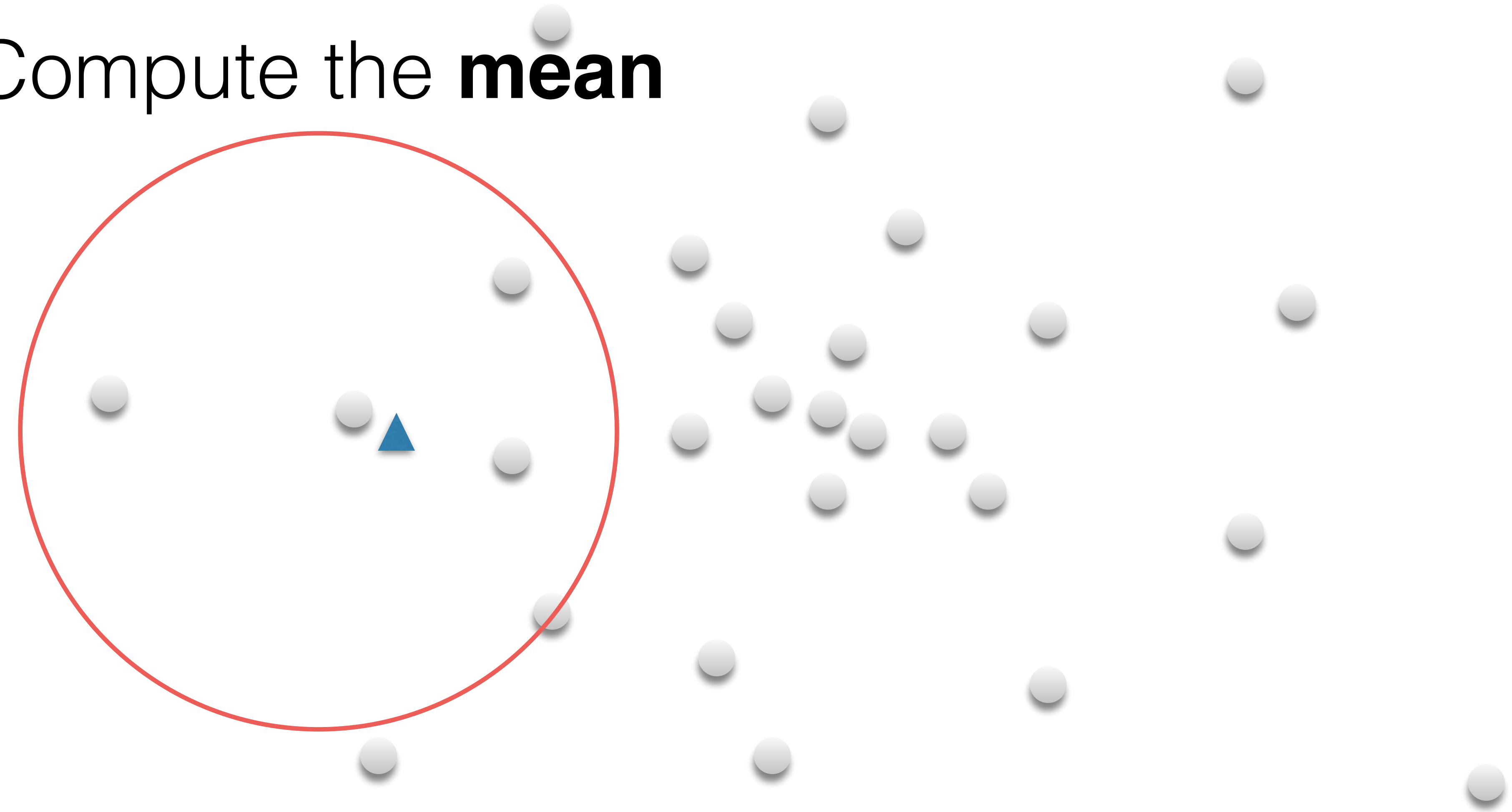
Compute the **mean**



Mean Shift Algorithm

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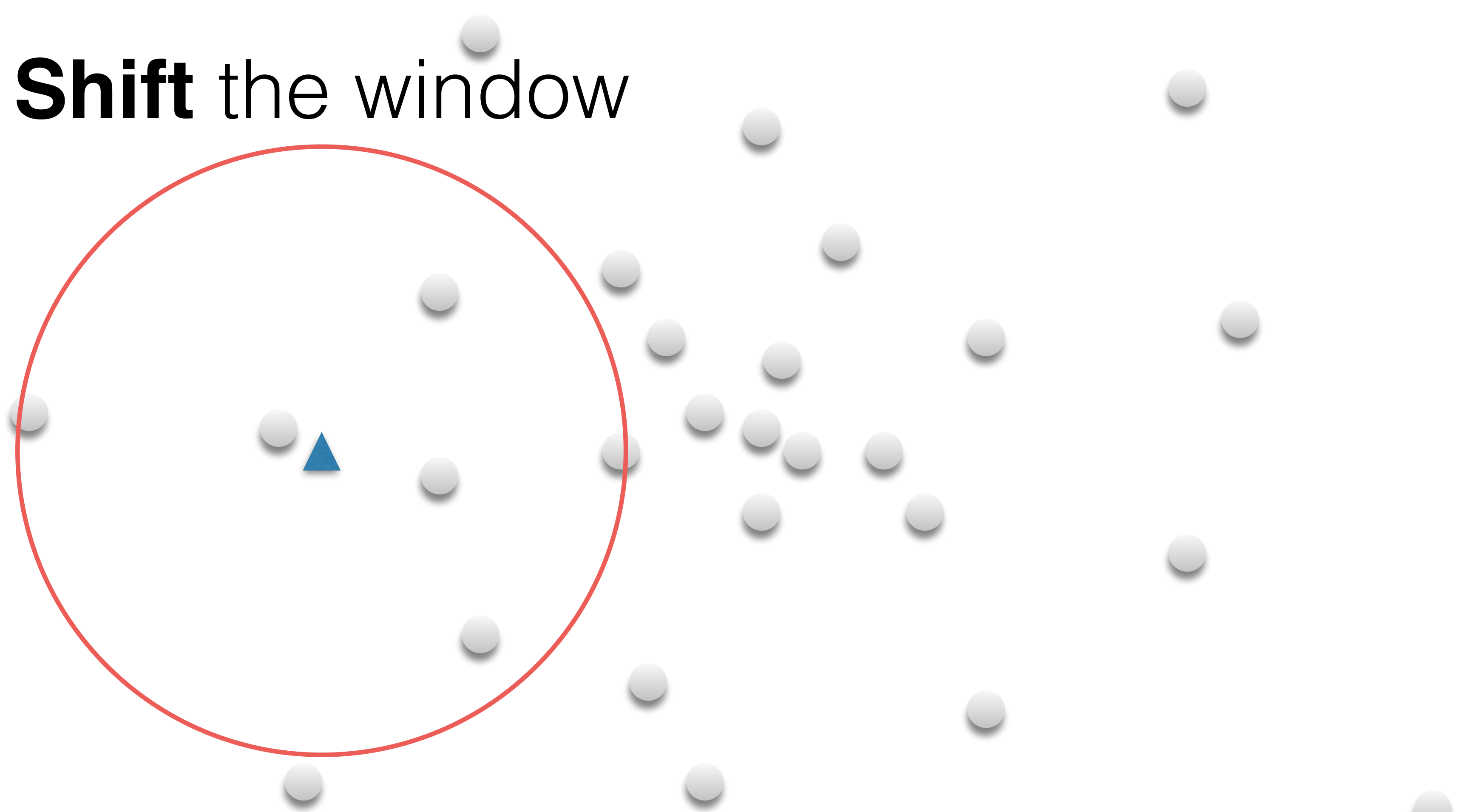
Shift the window



Mean Shift Algorithm

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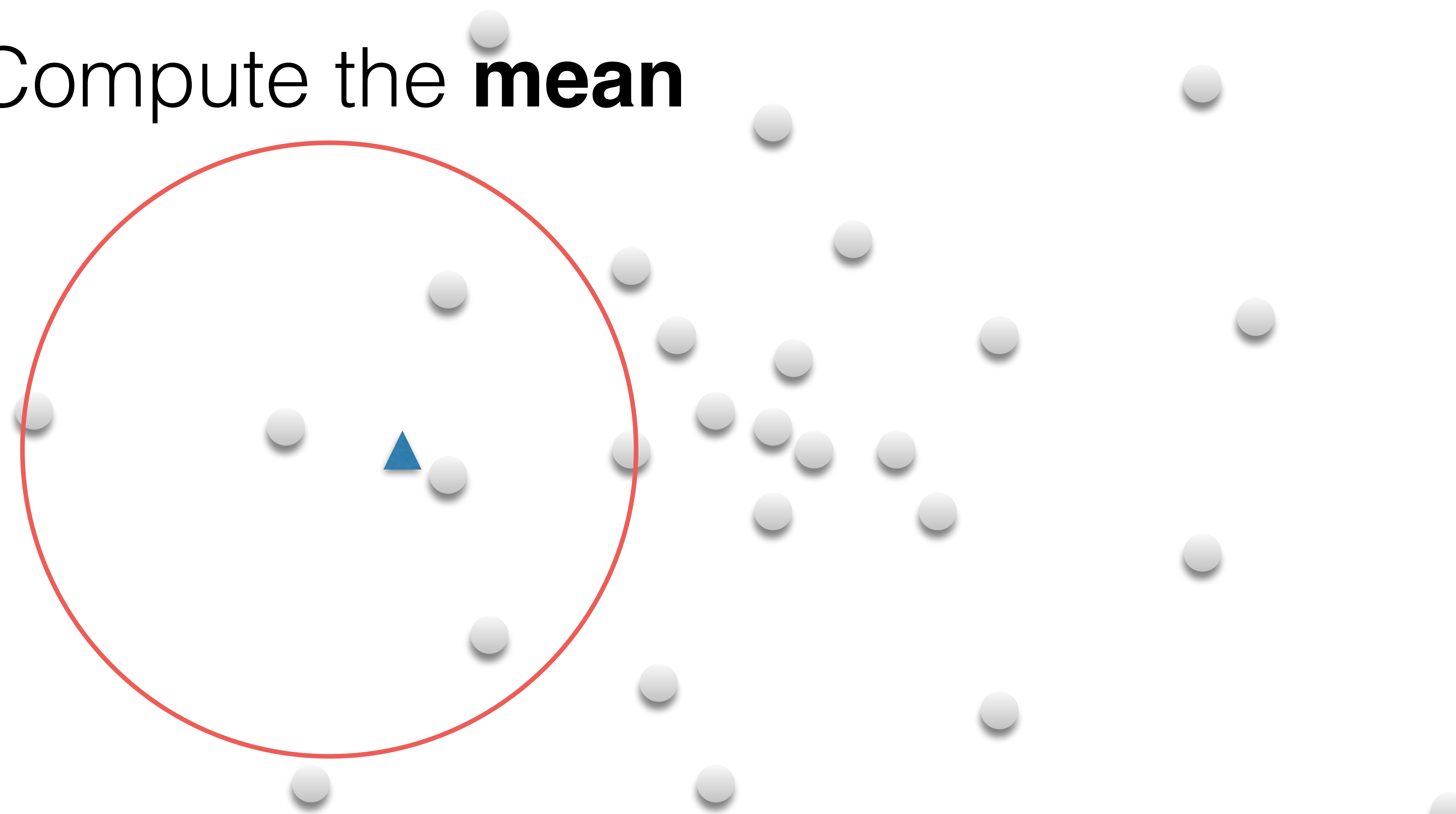

Mean Shift Algorithm

A ‘mode seeking’ algorithm

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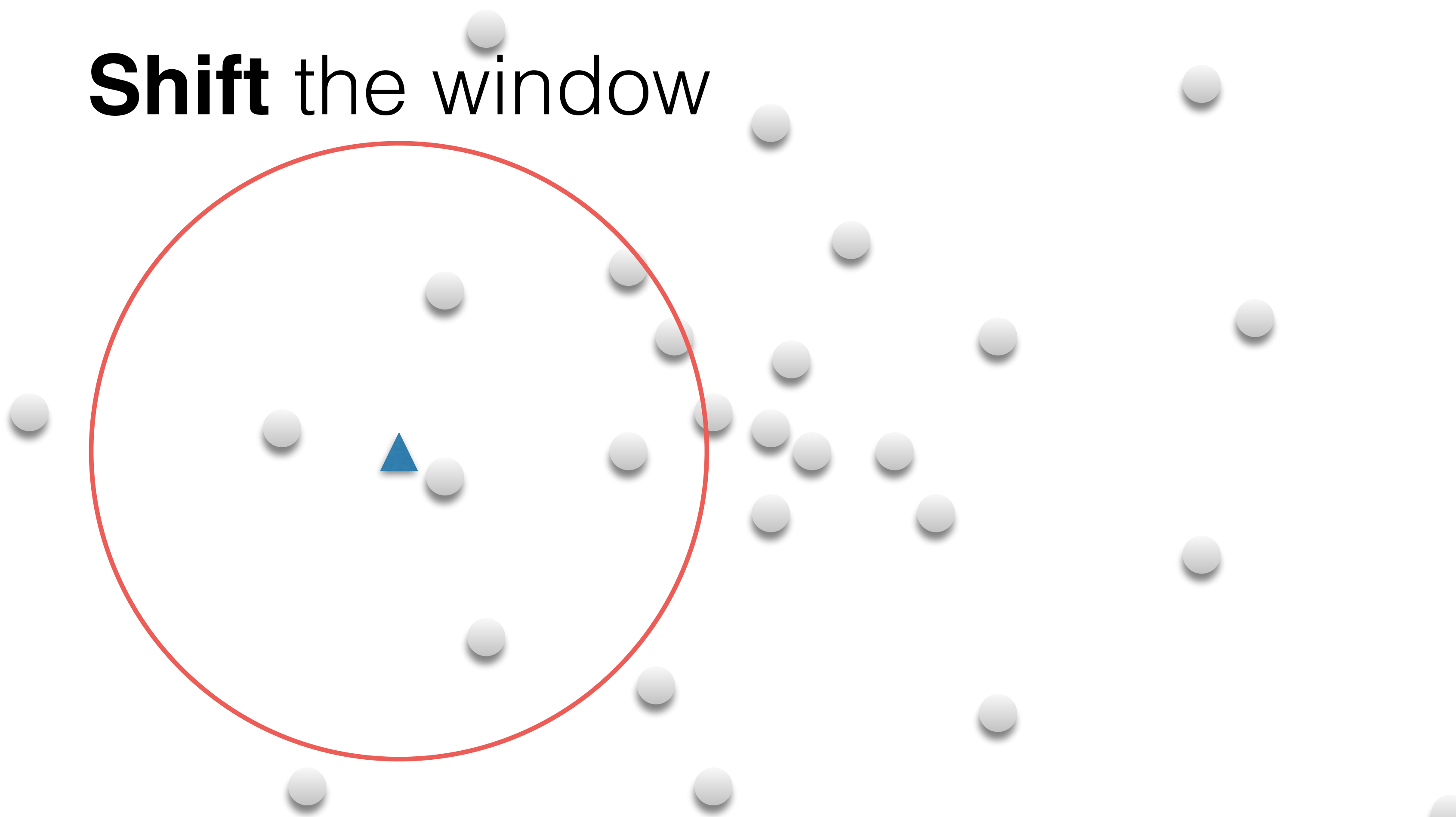

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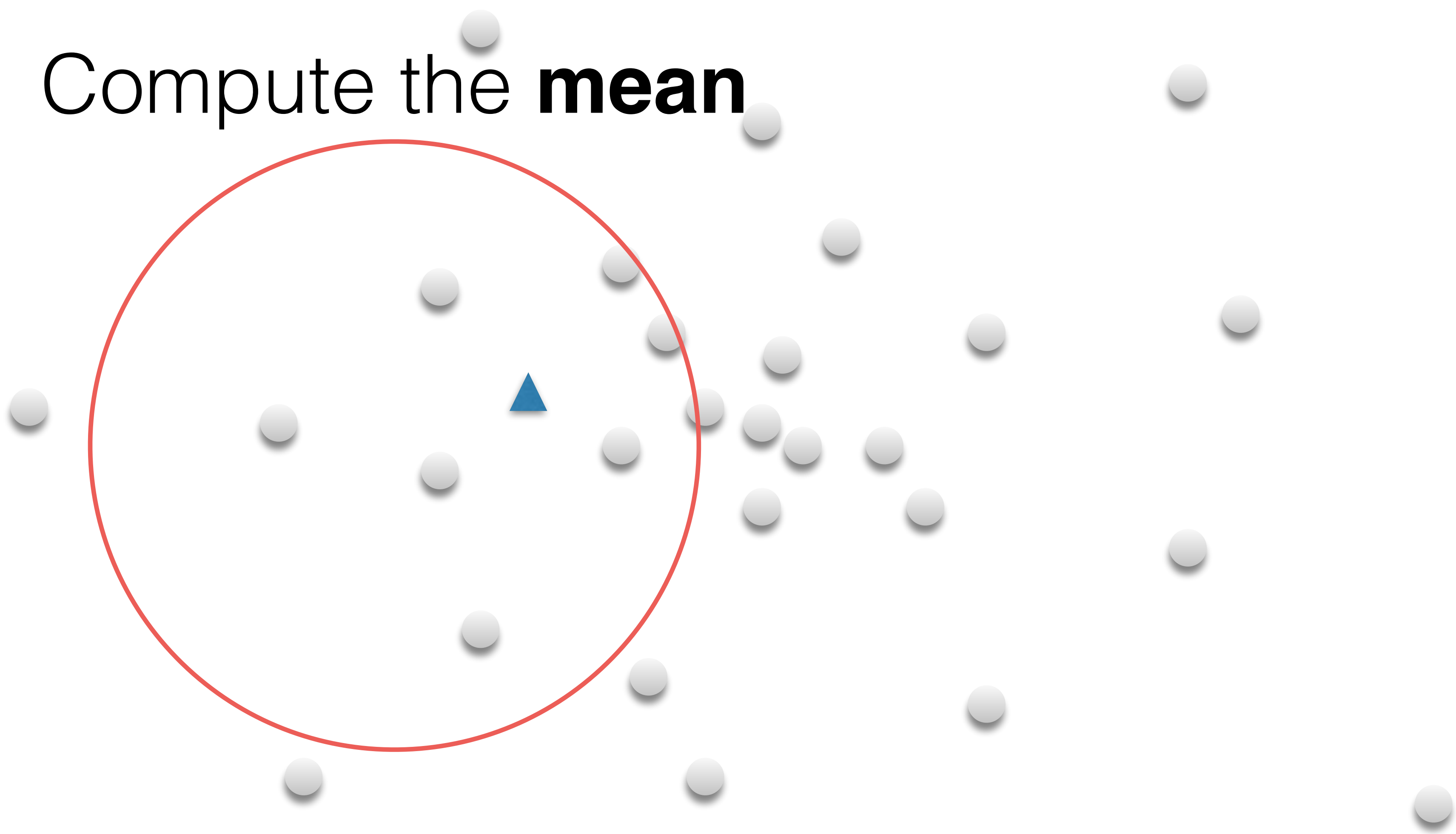

Mean Shift Algorithm

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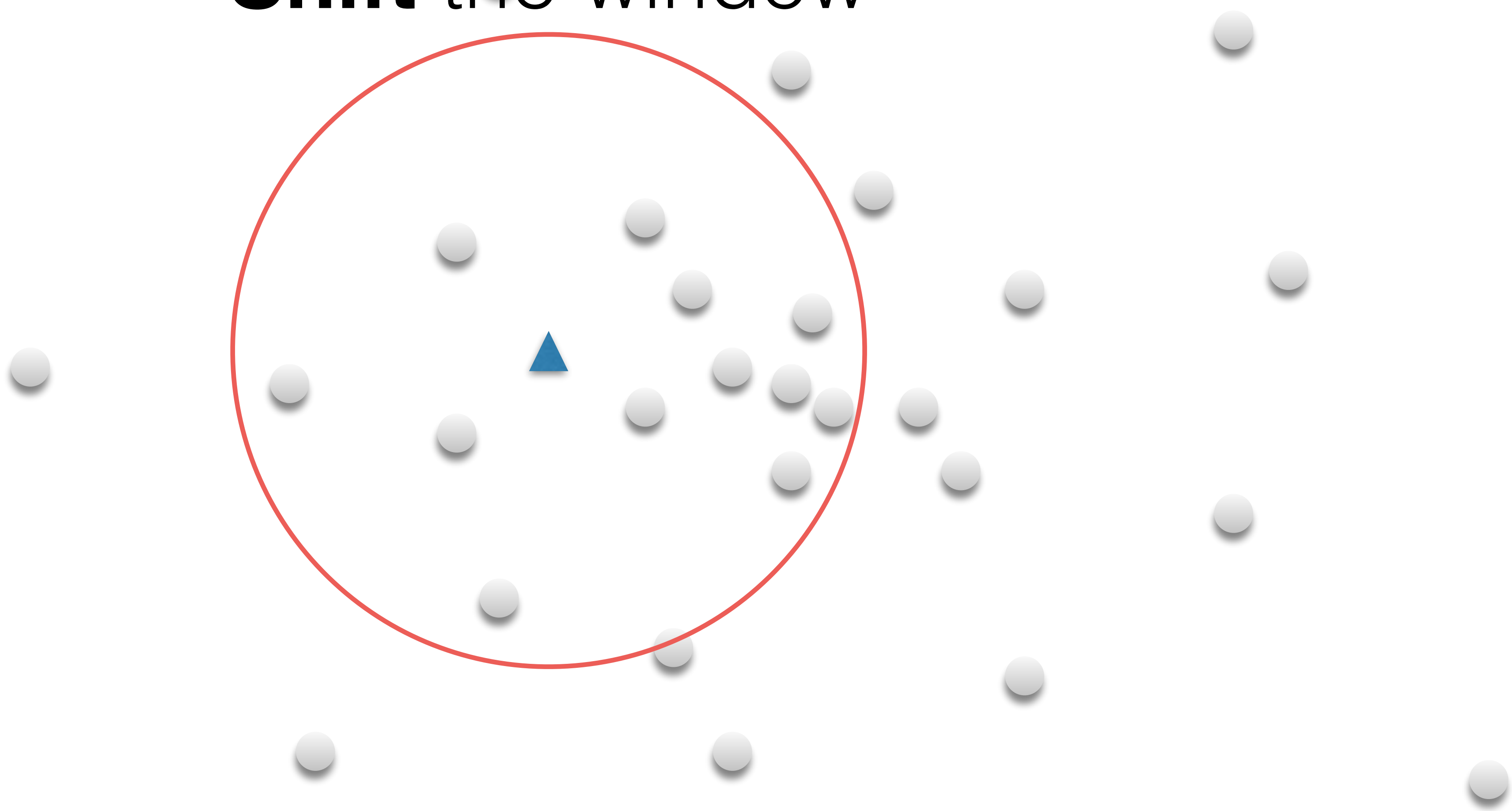
Shift the window



