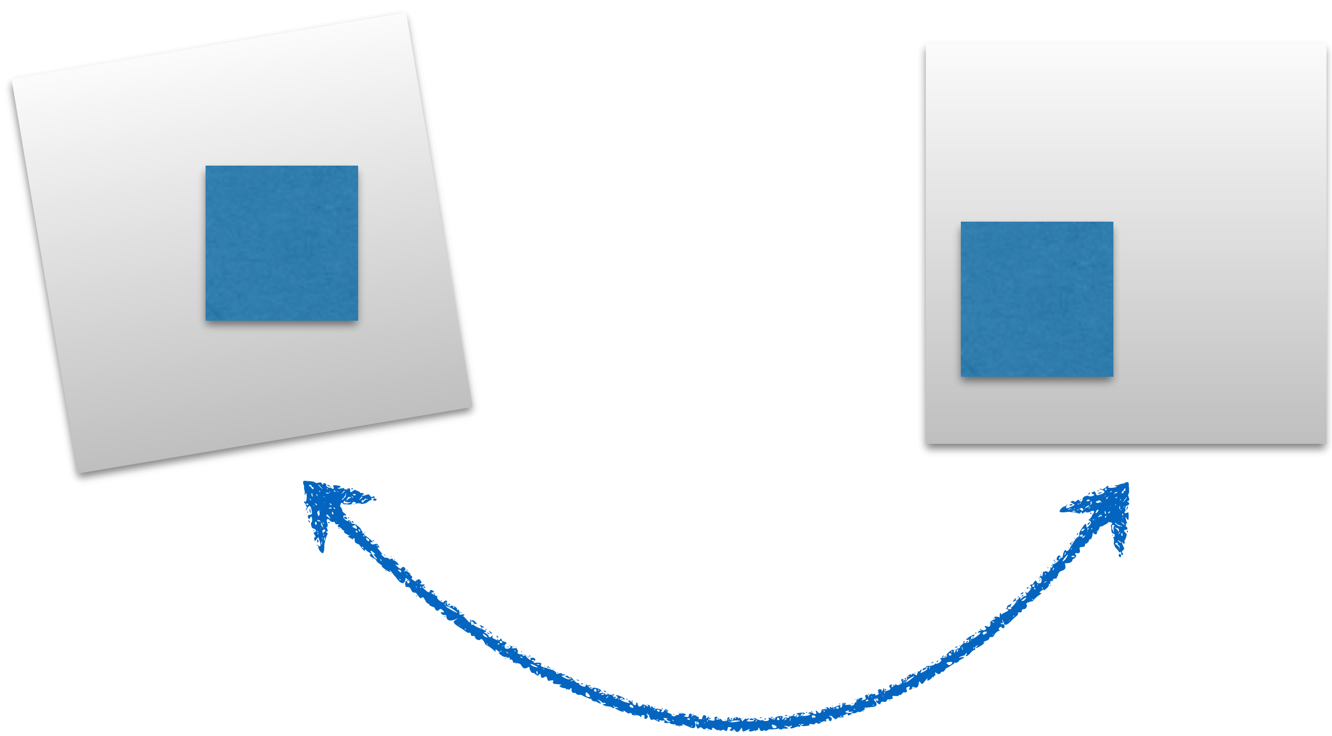
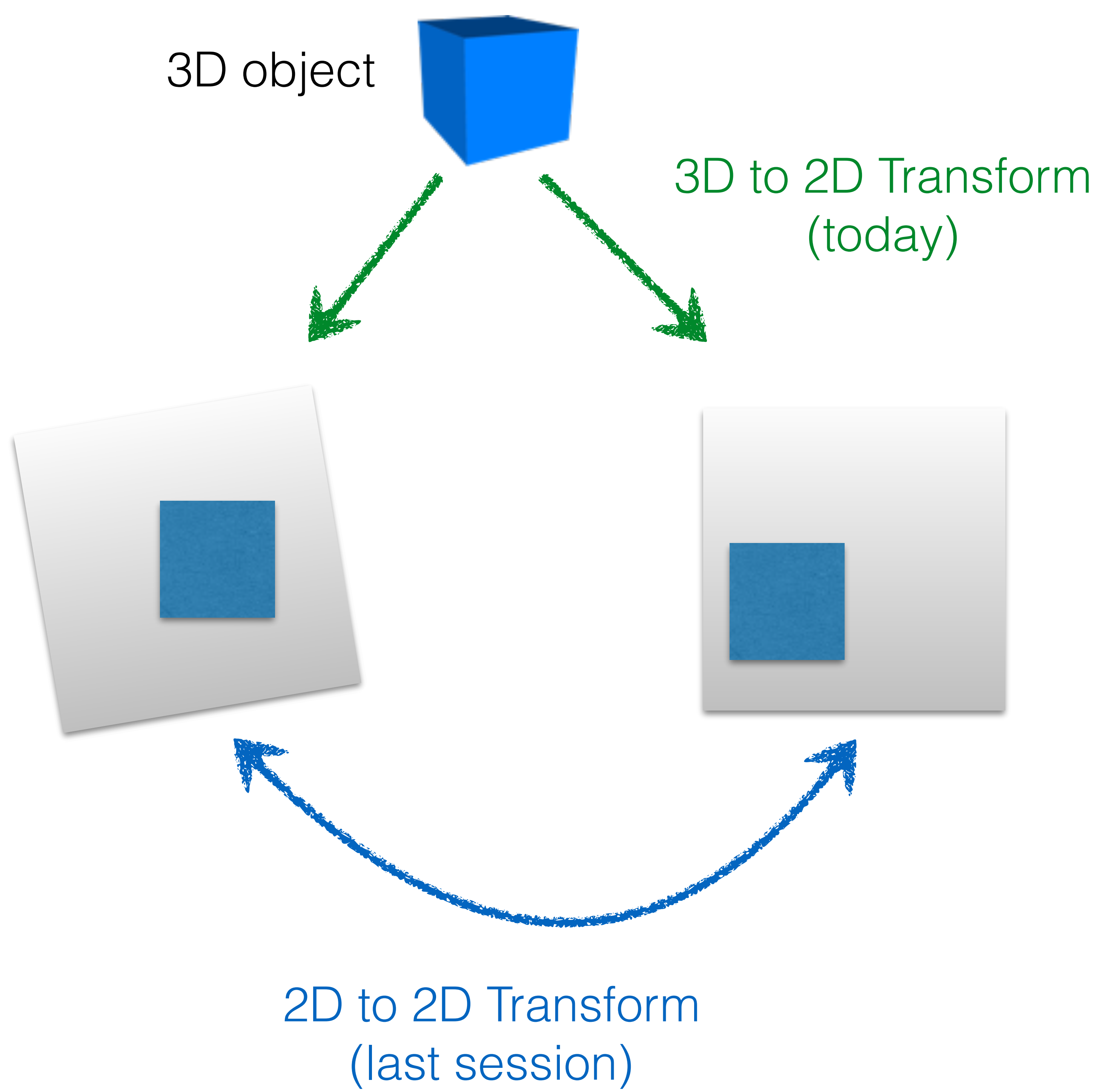