Mean Shift Algorithm

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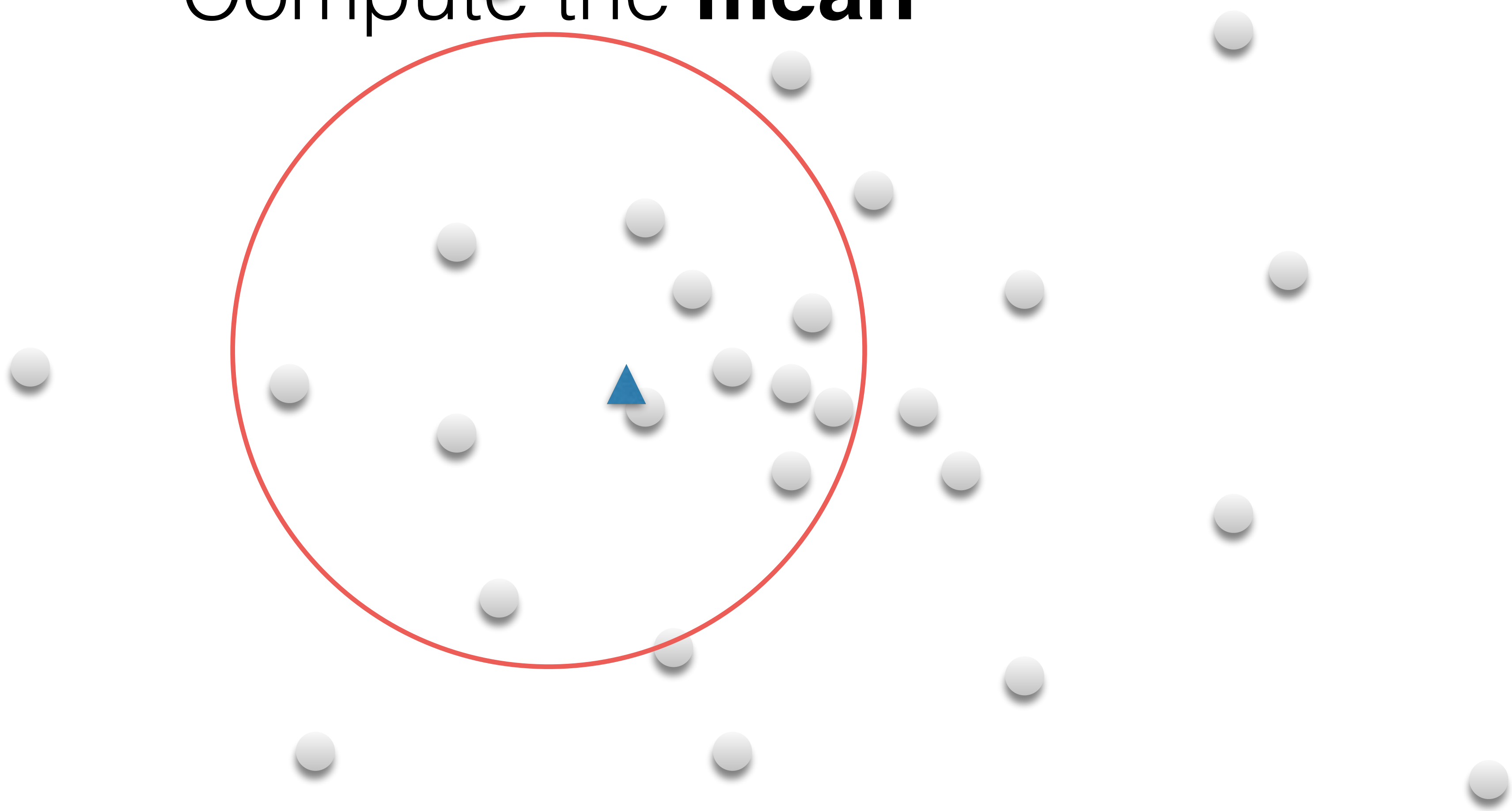
Compute the **mean**



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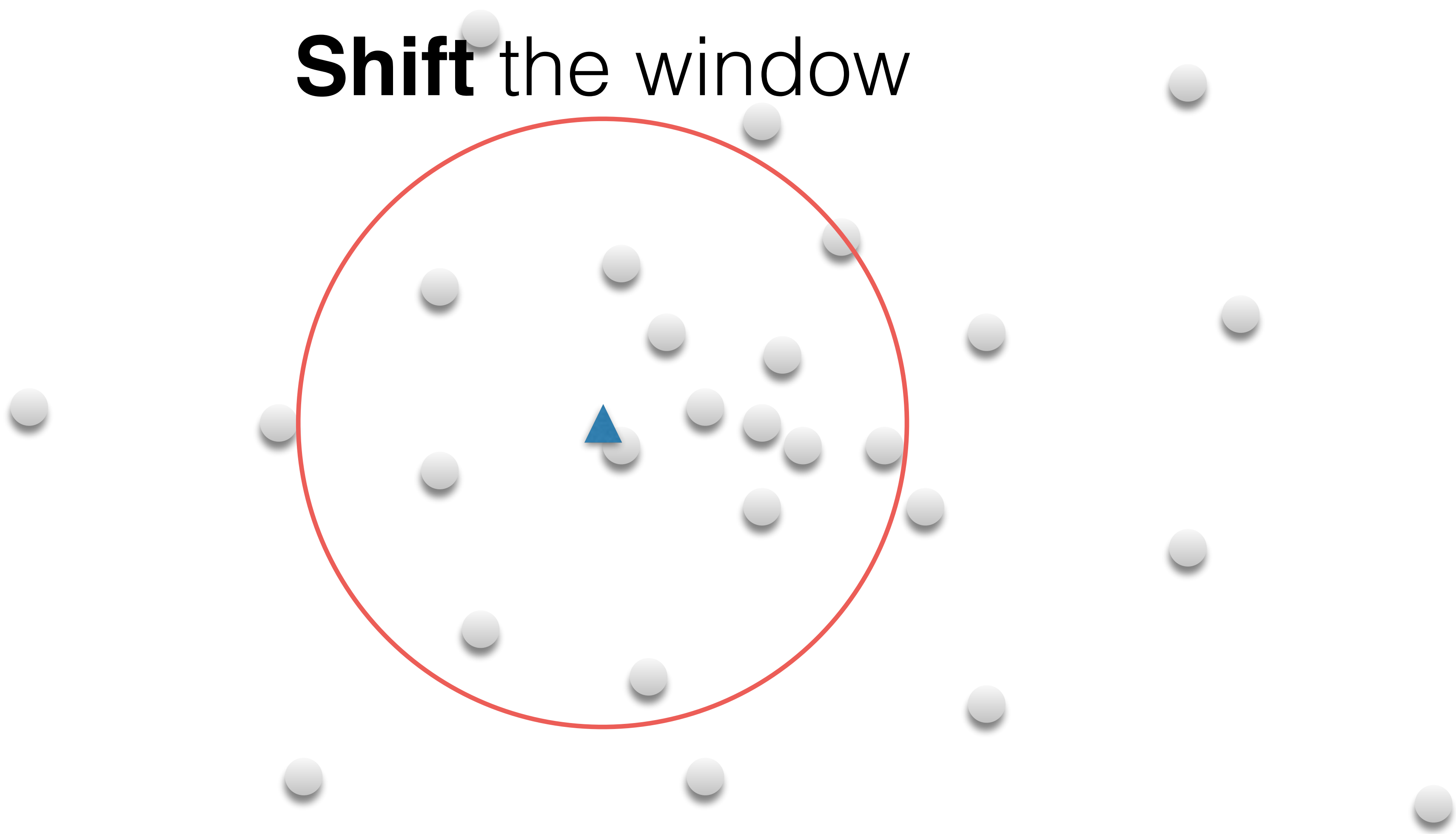

Mean Shift Algorithm

A ‘mode seeking’ algorithm

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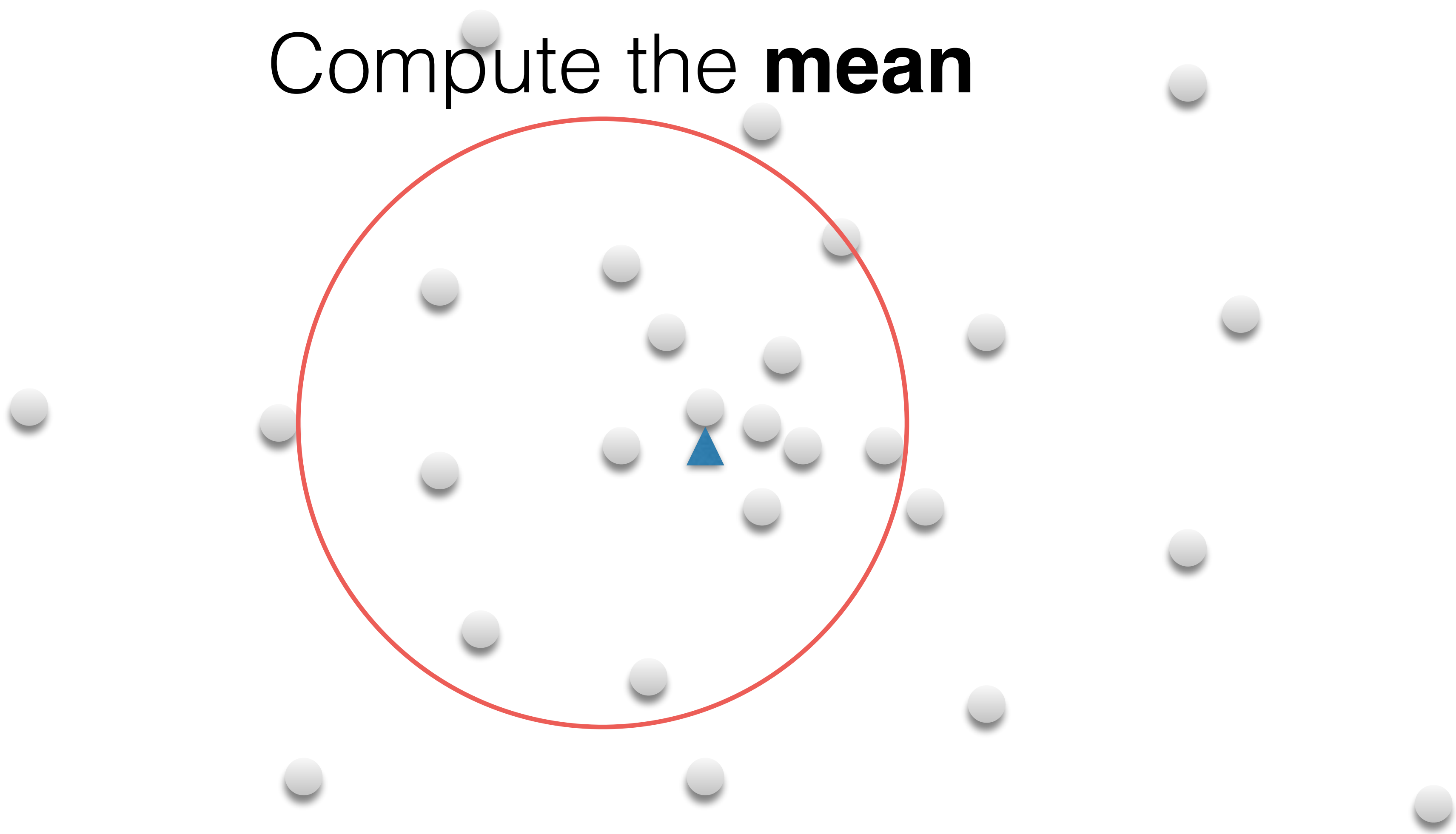
Compute the **mean**



Mean Shift Algorithm

A ‘mode seeking’ algorithm

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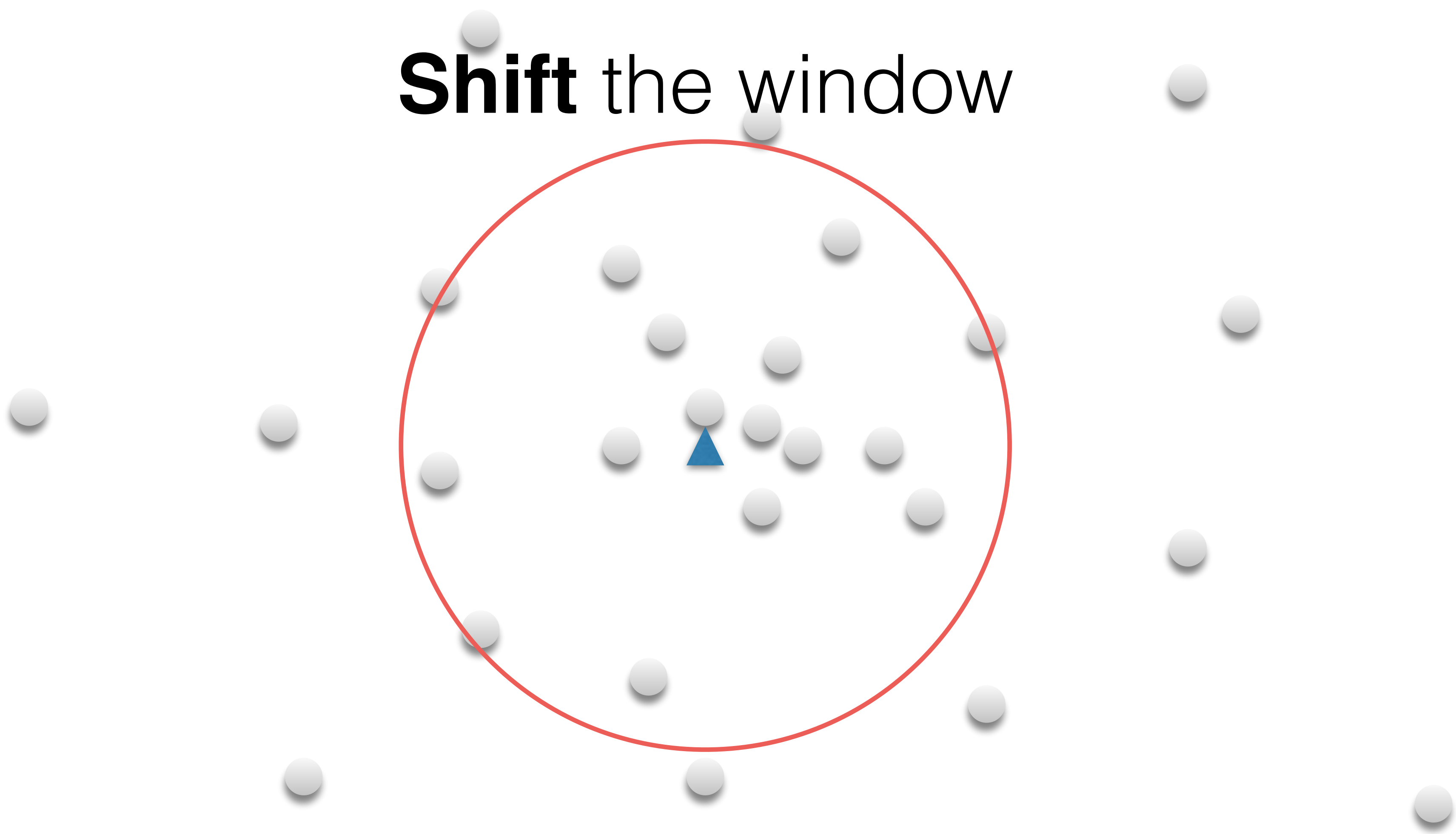
Shift the window



Mean Shift Algorithm

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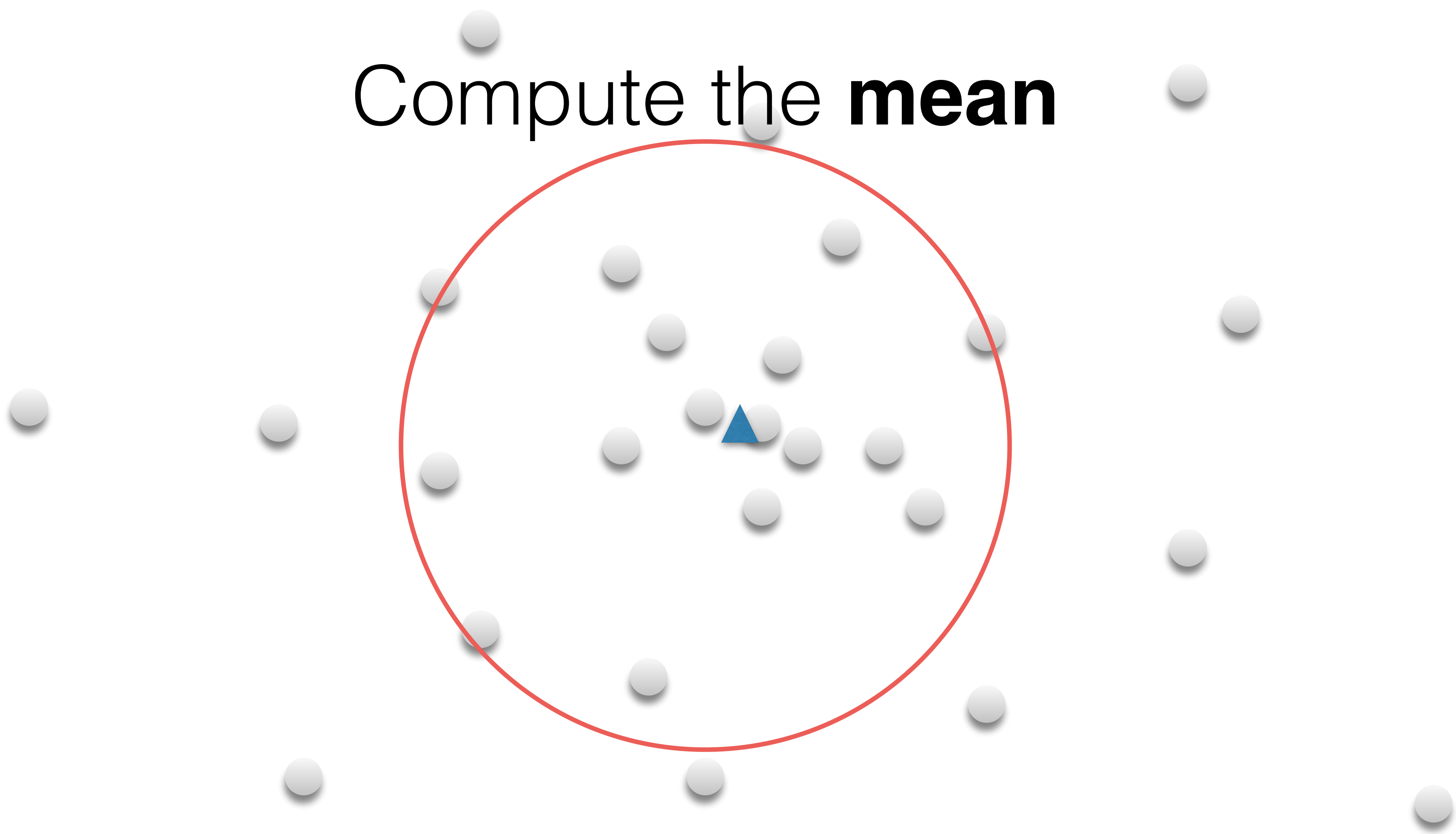

Mean Shift Algorithm

A ‘mode seeking’ algorithm

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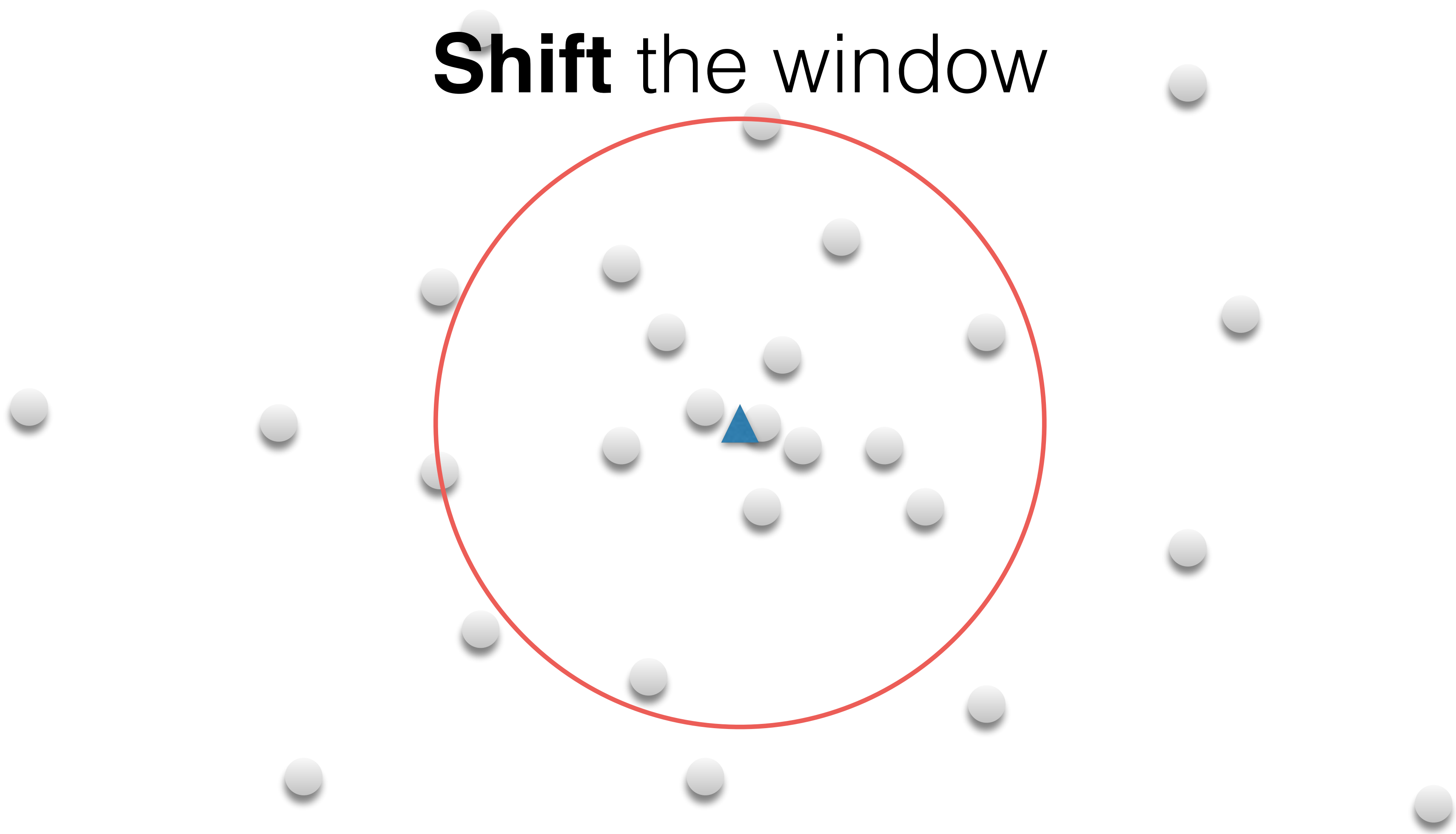
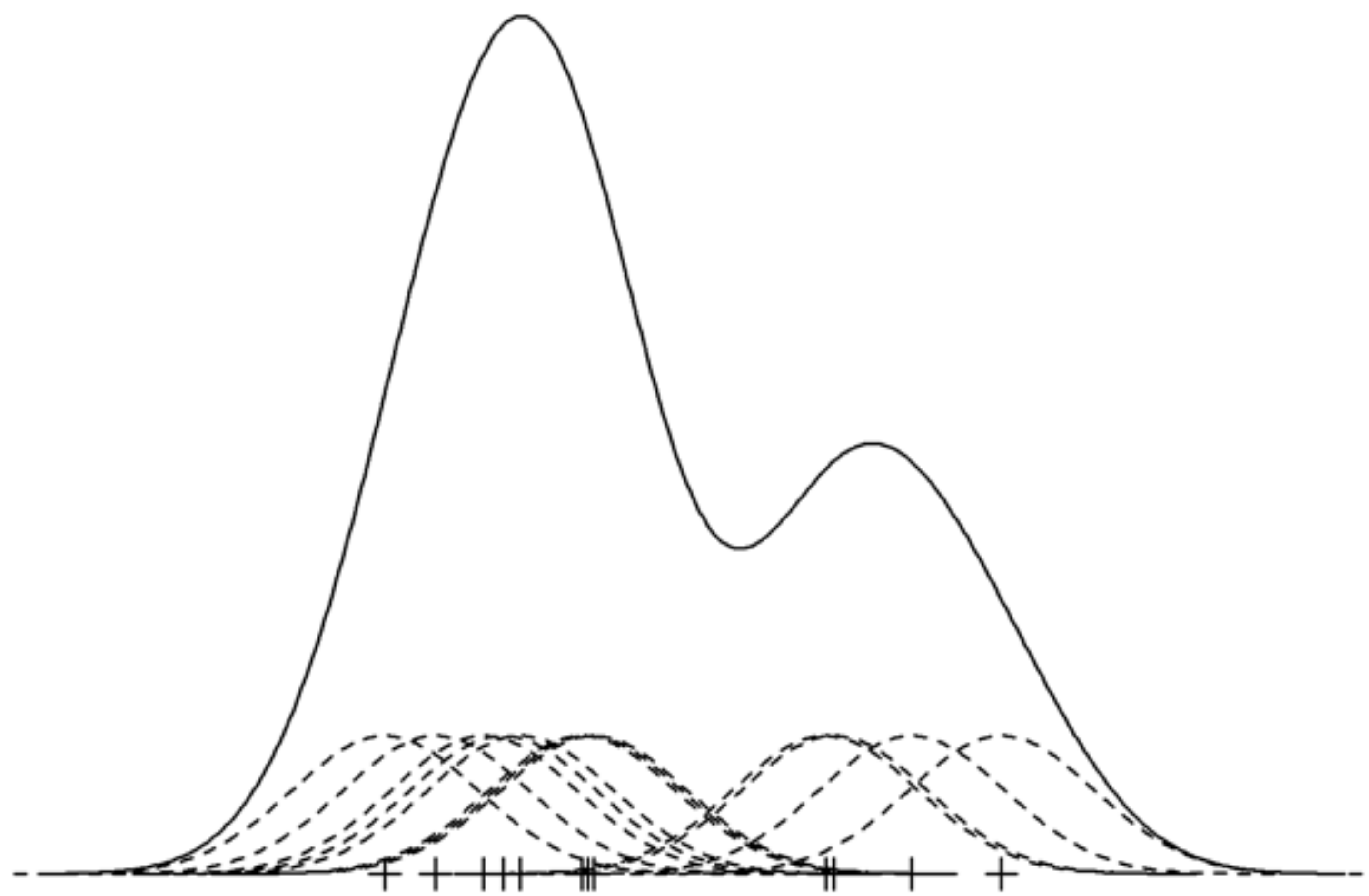
Shift the window



To understand the mean shift algorithm …

Kernel Density Estimation

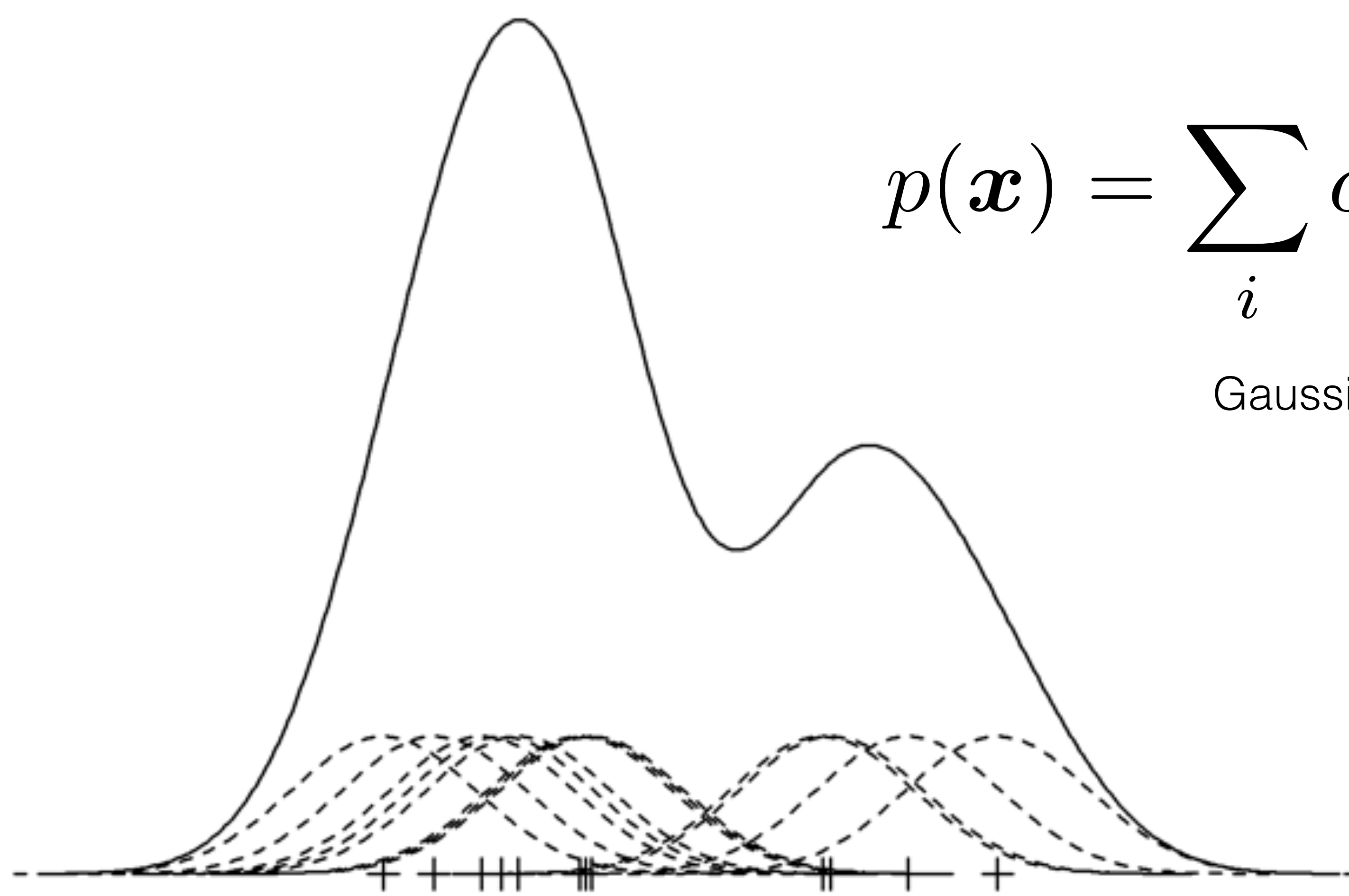
Approximate the underlying PDF from samples



Put 'bump' on every sample to approximate the PDF

Kernel Density Estimation

Approximate the underlying PDF from samples from it



$$p(\boldsymbol{x}) = \sum_i c_i e^{-\frac{(\boldsymbol{x}-\boldsymbol{x}_i)^2}{2\sigma^2}}$$

Gaussian ‘bump’ aka ‘kernel’

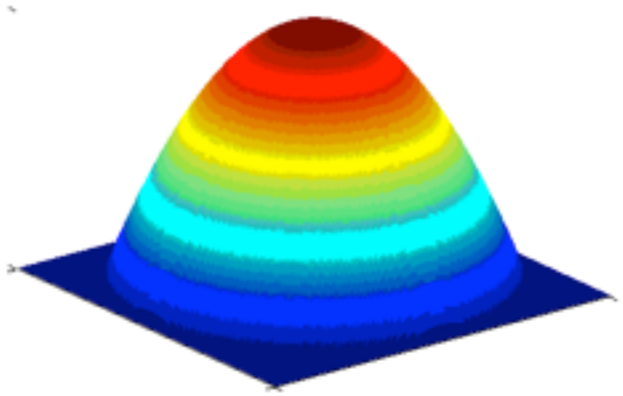
Put ‘bump’ on every sample to approximate the PDF

Kernel Function

$$K(\boldsymbol{x}, \boldsymbol{x}')$$

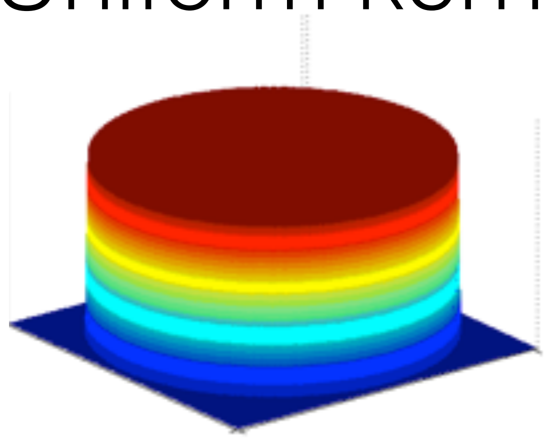
a ‘distance’ between two points

Epanechnikov kernel



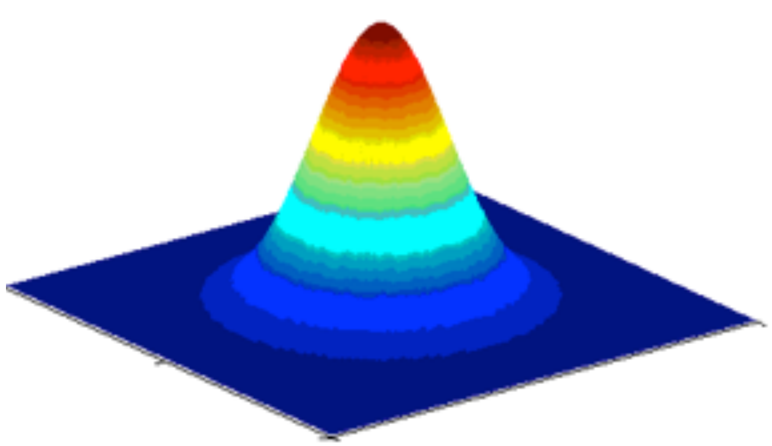
$$K(\boldsymbol{x}, \boldsymbol{x}') = \begin{cases} c(1 - \|\boldsymbol{x} - \boldsymbol{x}'\|^2) & \|\boldsymbol{x} - \boldsymbol{x}'\|^2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Uniform kernel



$$K(\boldsymbol{x}, \boldsymbol{x}') = \begin{cases} c & \|\boldsymbol{x} - \boldsymbol{x}'\|^2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Normal kernel



$$K(\boldsymbol{x}, \boldsymbol{x}') = c \exp\left(\frac{1}{2}\|\boldsymbol{x} - \boldsymbol{x}'\|^2\right)$$

Radially symmetric kernels

Radially symmetric kernels

…can be written in terms of its *profile*

$$K(\boldsymbol{x}, \boldsymbol{x}') = c \cdot k(\|\boldsymbol{x} - \boldsymbol{x}'\|^2)$$

profile

Connecting KDE and the Mean Shift Algorithm

Consider a set of points: $\{\boldsymbol{x}_s\}_{s=1}^{S}$ $\boldsymbol{x}_s \in \mathcal{R}^d$

Sample mean: $$m(\boldsymbol{x}) = \frac{\sum_s K(\boldsymbol{x}, \boldsymbol{x}_s)\boldsymbol{x}_s}{\sum_s K(\boldsymbol{x}, \boldsymbol{x}_s)}$$

Mean shift: $$m(\boldsymbol{x}) - \boldsymbol{x}$$

Mean shift algorithm

From each data point, move to its mean $\boldsymbol{x} \leftarrow m(\boldsymbol{x})$

Iterate until $\boldsymbol{x} = m(\boldsymbol{x})$

Where does this algorithm come from?

Consider a set of points: $\{\boldsymbol{x}_s\}_{s=1}^{S}$ $\boldsymbol{x}_s \in \mathcal{R}^d$

Sample mean: $$m(\boldsymbol{x}) = \frac{\sum_s K(\boldsymbol{x}, \boldsymbol{x}_s)\boldsymbol{x}_s}{\sum_s K(\boldsymbol{x}, \boldsymbol{x}_s)}$$

Mean shift: $m(\boldsymbol{x}) - \boldsymbol{x}$

Where does this come from?

Mean shift algorithm

From each data point, move to its mean $\boldsymbol{x} \leftarrow m(\boldsymbol{x})$

Iterate until $\boldsymbol{x} = m(\boldsymbol{x})$

Where does this algorithm come from?

How is the KDE related to the mean shift algorithm?

Kernel density estimate

(radially symmetric kernels)

$$P(\boldsymbol{x}) = \frac{1}{N} c \sum_{n} k(\|\boldsymbol{x} - \boldsymbol{x}_n\|^2)$$

Gradient of the PDF is related to the mean shift vector

$$\nabla P(\boldsymbol{x}) \propto m(\boldsymbol{x})$$

The mean shift is a 'step' in the direction of the gradient of the KDE

Derivation

$$P(\boldsymbol{x}) = \frac{1}{N} c \sum_n k(\|\boldsymbol{x} - \boldsymbol{x}_n\|^2)$$

Gradient

$$\nabla P(\boldsymbol{x}) = \frac{1}{N} c \sum_n \nabla k(\|\boldsymbol{x} - \boldsymbol{x}_n\|^2)$$

expand derivative

$$\nabla P(\boldsymbol{x}) = \frac{1}{N} 2c \sum_n (\boldsymbol{x} - \boldsymbol{x}_n) k'(\|\boldsymbol{x} - \boldsymbol{x}_n\|^2)$$

change of notation
(kernel-shadow pairs)

$$\nabla P(\boldsymbol{x}) = \frac{1}{N} 2c \sum_n (\boldsymbol{x}_n - \boldsymbol{x}) g(\|\boldsymbol{x} - \boldsymbol{x}_n\|^2)$$

$$k'(\cdot) = -g(\cdot)$$

$$\nabla P(\boldsymbol{x}) = \frac{1}{N} 2c \sum_n (\boldsymbol{x}_n - \boldsymbol{x}) g(\|\boldsymbol{x} - \boldsymbol{x}_n\|^2)$$

multiply it out

$$\nabla P(\boldsymbol{x}) = \frac{1}{N} 2c \sum_n \boldsymbol{x}_n g(\|\boldsymbol{x} - \boldsymbol{x}_n\|^2) - \frac{1}{N} 2c \sum_n \boldsymbol{x} g(\|\boldsymbol{x} - \boldsymbol{x}_n\|^2)$$

too long (enter short hand notation)

$$\nabla P(\boldsymbol{x}) = \frac{1}{N} 2c \sum_n \boldsymbol{x}_n g_n - \frac{1}{N} 2c \sum_n \boldsymbol{x} g_n$$

$$\nabla P(\boldsymbol{x}) = \frac{1}{N} 2c \sum_n \boldsymbol{x}_n g_n - \frac{1}{N} 2c \sum_n \boldsymbol{x} g_n$$

multiply by one!

$$\nabla P(\boldsymbol{x}) = \frac{1}{N} 2c \sum_n \boldsymbol{x}_n g_n \left(\frac{\sum_n g_n}{\sum_n g_n} \right) - \frac{1}{N} 2c \sum_n \boldsymbol{x} g_n$$

collecting like terms…

$$\nabla P(\boldsymbol{x}) = \frac{1}{N} 2c \sum_n g_n \left(\frac{\sum_n \boldsymbol{x}_n g_n}{\sum_n g_n} - \boldsymbol{x} \right)$$

Does this look familiar?

mean shift

$$\nabla P(\boldsymbol{x}) = \frac{1}{N} 2c \sum_n g_n \underbrace{\left(\frac{\sum_n \boldsymbol{x}_n g_n}{\sum_n g_n} - \boldsymbol{x} \right)}_{\text{mean shift!}}$$