# Two-View Geometry

16-385 Computer Vision

Carnegie Mellon University (Kris Kitani)



2D to 2D Transform
(last session)

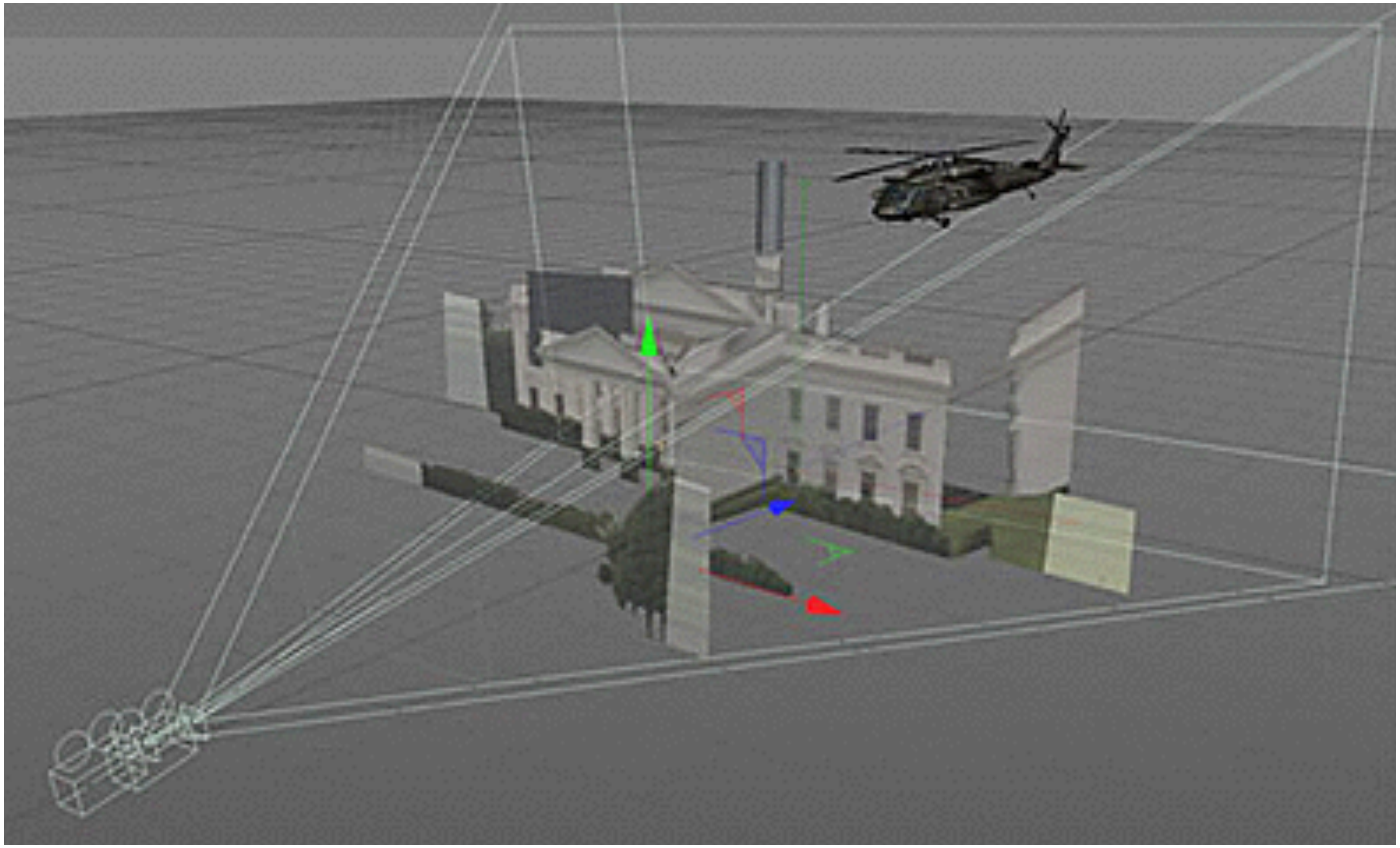

3D object
3D to 2D Transform
(today)
2D to 2D Transform
(last session)

What can we do if we can estimate 3D pose?