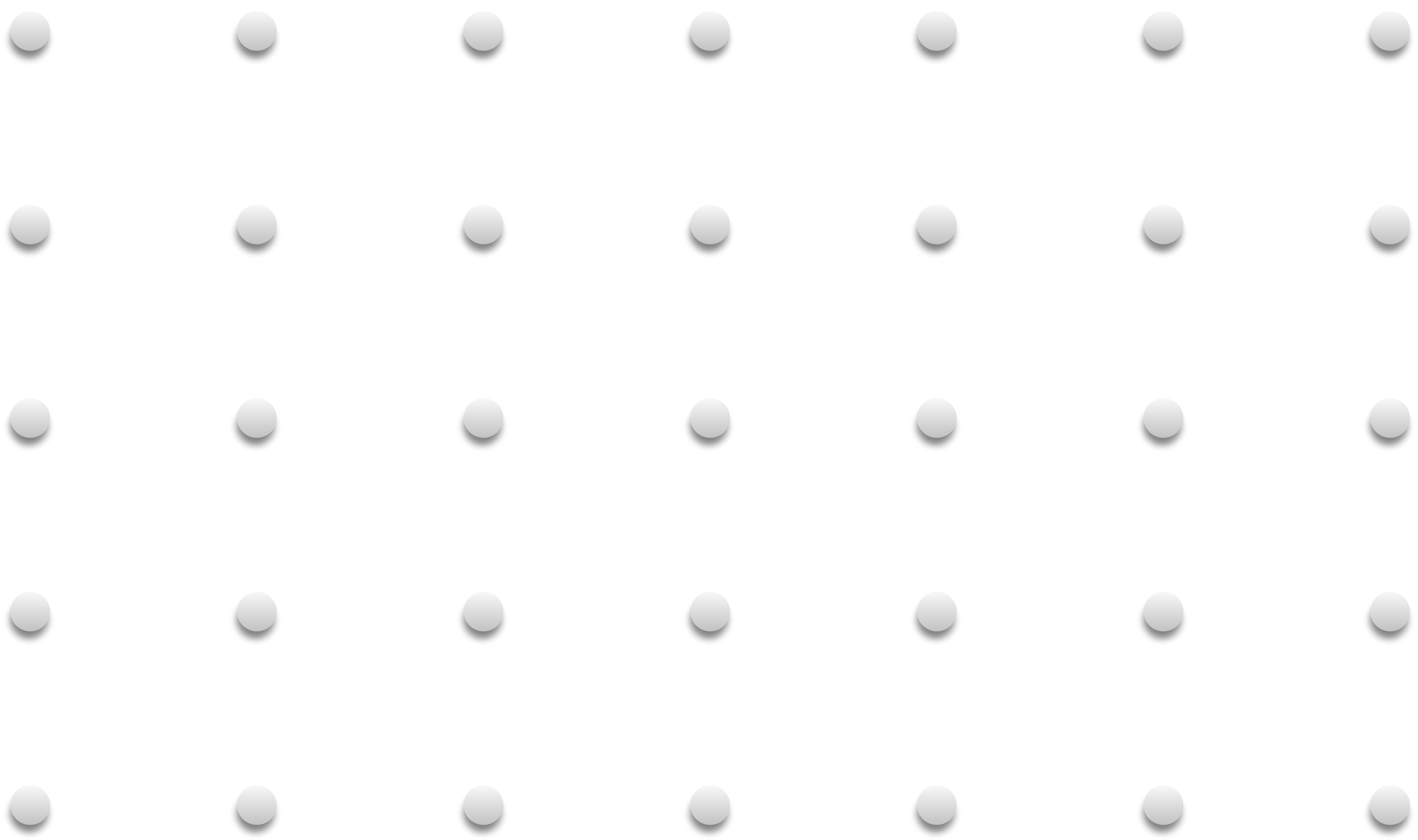
The mean shift is a 'step' in the direction of the gradient of the KDE

$$m(\boldsymbol{x}) = \left(\frac{\sum_n \boldsymbol{x}_n g_n}{\sum_n g_n} - \boldsymbol{x} \right) = \frac{\nabla P(\boldsymbol{x})}{\frac{1}{N} 2c \sum_n g_n}$$

Gradient ascent with adaptive step size

Dealing with images

Pixels for a lattice, spatial density is the same everywhere!



What can we do?

Consider a set of points: $\{\boldsymbol{x}_s\}_{s=1}^{S}$ $\boldsymbol{x}_s \in \mathcal{R}^d$

Associated weights: $w(\boldsymbol{x}_s)$

Sample mean: $$m(\boldsymbol{x}) = \frac{\sum_s K(\boldsymbol{x}, \boldsymbol{x}_s) w(\boldsymbol{x}_s) \boldsymbol{x}_s}{\sum_s K(\boldsymbol{x}, \boldsymbol{x}_s) w(\boldsymbol{x}_s)}$$

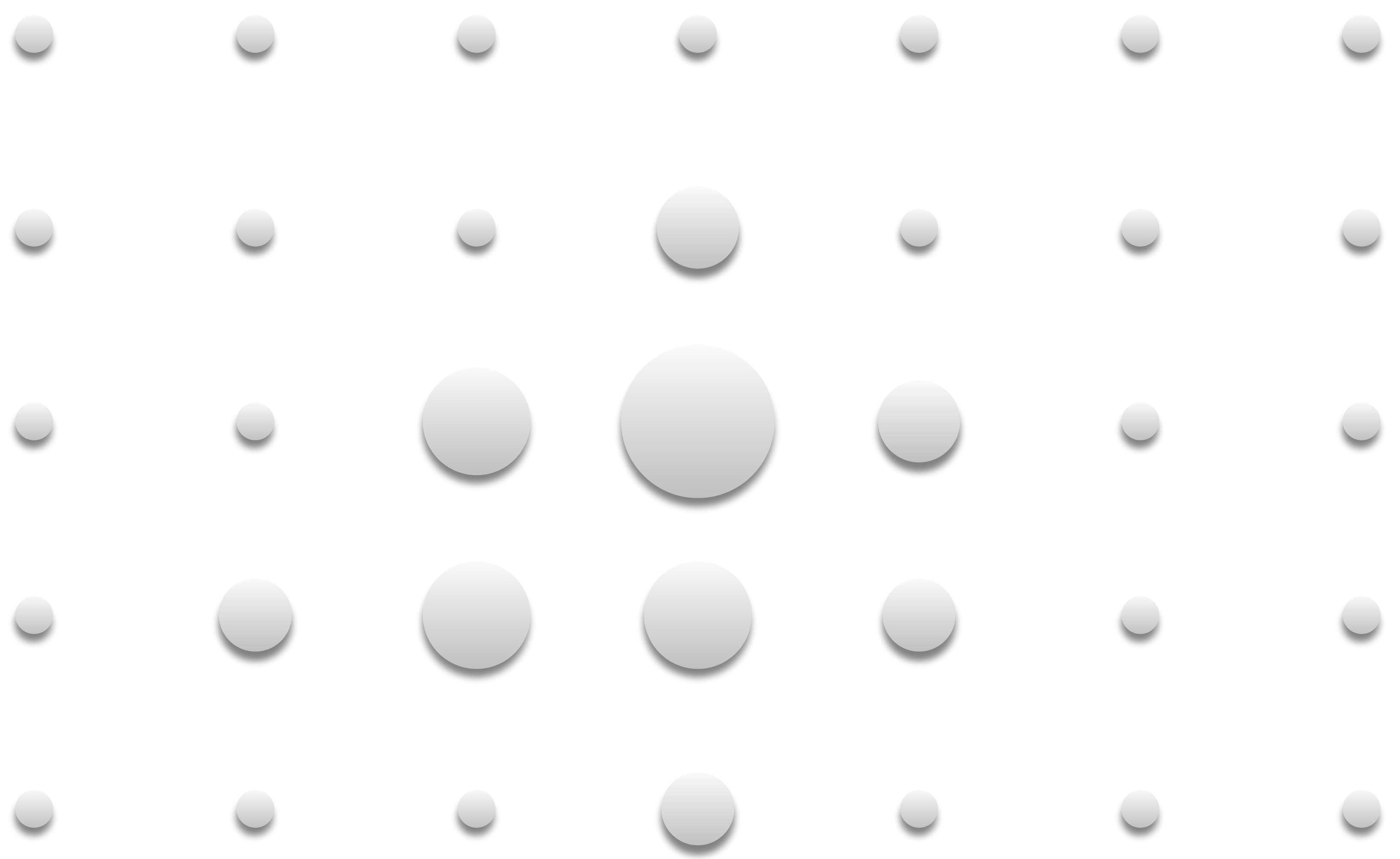
Mean shift: $m(\boldsymbol{x}) - \boldsymbol{x}$

Mean shift algorithm

From each data point, move to its mean $\boldsymbol{x} \leftarrow m(\boldsymbol{x})$

Iterate until $\boldsymbol{x} = m(\boldsymbol{x})$

For images, each pixel is point with a weight



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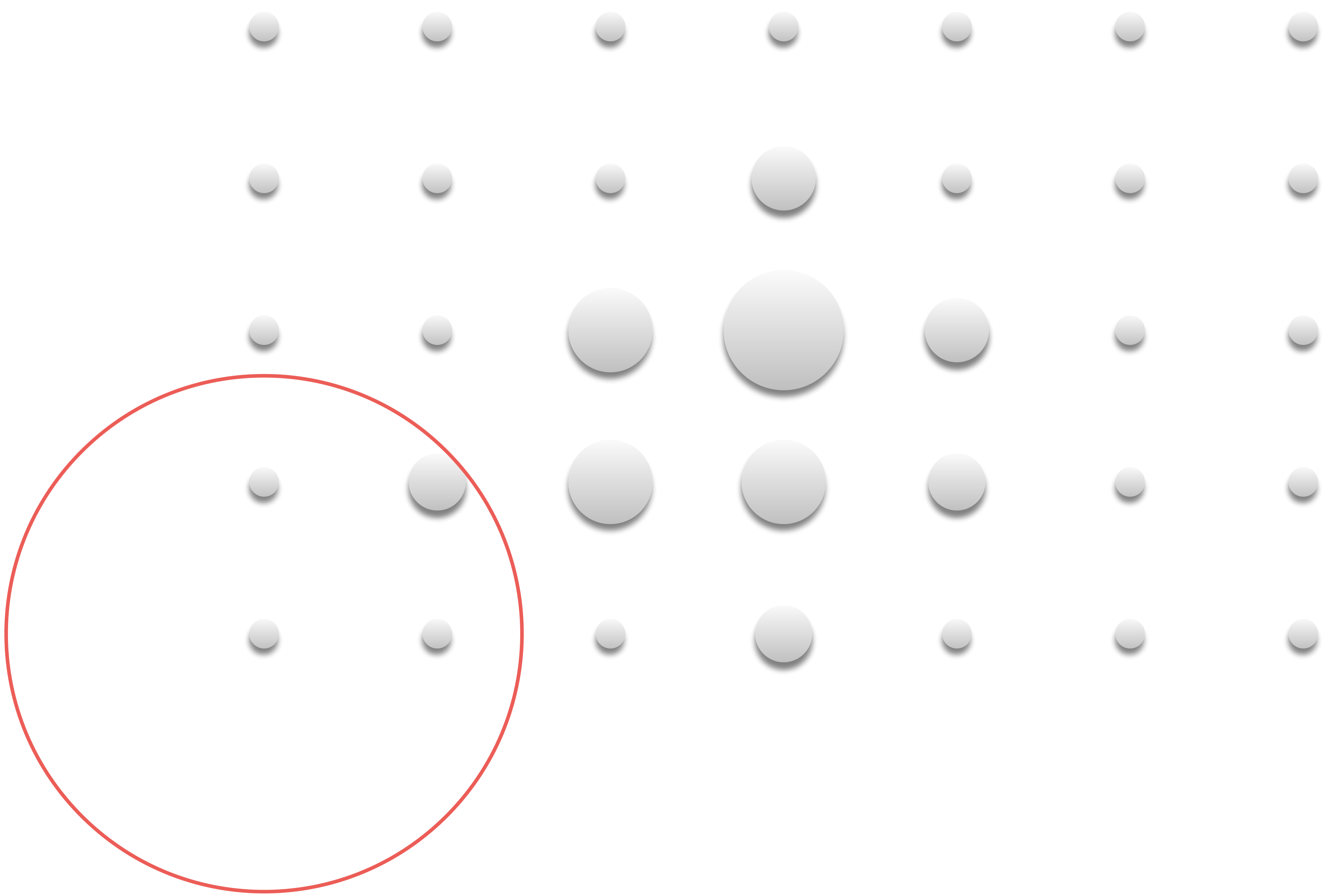

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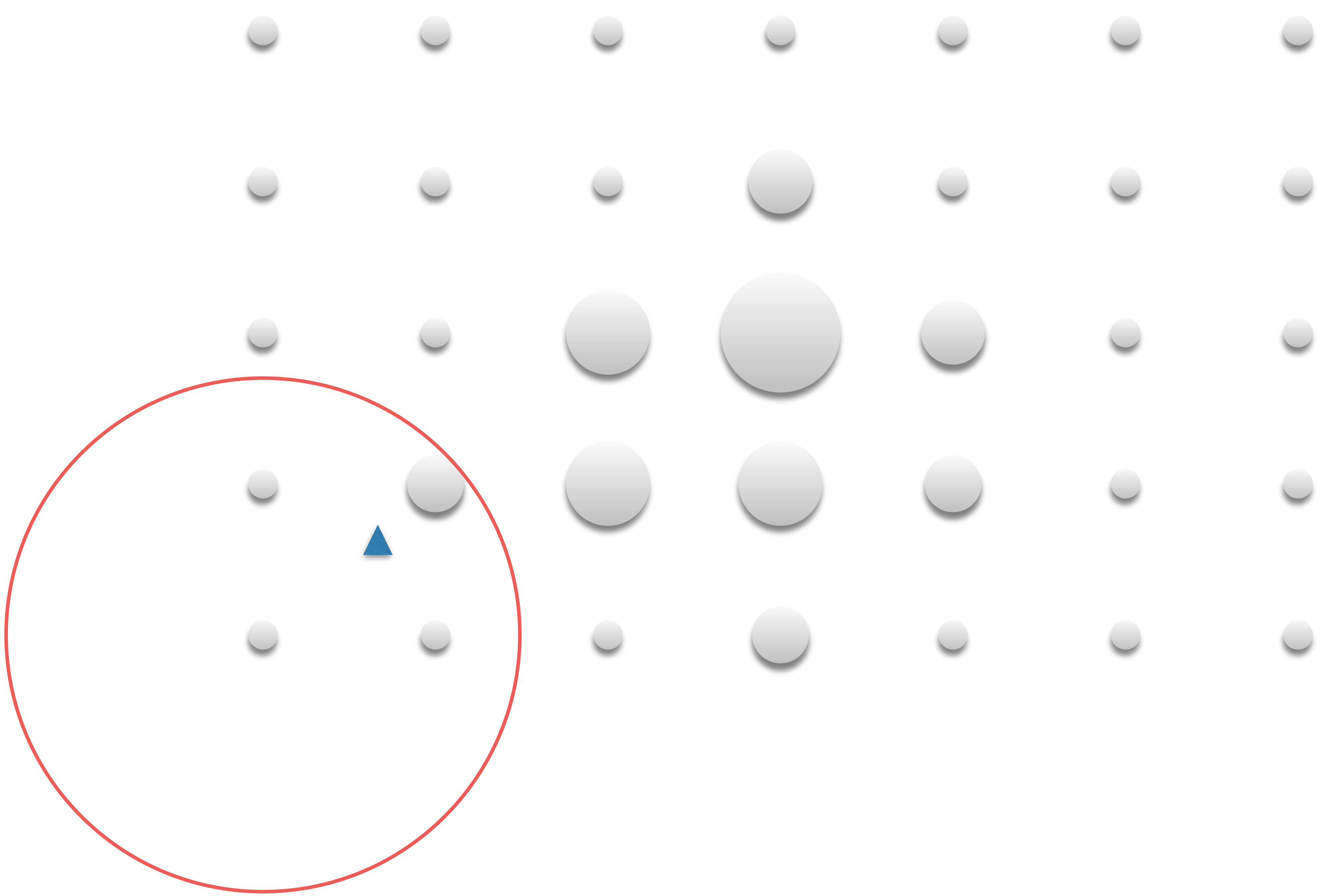

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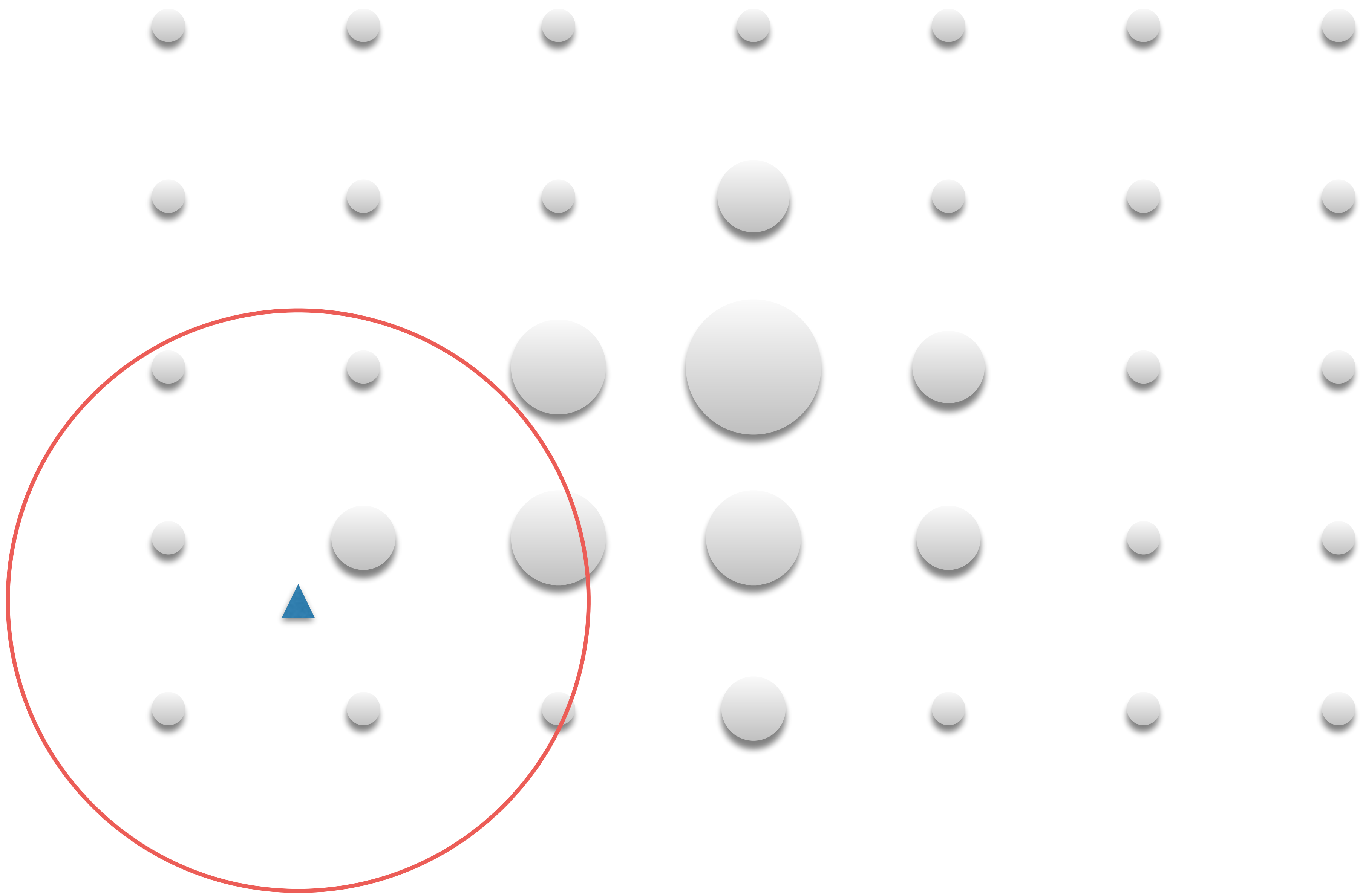

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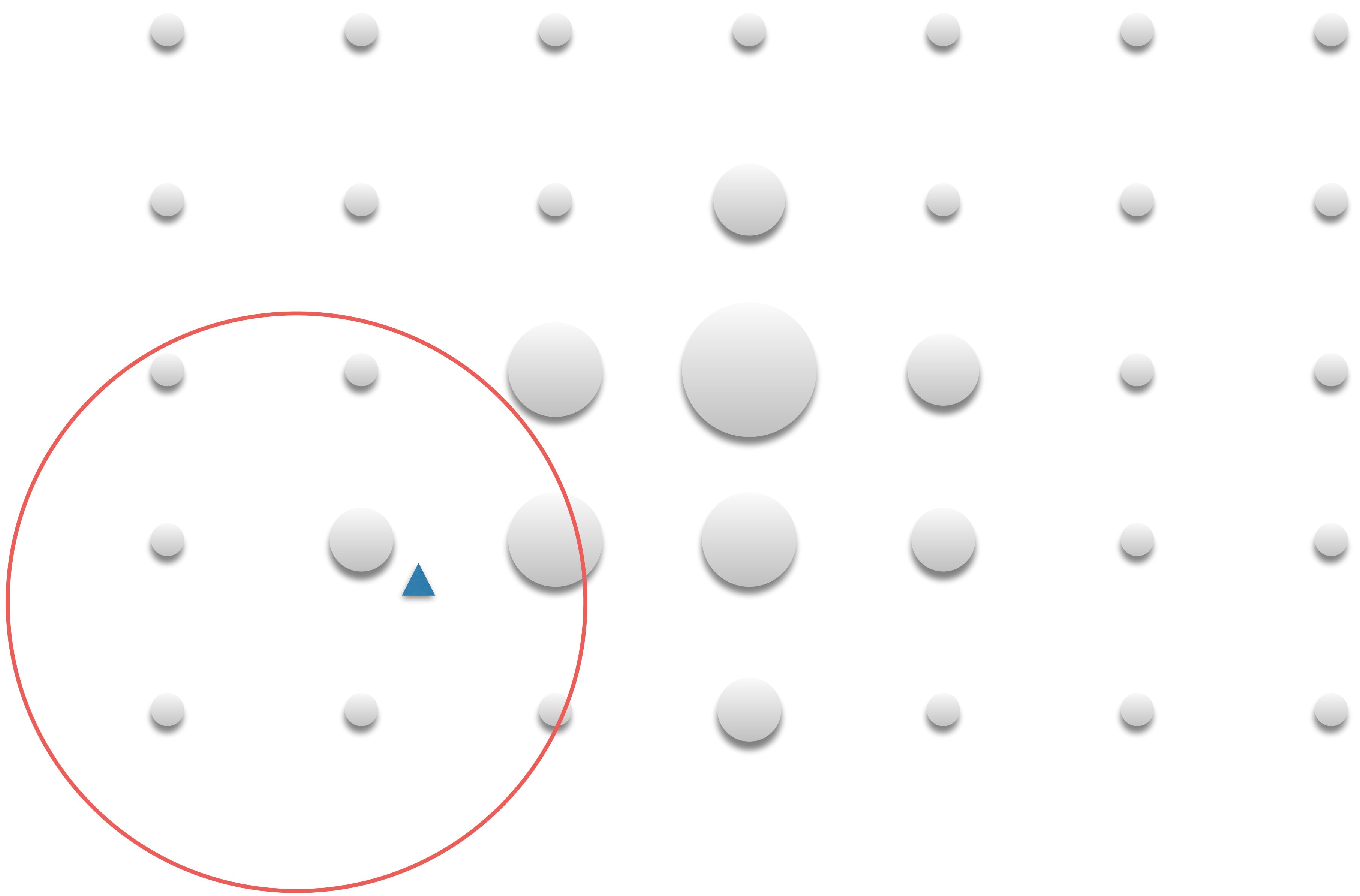

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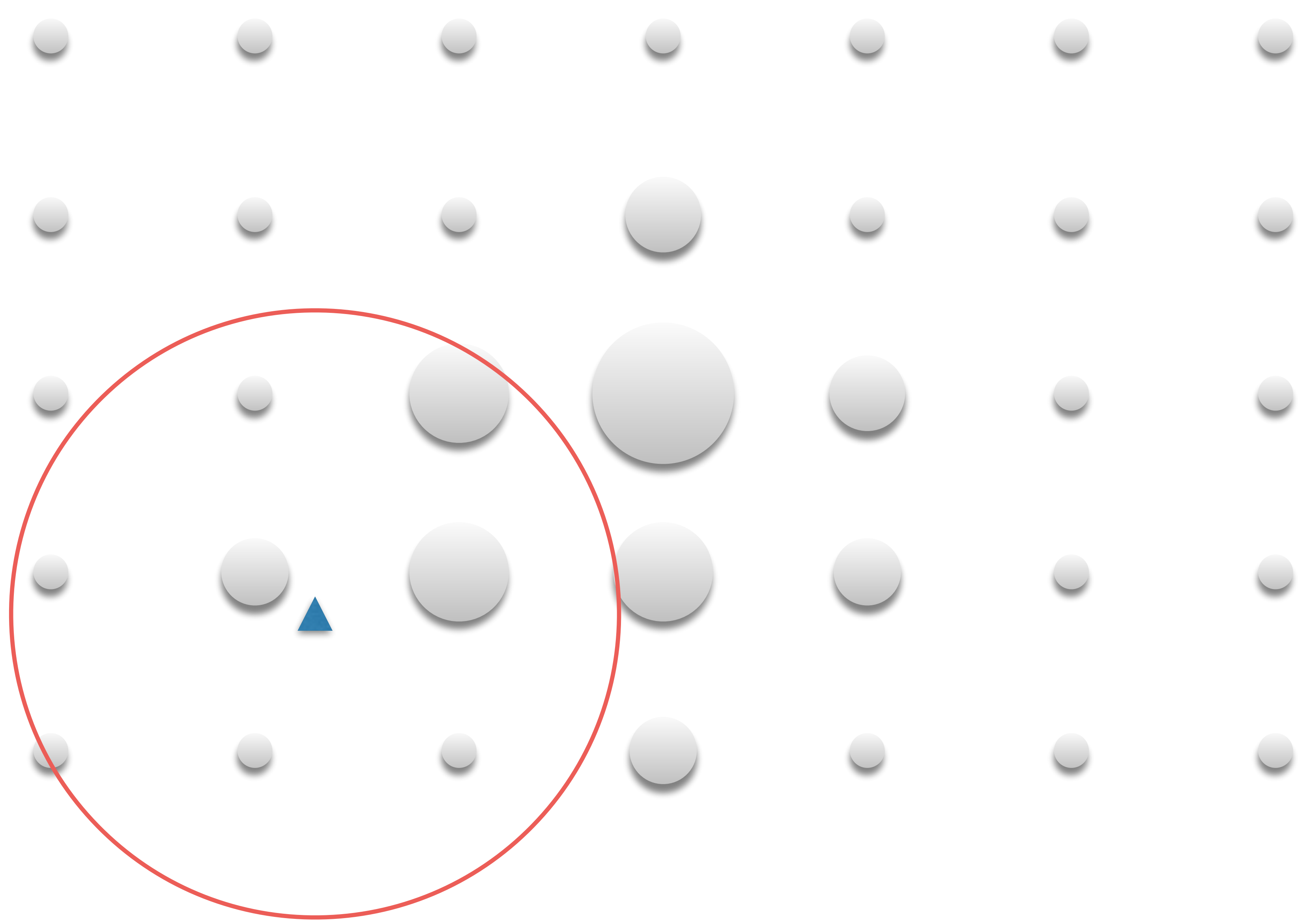

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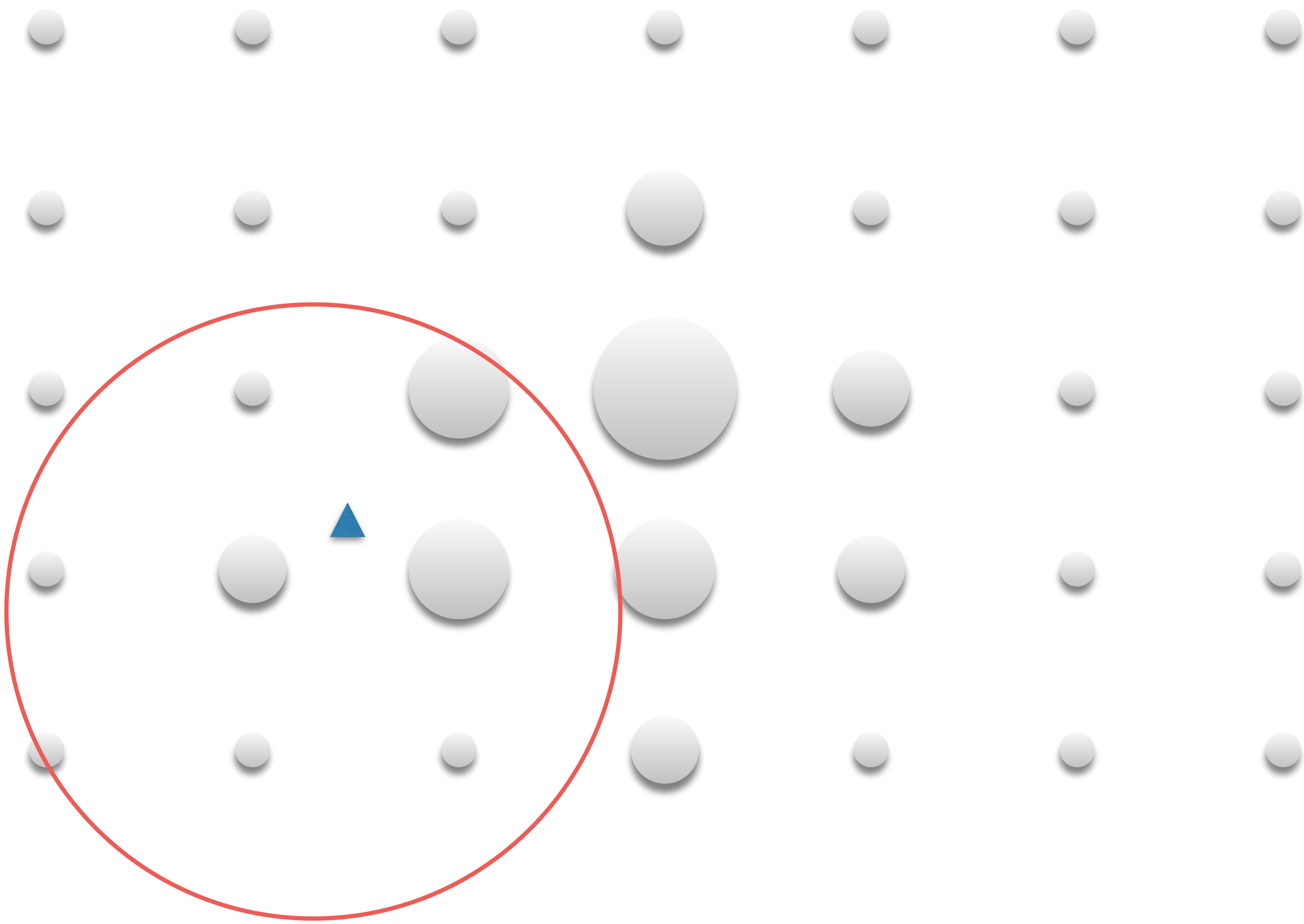

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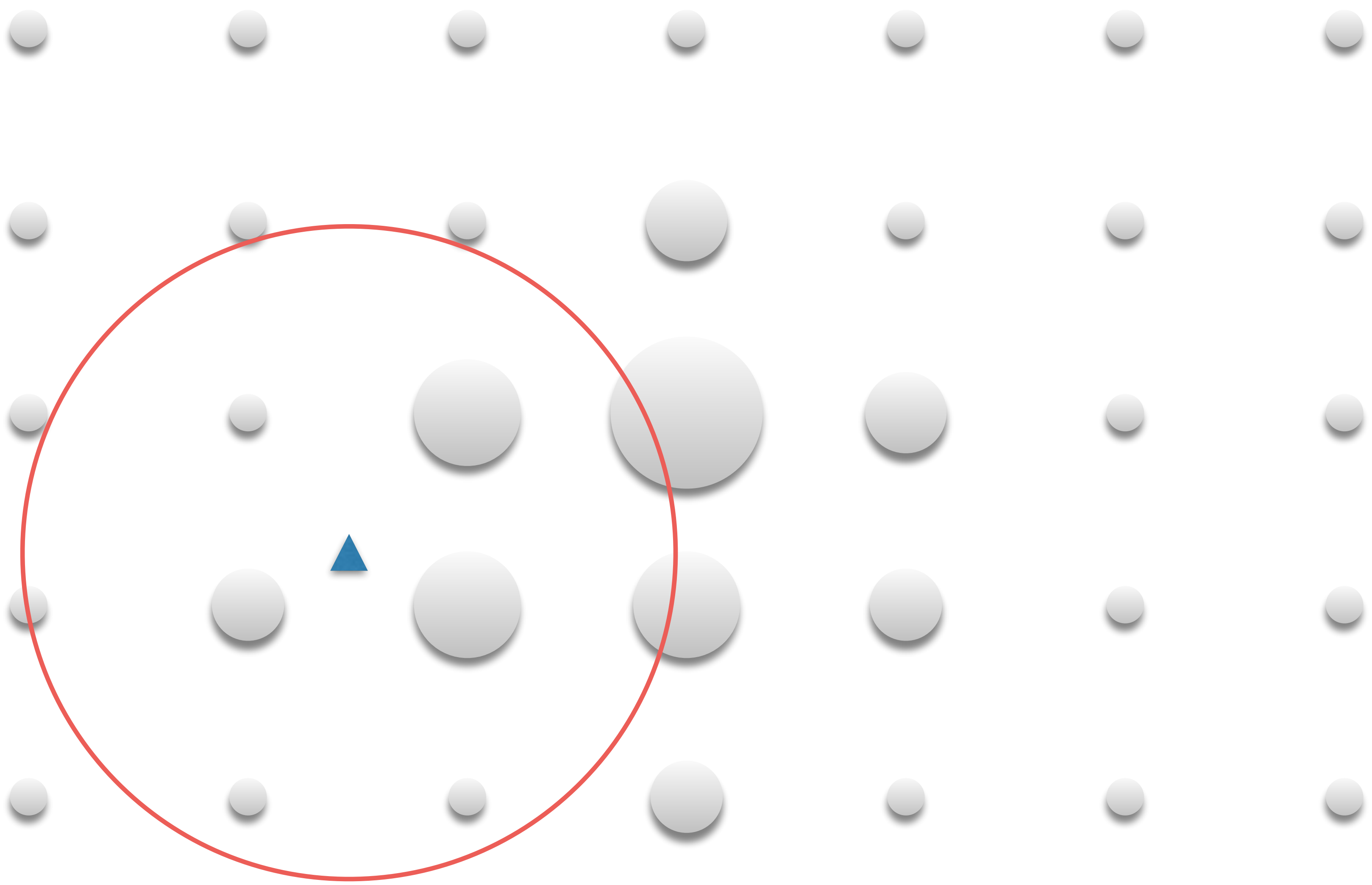

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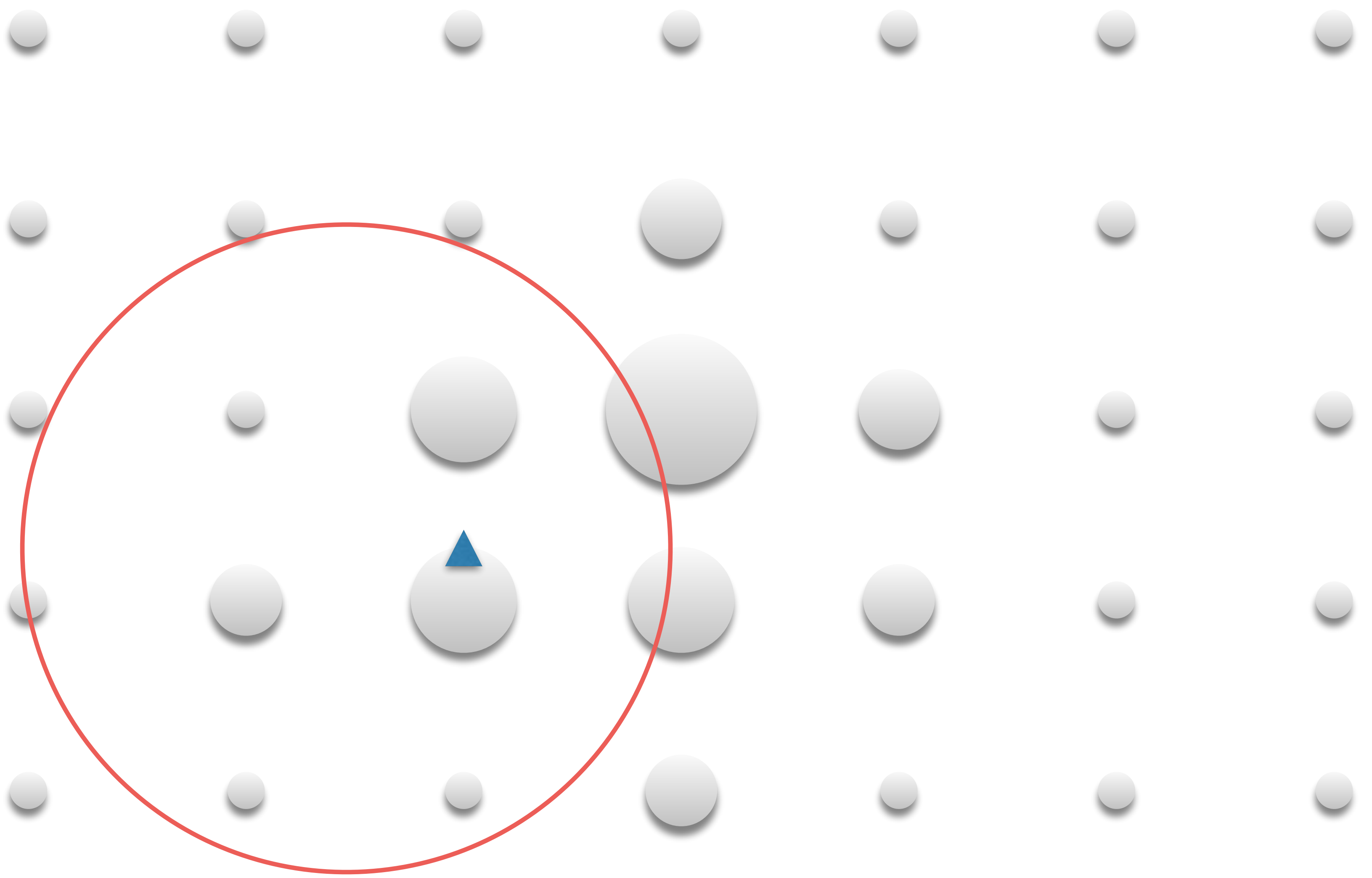

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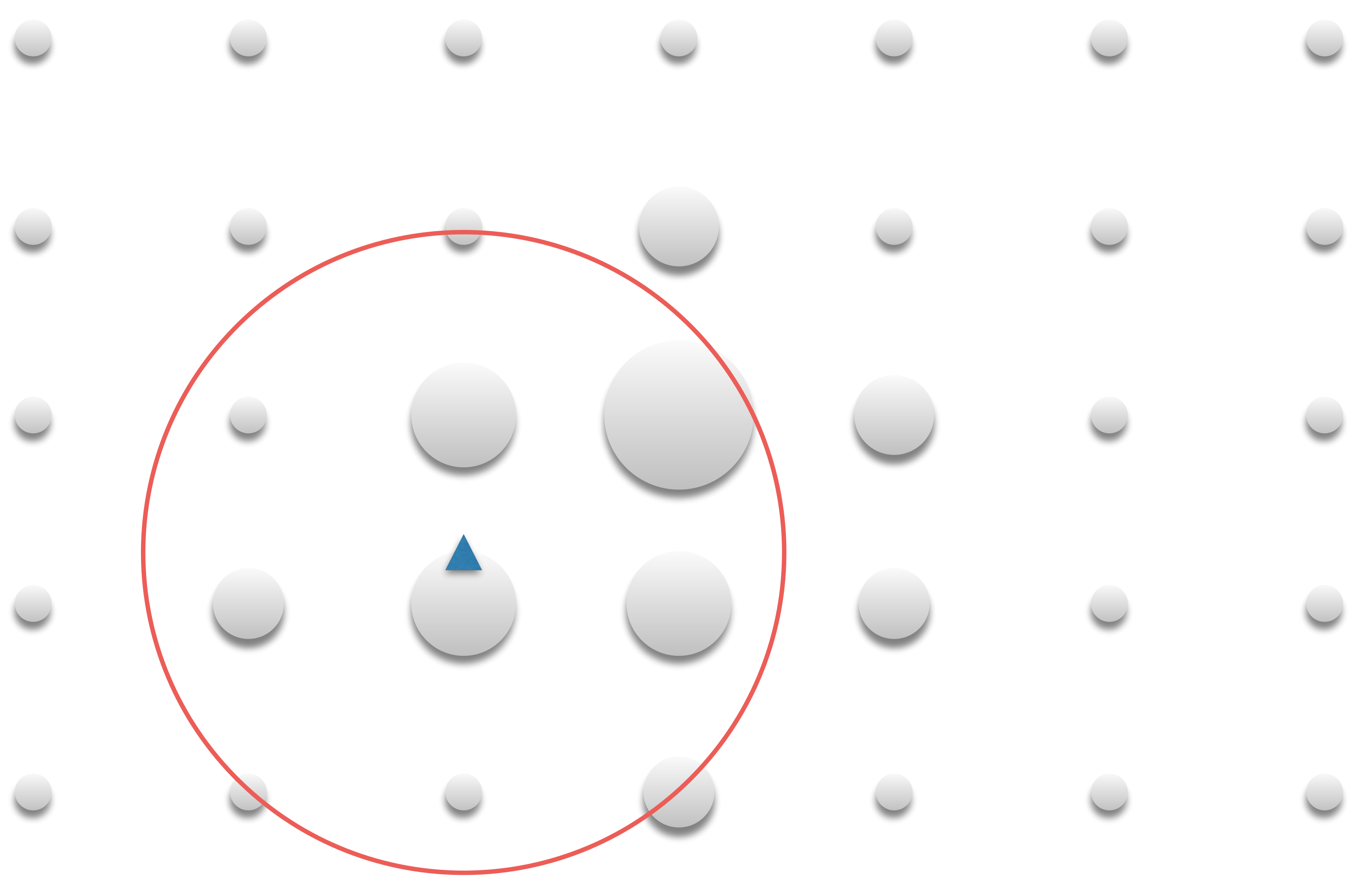

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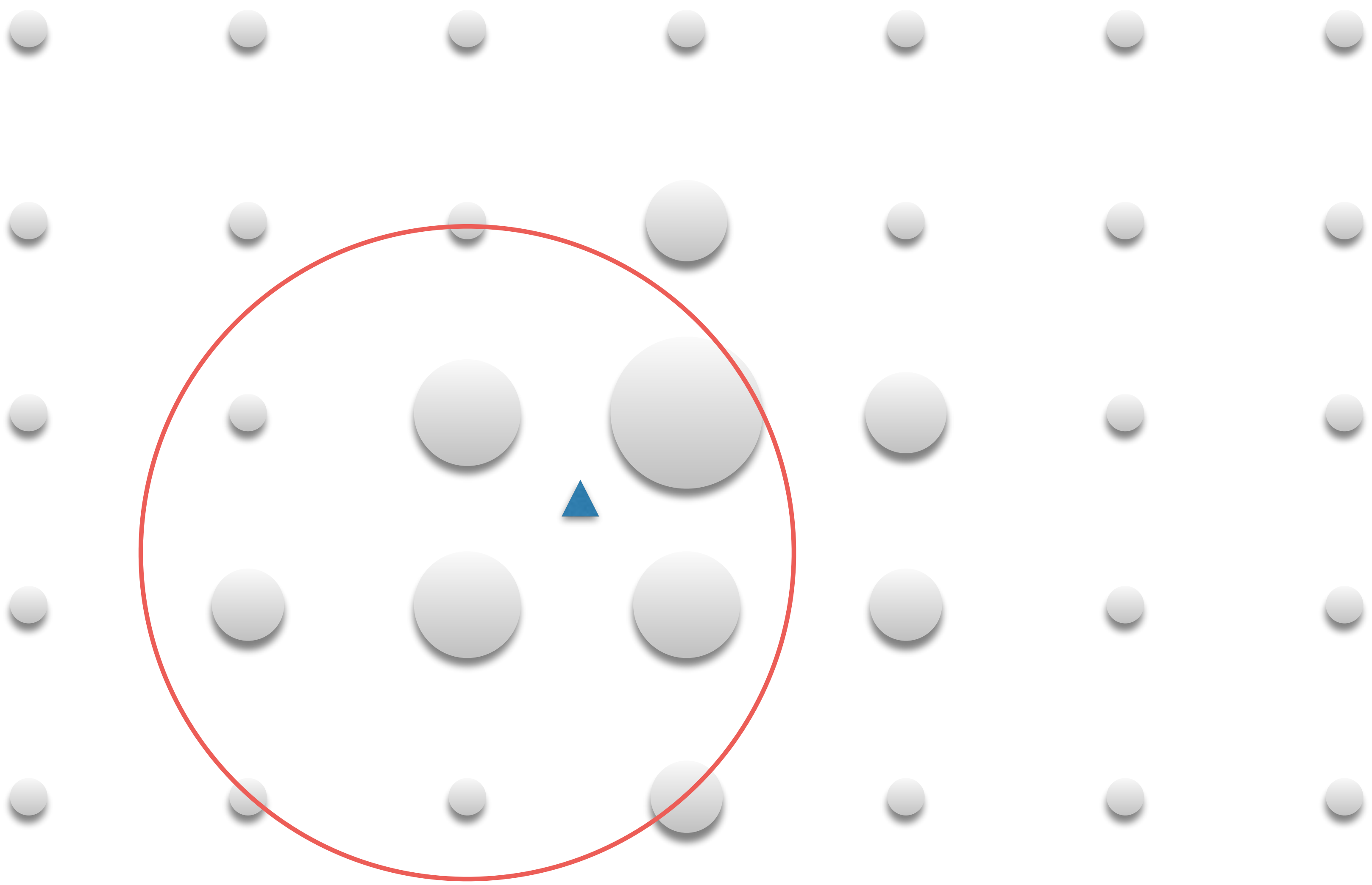

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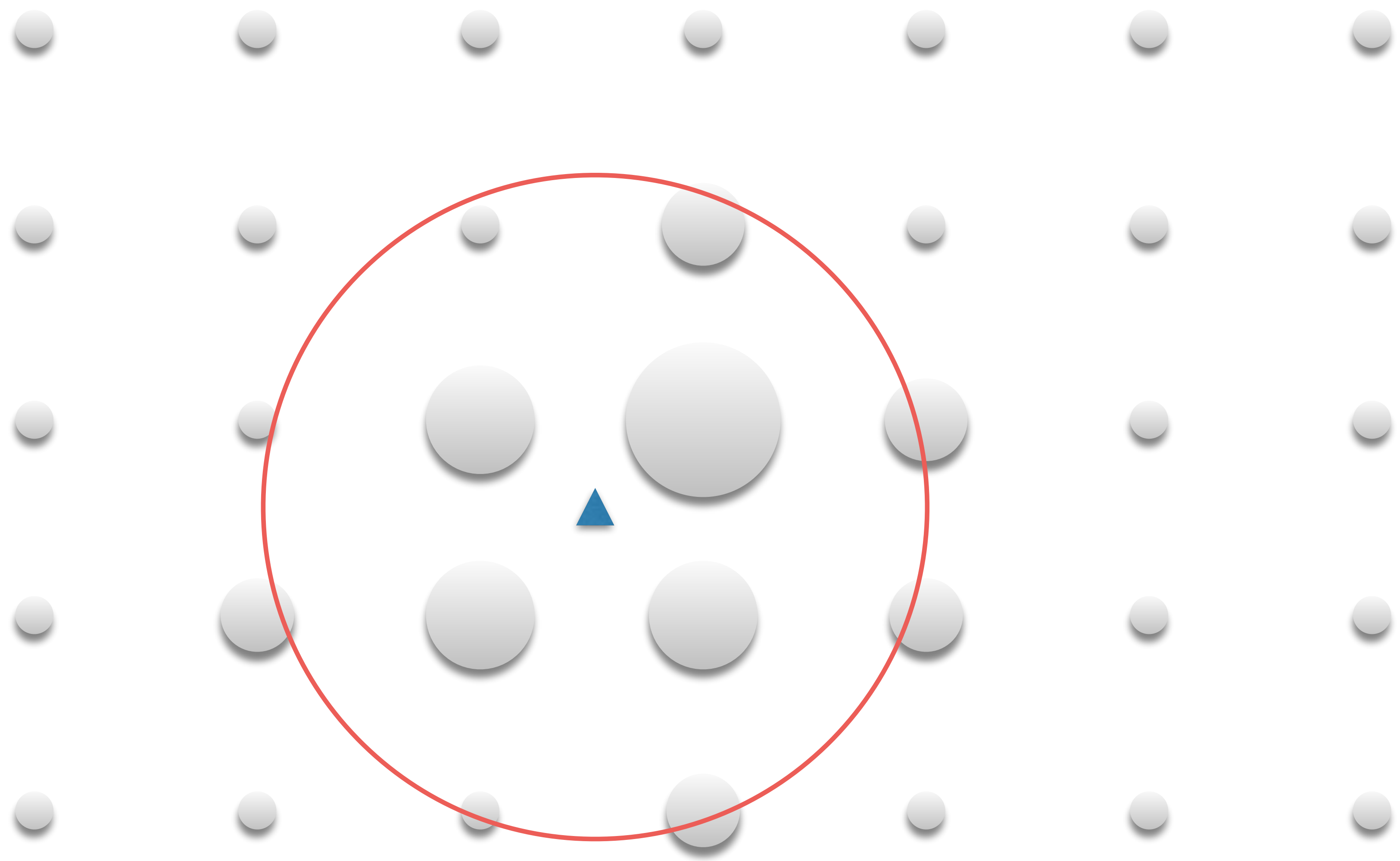

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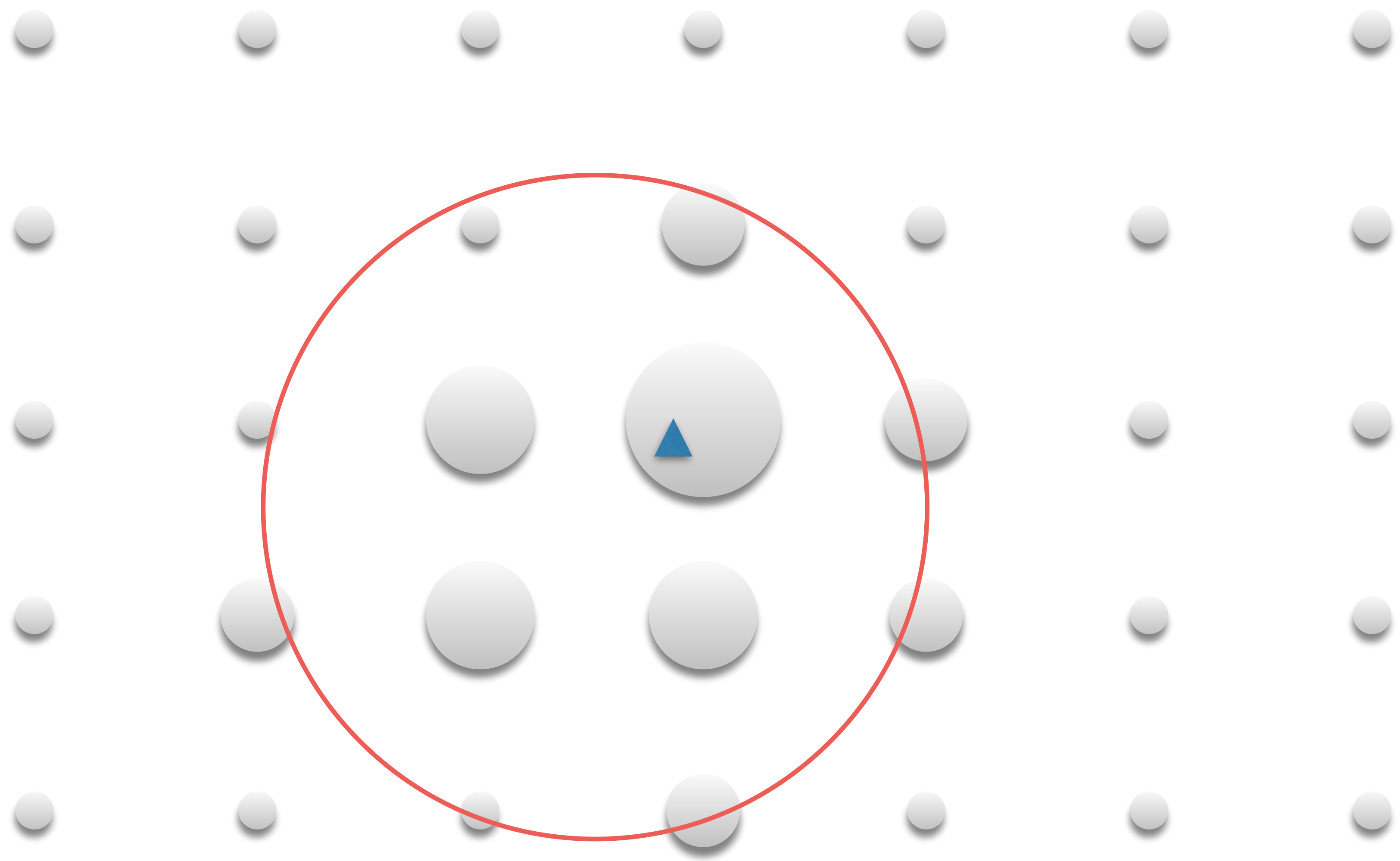

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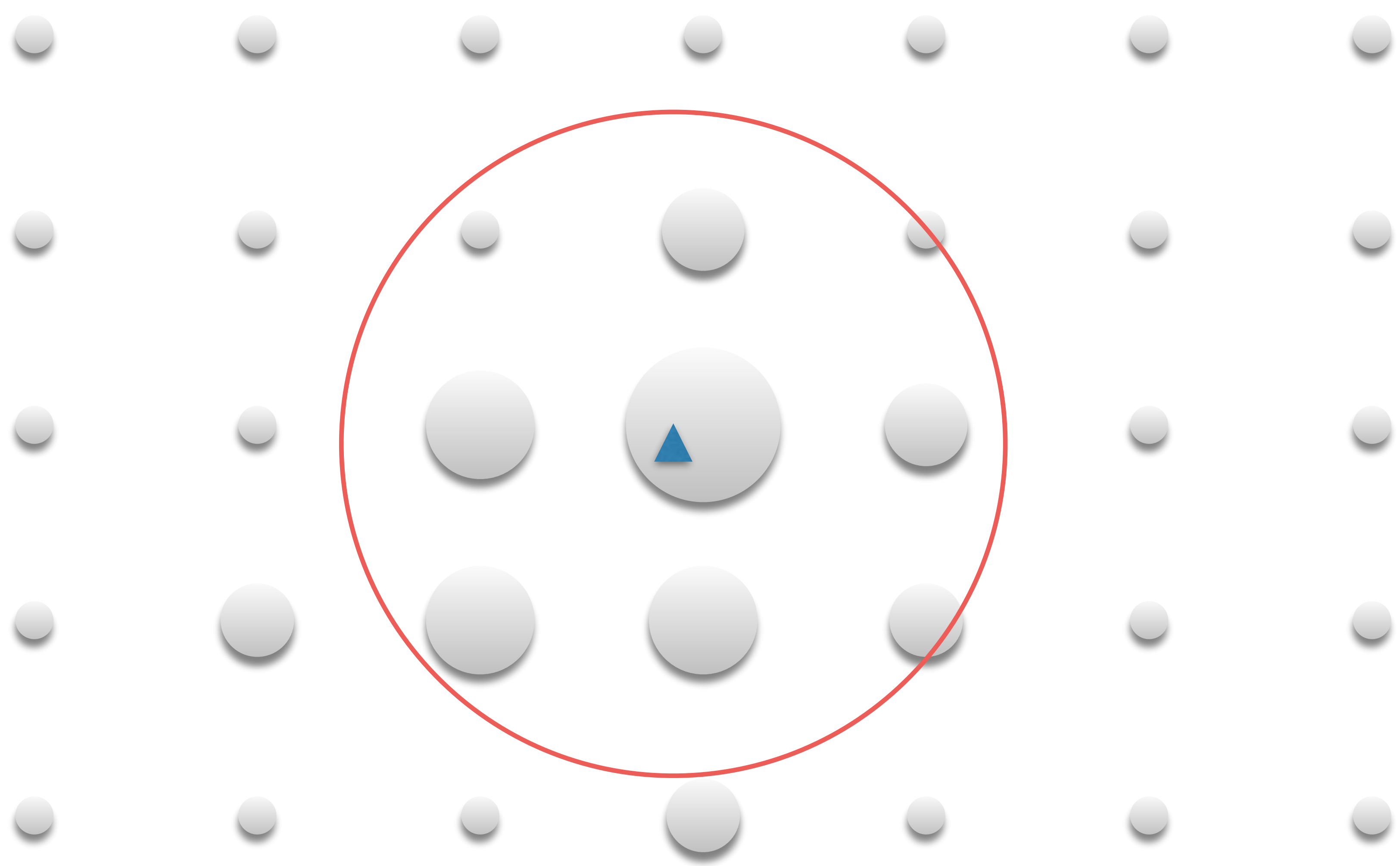
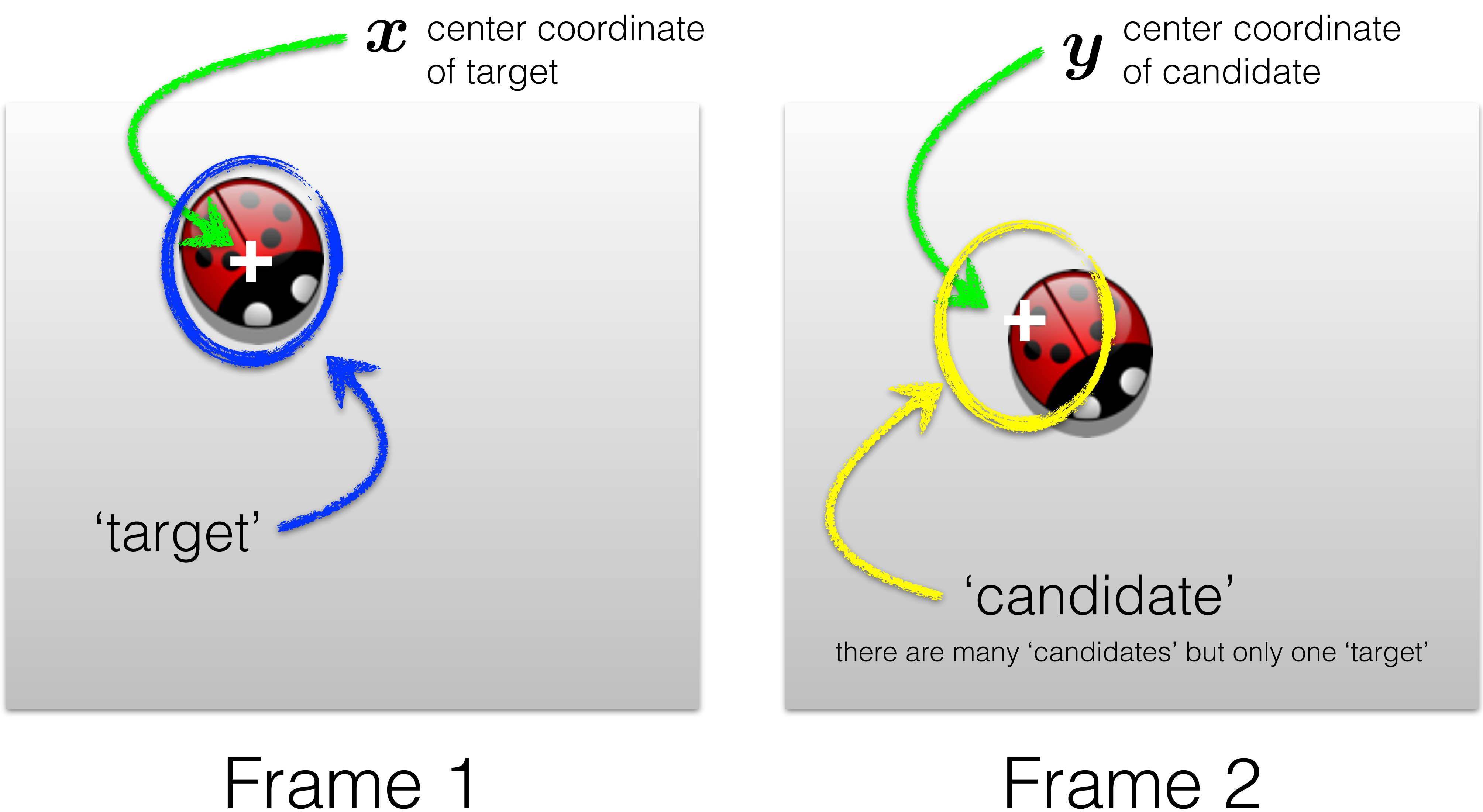


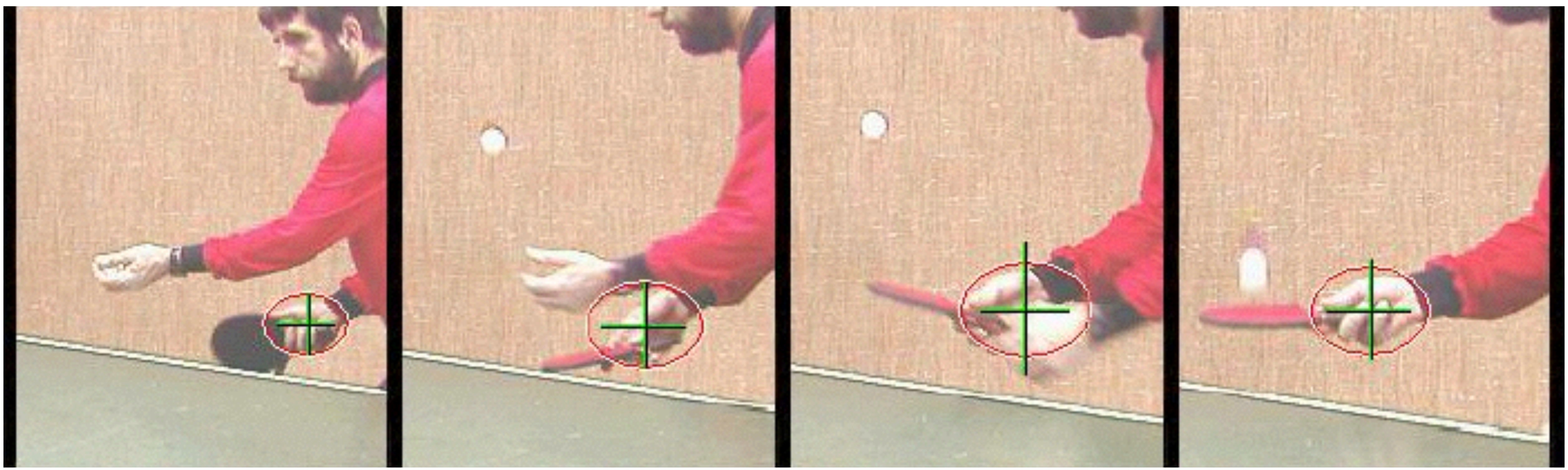
Finally… mean shift tracking in video

Goal: find the best candidate location in frame 2

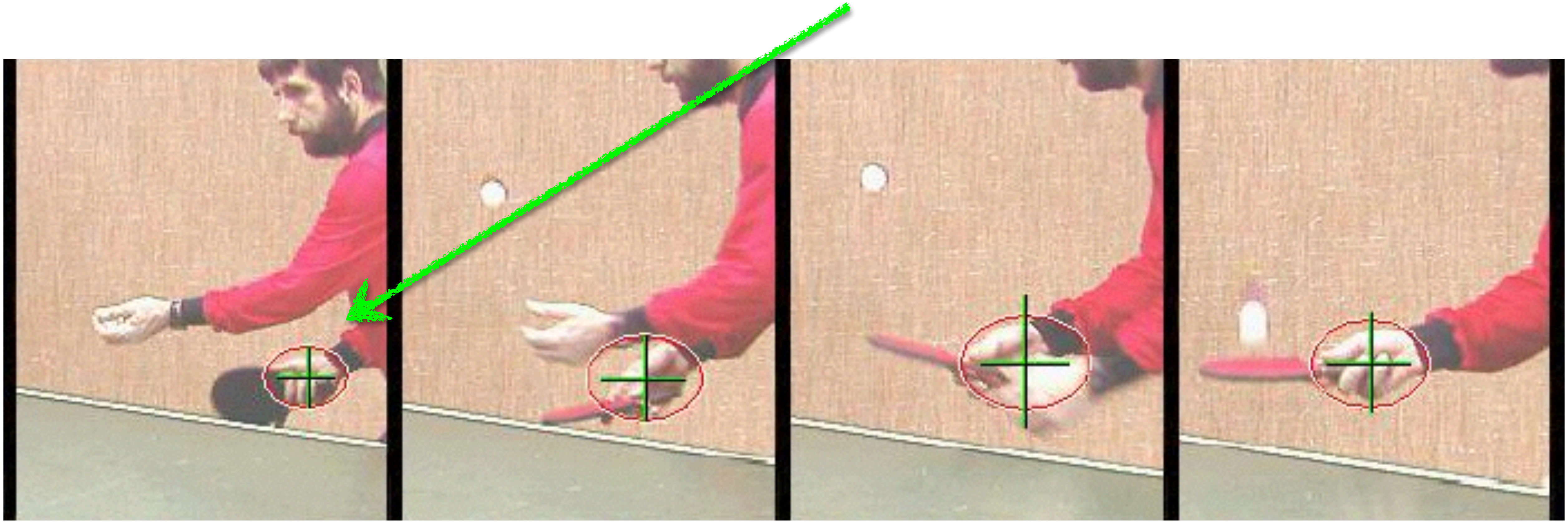


Use the mean shift algorithm
to find the best candidate location

Non-rigid object tracking

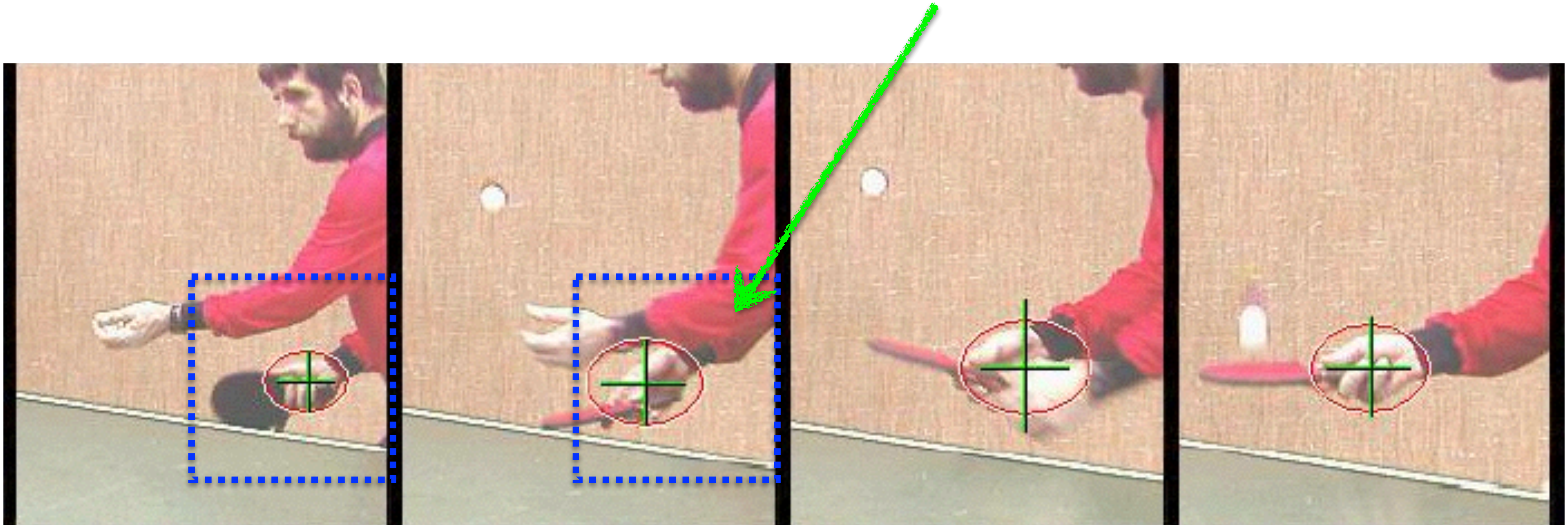


Compute a descriptor for the target



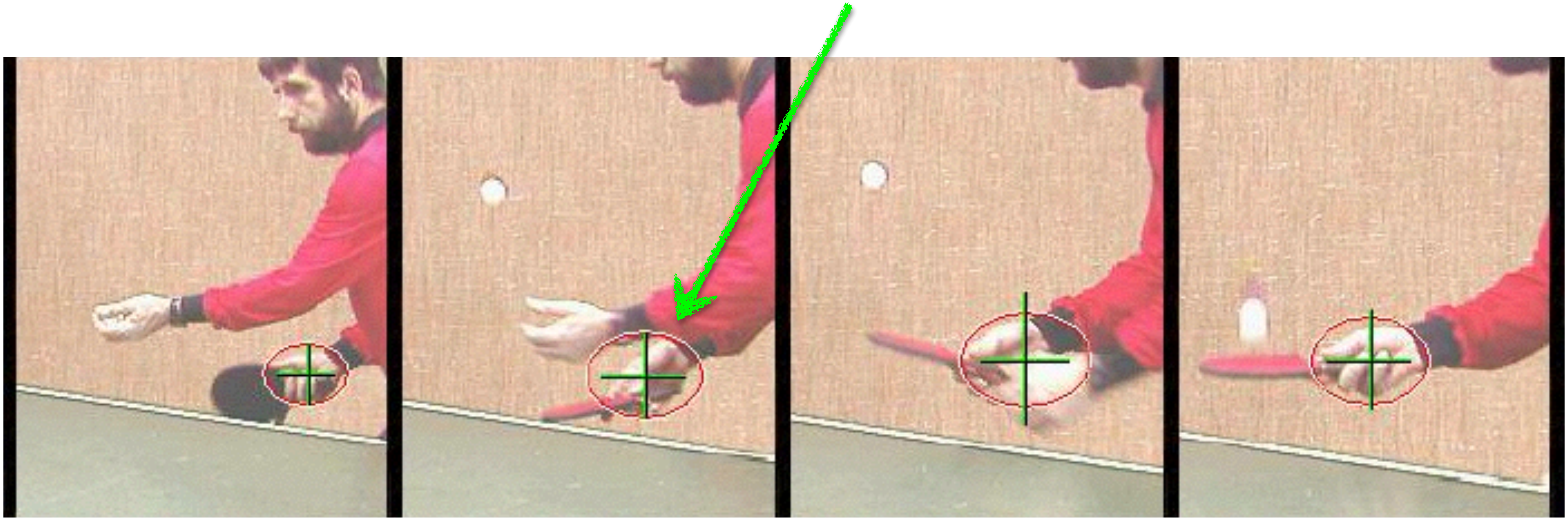
Target

Search for similar descriptor in neighborhood in next frame



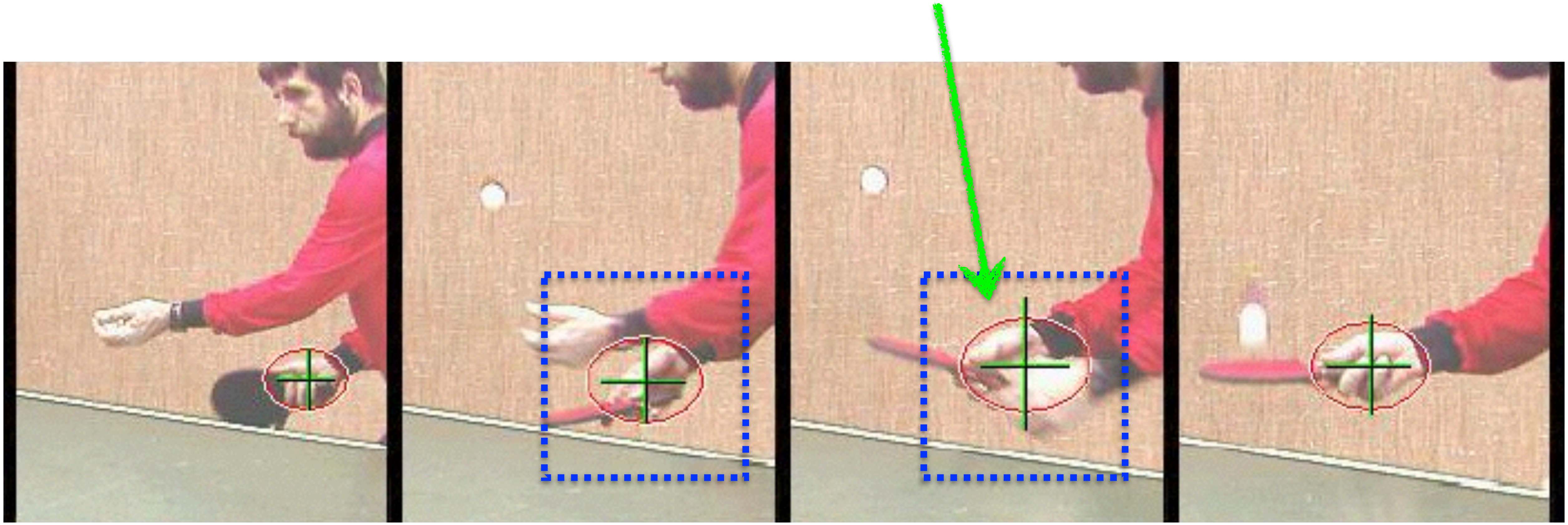
Target Candidate

Compute a descriptor for the new target



Target

Search for similar descriptor in neighborhood in next frame



Target Candidate

How do we model the target and candidate regions?

Modeling the target

M-dimensional **target** descriptor

$$\boldsymbol{q} = \{q_1, \ldots, q_M\}$$

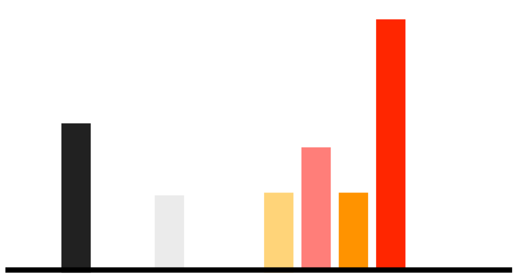
(centered at target center)

Normalization factor

Kronecker delta function

$$q_m = C \sum_n k(\|\boldsymbol{x}_n\|^2)\delta[b(\boldsymbol{x}_n) - m]$$

function of inverse distance (weight)



A normalized color histogram (weighted by distance)

Modeling the candidate

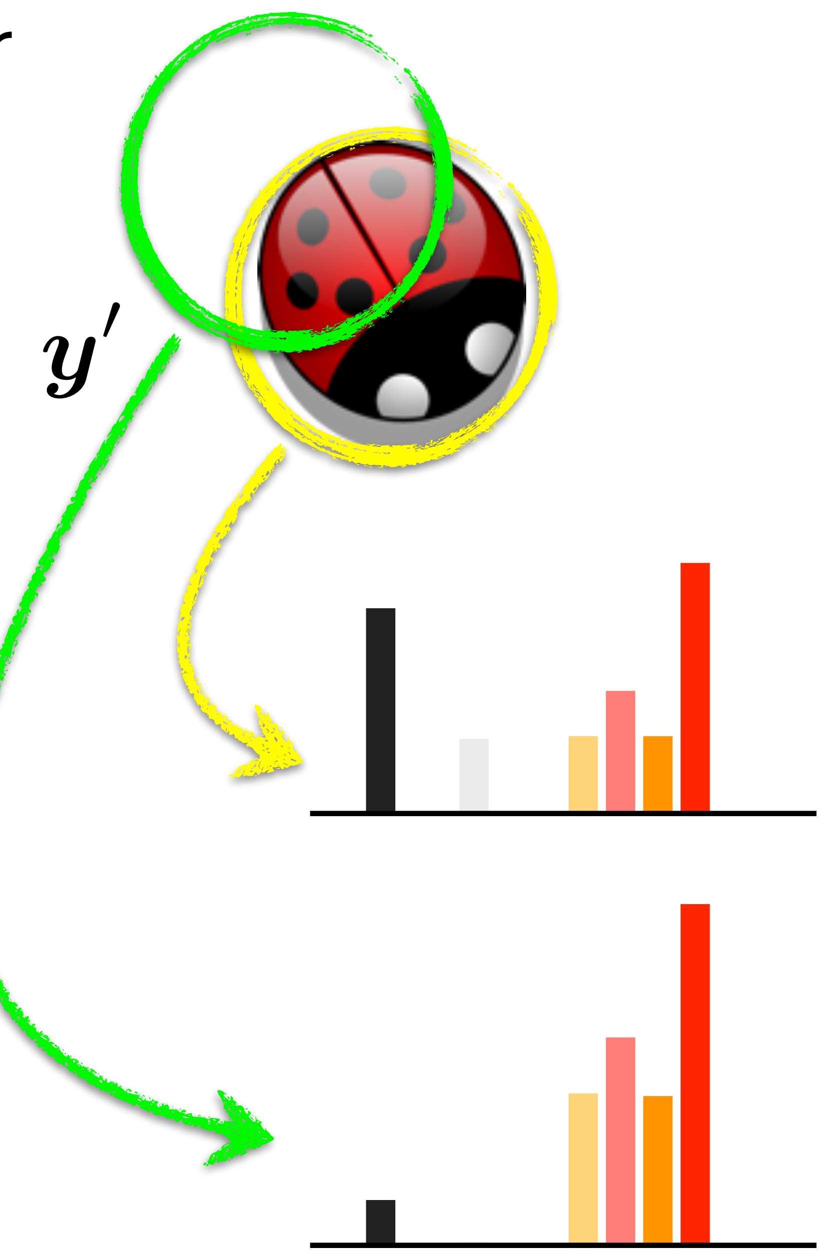
M-dimensional **candidate** descriptor

$$\boldsymbol{p}(\boldsymbol{y}) = \{p_1(\boldsymbol{y}), \ldots, p_M(\boldsymbol{y}\}$$

(centered at location **y**)

$$p_m = C_h \sum_n k\left(\left\|\frac{\boldsymbol{y} - \boldsymbol{x}_n}{h}\right\|^2\right)\delta[b(\boldsymbol{x}_n) - m]$$

bandwidth



Similarity between the target and candidate

Distance function

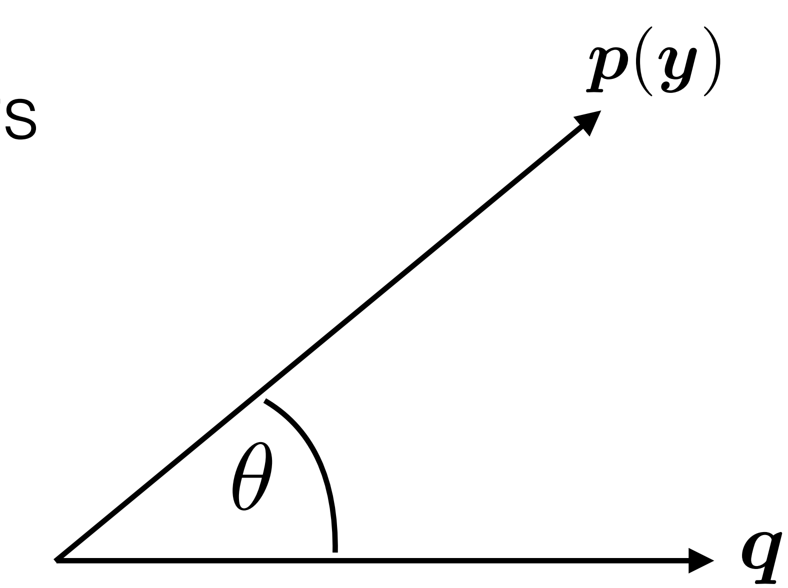
$$d(\boldsymbol{y}) = \sqrt{1 - \rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]}$$

Bhattacharyya Coefficient

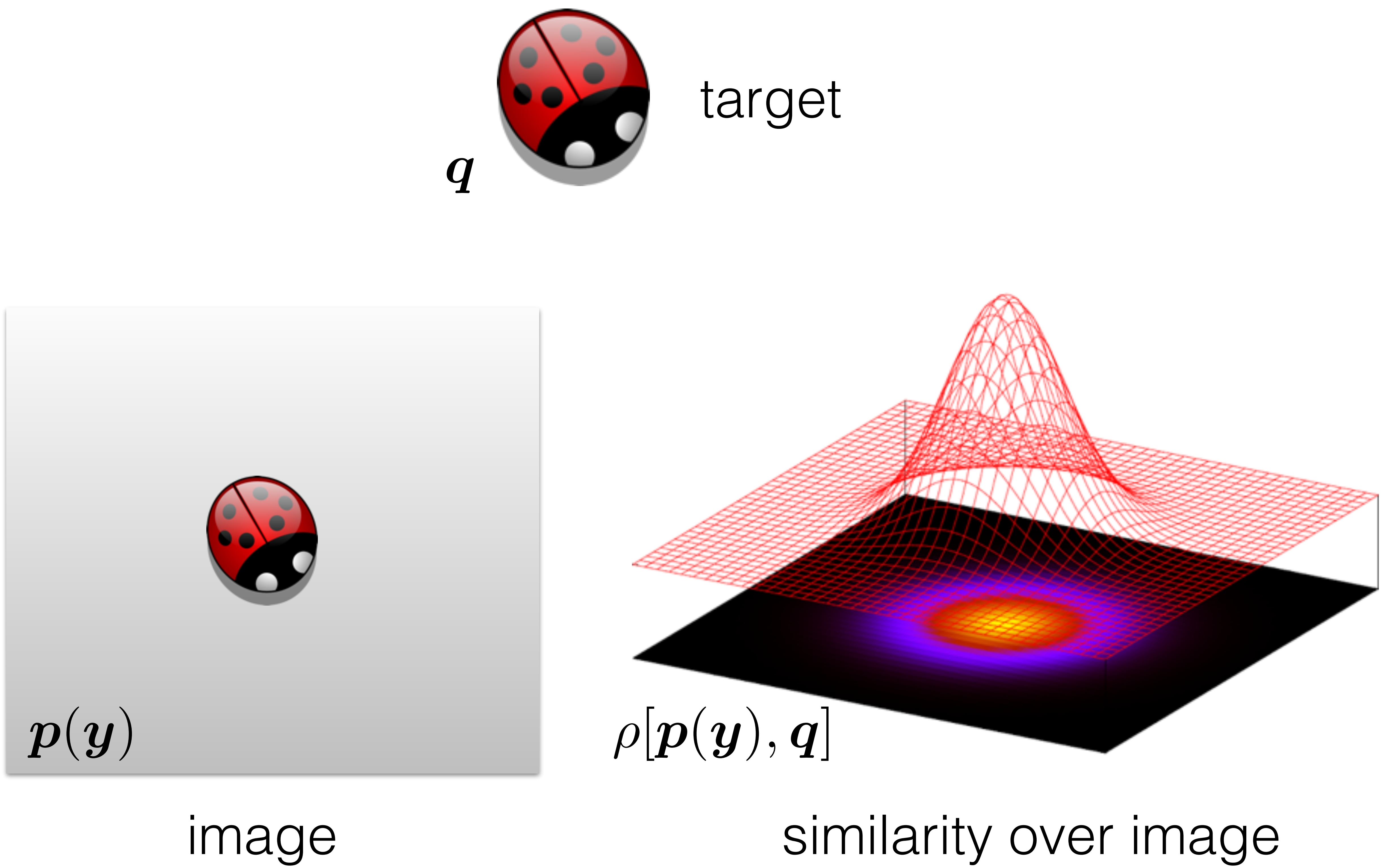
$$\rho(y) \equiv \rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}] = \sum_m \sqrt{p_m(\boldsymbol{y}) q_u}$$

Just the Cosine distance between two unit vectors

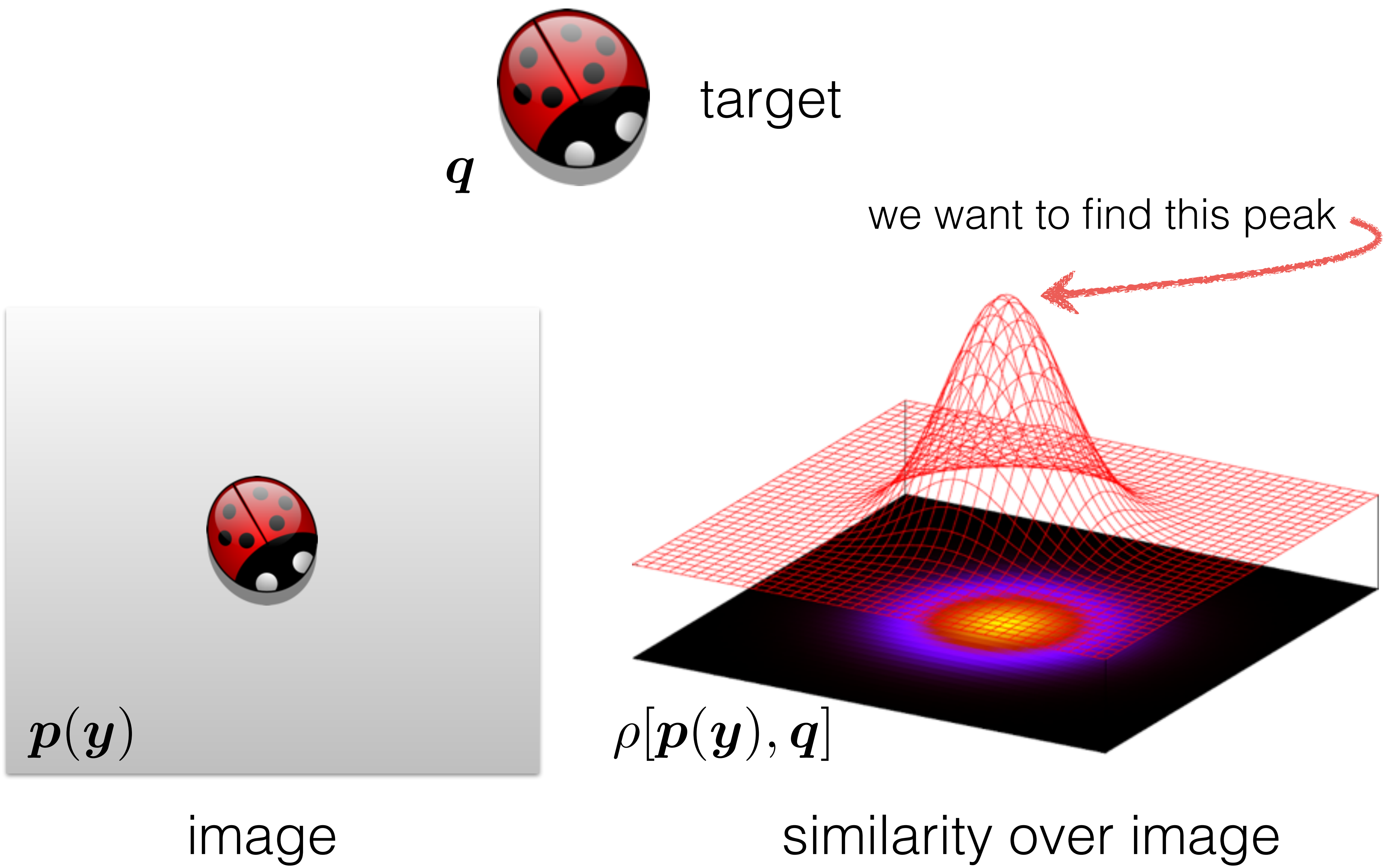
$$\rho(\boldsymbol{y}) = \cos\theta_{\boldsymbol{y}} = \frac{\boldsymbol{p}(\boldsymbol{y})^\top \boldsymbol{q}}{\|\boldsymbol{p}\| \|\boldsymbol{q}\|} = \sum_m \sqrt{p_m(\boldsymbol{y}) q_m}$$



Now we can compute the similarity between a target and multiple candidate regions



target
$\boldsymbol{q}$
$\boldsymbol{p}(\boldsymbol{y})$
$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]$
image
similarity over image



target
$\boldsymbol{q}$
we want to find this peak
$\boldsymbol{p}(\boldsymbol{y})$
$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]$
image
similarity over image

Objective function

$$\min_{\boldsymbol{y}} d(\boldsymbol{y}) \quad \text{same as} \quad \max_{\boldsymbol{y}} \rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]$$

Assuming a good initial guess

$$\rho[\boldsymbol{p}(\boldsymbol{y}_0 + \boldsymbol{y}), \boldsymbol{q}]$$

Linearize around the initial guess (Taylor series expansion)

$$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}] \approx \frac{1}{2} \sum_m \sqrt{p_m(\boldsymbol{y}_0) q_m} + \frac{1}{2} \sum_m p_m(\boldsymbol{y}) \sqrt{\frac{q_m}{p_m(\boldsymbol{y}_0)}}$$

function at specified value

derivative

Linearized objective

$$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}] \approx \frac{1}{2} \sum_m \sqrt{p_m(\boldsymbol{y}_0) q_m} + \frac{1}{2} \sum_m p_m(\boldsymbol{y}) \sqrt{\frac{q_m}{p_m(\boldsymbol{y}_0)}}$$

$$p_m = C_h \sum_n k \left(\left\| \frac{\boldsymbol{y} - \boldsymbol{x}_n}{h} \right\|^2 \right) \delta[b(\boldsymbol{x}_n) - m]$$

Remember definition of this?

Fully expanded

$$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}] \approx \frac{1}{2} \sum_m \sqrt{p_m(\boldsymbol{y}_0) q_m} + \frac{1}{2} \sum_m \left\{ C_h \sum_n k \left(\left\| \frac{\boldsymbol{y} - \boldsymbol{x}_n}{h} \right\|^2 \right) \delta[b(\boldsymbol{x}_n) - m] \right\} \sqrt{\frac{q_m}{p_m(\boldsymbol{y}_0)}}$$

Fully expanded linearized objective

$$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}] \approx \frac{1}{2} \sum_m \sqrt{p_m(\boldsymbol{y}_0) q_m} + \frac{1}{2} \sum_m \left\{ C_h \sum_n k \left(\left\| \frac{\boldsymbol{y} - \boldsymbol{x}_n}{h} \right\|^2 \right) \delta[b(\boldsymbol{x}_n) - m] \right\} \sqrt{\frac{q_m}{p_m(\boldsymbol{y}_0)}}$$

Moving terms around…

$$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}] \approx \frac{1}{2} \sum_m \sqrt{p_m(\boldsymbol{y}_0) q_m} + \frac{C_h}{2} \sum_n w_n k \left(\left\| \frac{\boldsymbol{y} - \boldsymbol{x}_n}{h} \right\|^2 \right)$$

Does not depend on unknown **y**

Weighted kernel density estimate

where $$w_n = \sum_m \sqrt{\frac{q_m}{p_m(\boldsymbol{y}_0)}} \delta[b(\boldsymbol{x}_n) - m]$$

Weight is bigger when $q_m > p_m(\boldsymbol{y}_0)$

OK, why are we doing all this math?

We want to maximize this

$$\max_{\boldsymbol{y}} \rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]$$

We want to maximize this

$$\max_{\boldsymbol{y}} \rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]$$

Fully expanded linearized objective

$$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}] \approx \frac{1}{2} \sum_m \sqrt{p_m(\boldsymbol{y}_0) q_m} + \frac{C_h}{2} \sum_n w_n k \left(\left\| \frac{\boldsymbol{y} - \boldsymbol{x}_n}{h} \right\|^2 \right)$$

where $\quad w_n = \sum_m \sqrt{\frac{q_m}{p_m(\boldsymbol{y}_0)}} \delta[b(\boldsymbol{x}_n) - m]$

We want to maximize this

$$\max_{\boldsymbol{y}} \rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]$$

Fully expanded linearized objective

$$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}] \approx \frac{1}{2}\sum_m \sqrt{p_m(\boldsymbol{y}_0) q_m} + \frac{C_h}{2}\sum_n w_n k\left(\left\|\frac{\boldsymbol{y} - \boldsymbol{x}_n}{h}\right\|^2\right)$$

doesn't depend on unknown **y**

where $$w_n = \sum_m \sqrt{\frac{q_m}{p_m(\boldsymbol{y}_0)}} \delta[b(\boldsymbol{x}_n) - m]$$

We want to maximize this

$$\max_{\boldsymbol{y}} \rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]$$

only need to maximize this!

Fully expanded linearized objective

$$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}] \approx \frac{1}{2}\sum_m \sqrt{p_m(\boldsymbol{y}_0) q_m} + \frac{C_h}{2}\sum_n w_n k\left(\left\|\frac{\boldsymbol{y} - \boldsymbol{x}_n}{h}\right\|^2\right)$$

doesn't depend on unknown **y**

where $w_n = \sum_m \sqrt{\frac{q_m}{p_m(\boldsymbol{y}_0)}} \delta[b(\boldsymbol{x}_n) - m]$

We want to maximize this

$$\max_{\boldsymbol{y}} \rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]$$

Fully expanded linearized objective

$$\rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}] \approx \frac{1}{2}\sum_m \sqrt{p_m(\boldsymbol{y}_0) q_m} + \frac{C_h}{2}\sum_n w_n k\left(\left\|\frac{\boldsymbol{y}-\boldsymbol{x}_n}{h}\right\|^2\right)$$

doesn't depend on unknown **y**

where $$w_n = \sum_m \sqrt{\frac{q_m}{p_m(\boldsymbol{y}_0)}}\delta[b(\boldsymbol{x}_n) - m]$$

what can we use to solve this weighted KDE?

Mean Shift Algorithm!

$$\frac{C_h}{2} \sum_n w_n k \left(\left\| \frac{\boldsymbol{y} - \boldsymbol{x}_n}{h} \right\|^2 \right)$$

the sample of mean of this KDE is

$$\boldsymbol{y}_1 = \frac{\sum_n \boldsymbol{x}_n w_n g \left(\left\| \frac{\boldsymbol{y}_0 - \boldsymbol{x}_n}{h} \right\|^2 \right)}{\sum_n w_n g \left(\left\| \frac{\boldsymbol{y}_0 - \boldsymbol{x}_n}{h} \right\|^2 \right)}$$

(new candidate location)

(this was derived earlier)

Mean Shift Tracking procedure

1. Initialize location $\boldsymbol{y}_0$
   Compute $\boldsymbol{q}$
   Compute $\boldsymbol{p}(\boldsymbol{y}_0)$

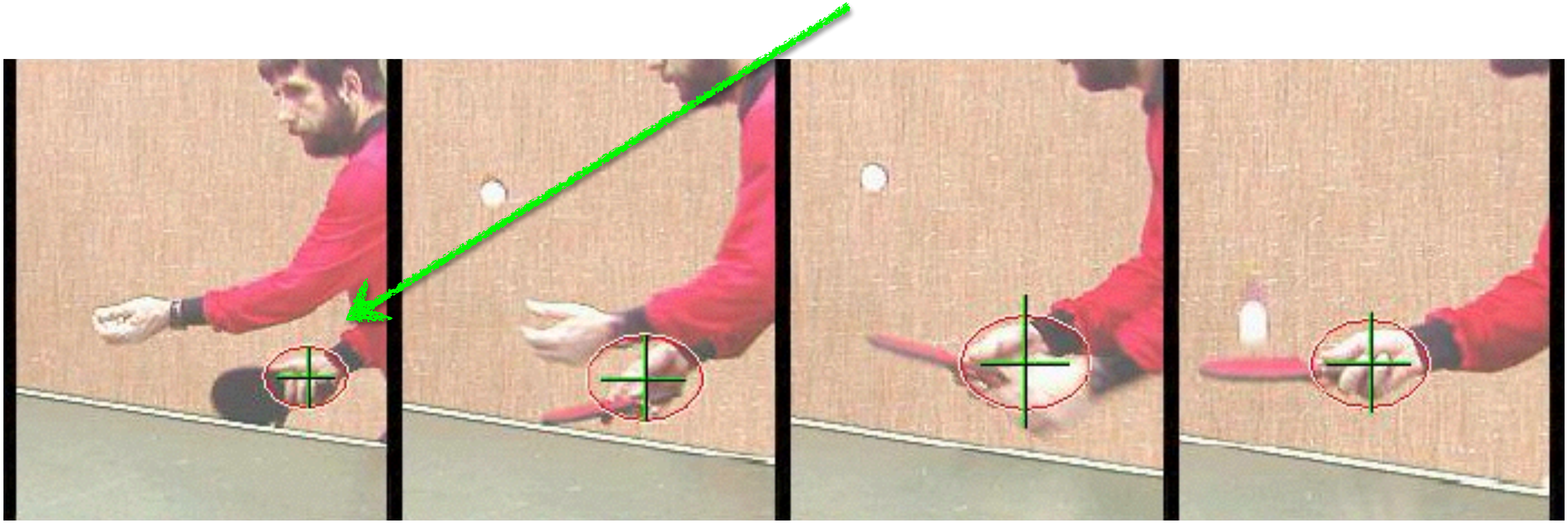
2. Derive weights w_n

3. Shift to new candidate location (mean shift) $\boldsymbol{y}_1$

4. Compute $\boldsymbol{p}(\boldsymbol{y}_1)$

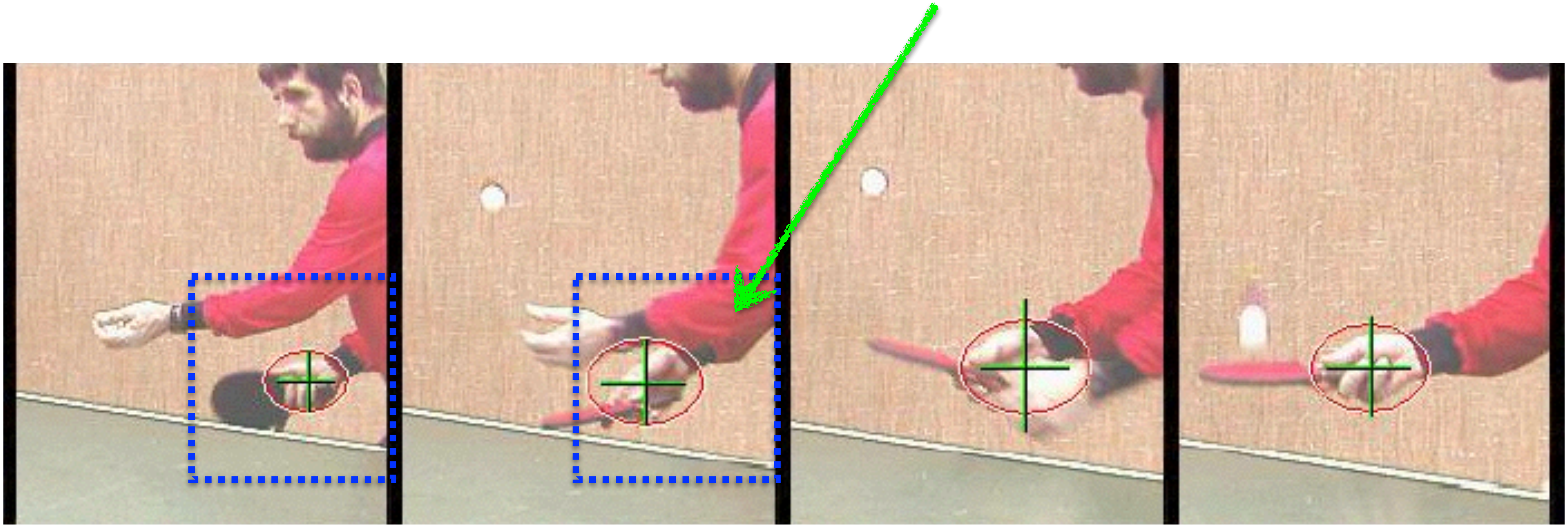
5. If $\|\boldsymbol{y}_0 - \boldsymbol{y}_1\| < \epsilon$ return
   Otherwise $\boldsymbol{y}_0 \leftarrow \boldsymbol{y}_1$ and go back to 2

Compute a descriptor for the target



Target
$\boldsymbol{q}$

Search for similar descriptor in neighborhood in next frame

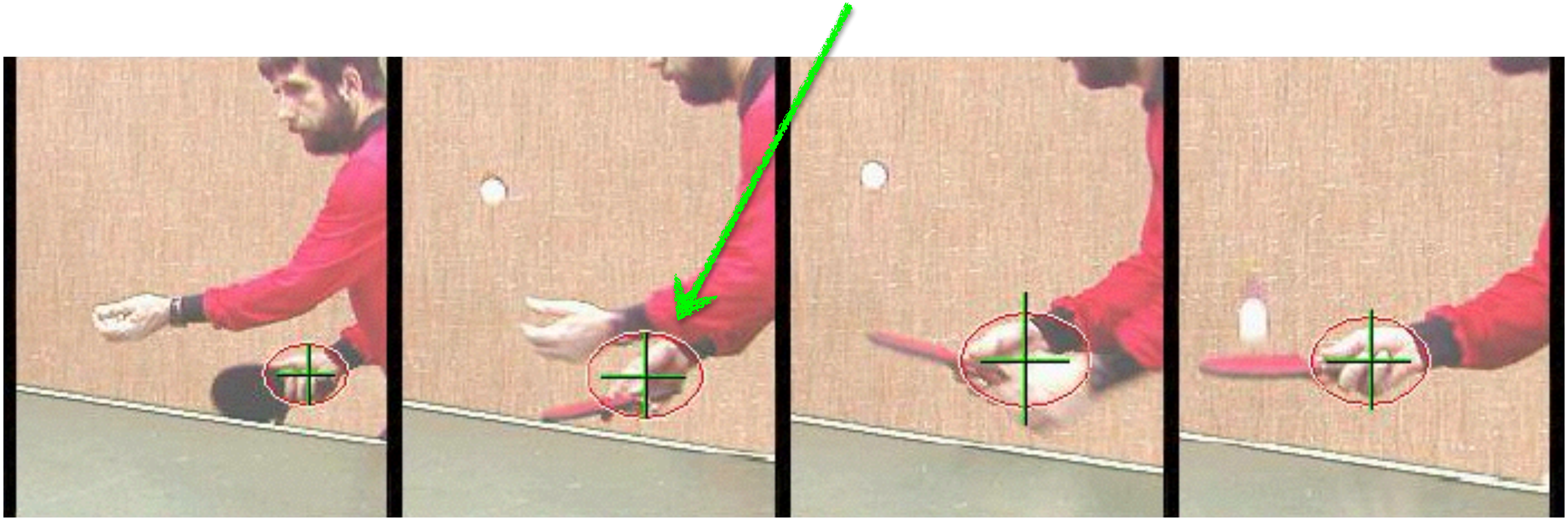


Target

Candidate

$$\max_{\boldsymbol{y}} \rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]$$

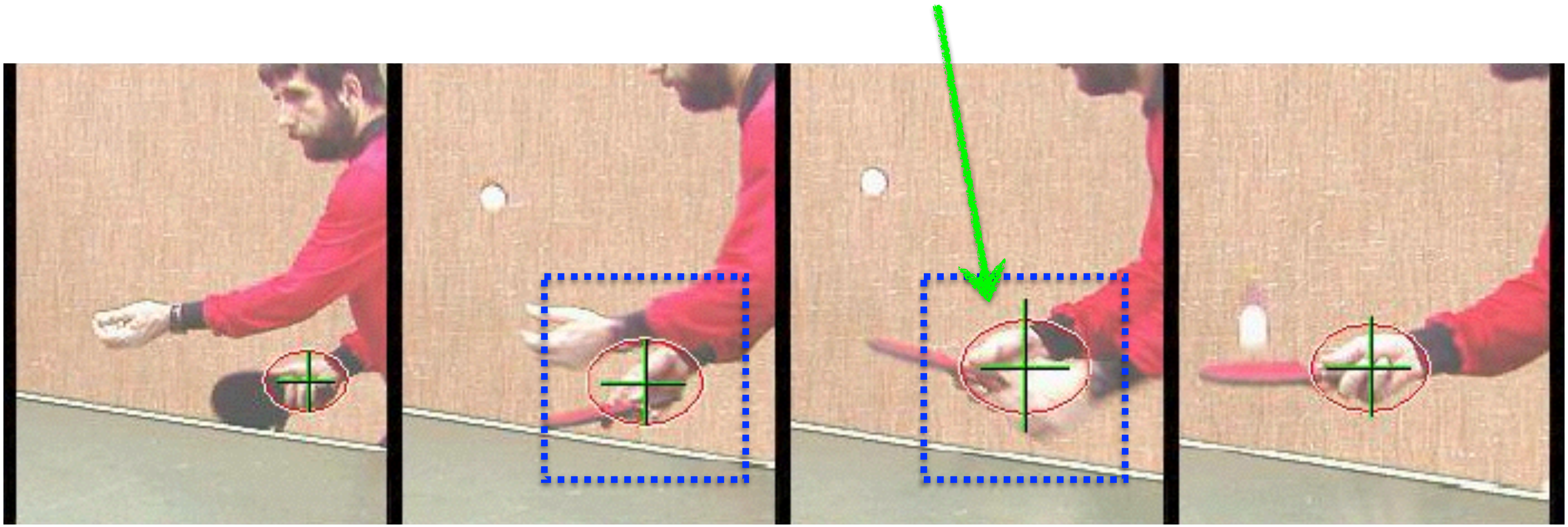
Compute a descriptor for the new target



Target

$\boldsymbol{q}$

Search for similar descriptor in neighborhood in next frame



Target Candidate

$$\max_{\boldsymbol{y}} \rho[\boldsymbol{p}(\boldsymbol{y}), \boldsymbol{q}]$$