# Head Tracking for Desktop Virtual Reality Displays using the Wii Remote

Johnny Chung Lee
Human-Computer Interaction Institute
Carnegie Mellon University


g.co/ProjectTango



# Camera Matrix

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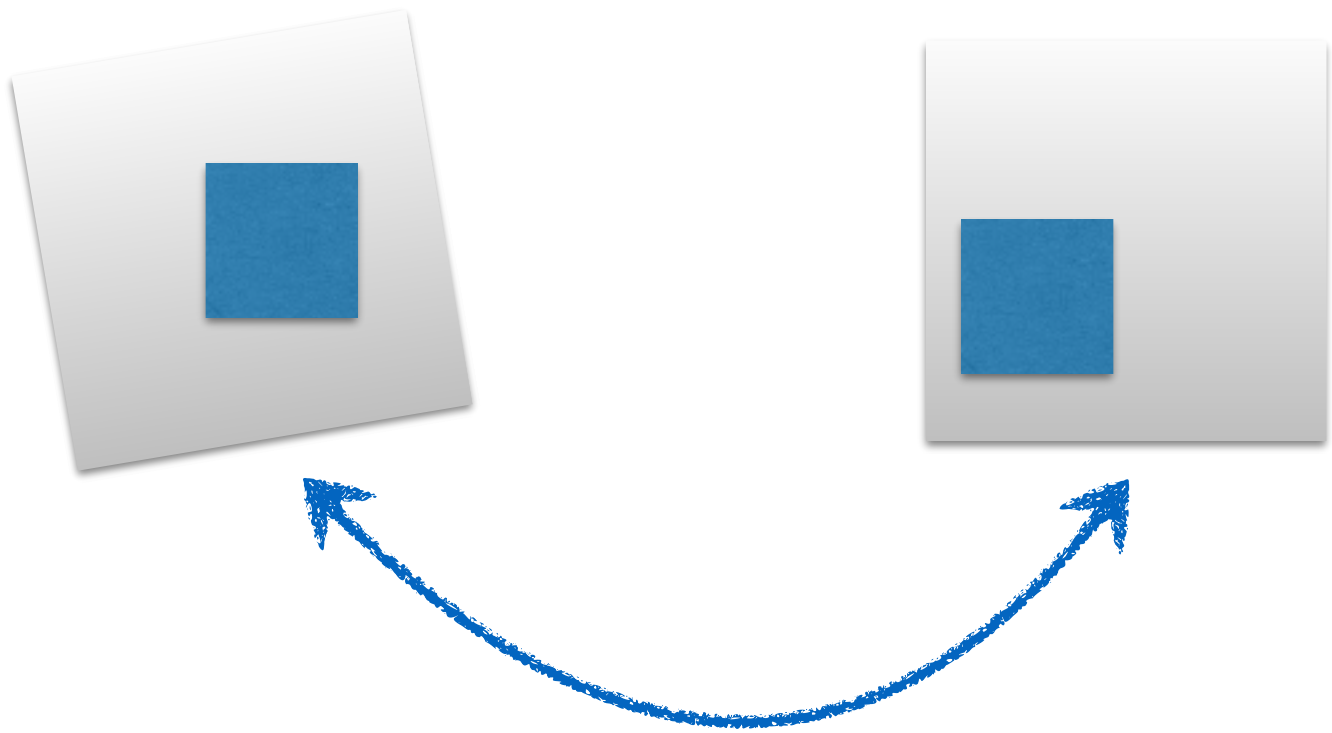
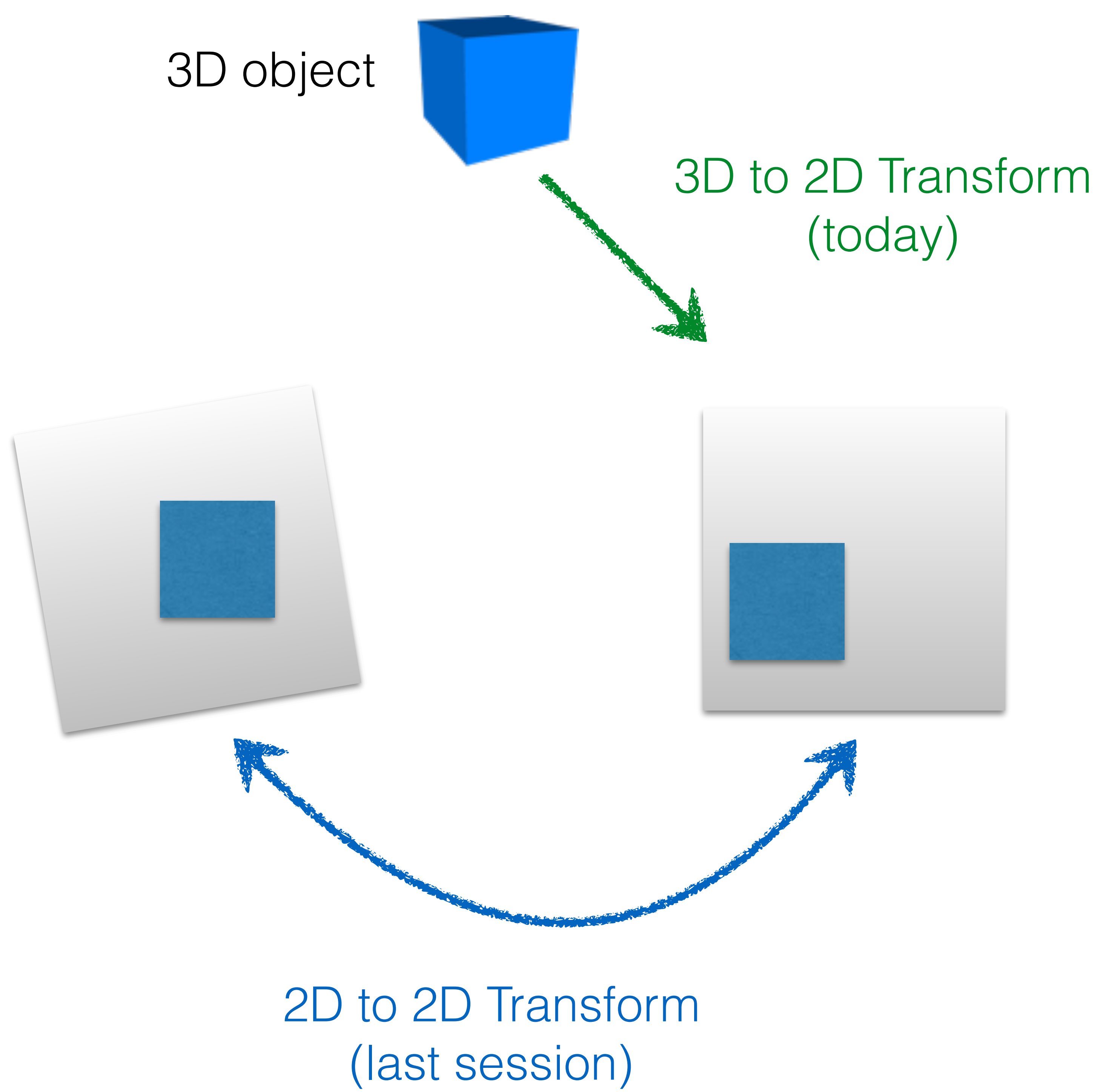


2D to 2D Transform
(last session)


3D object
3D to 2D Transform
(today)
2D to 2D Transform
(last session)

A camera is a mapping between

the **3D world**

and

a **2D image**

A camera is a mapping between
the 3D world and a 2D image

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

2D image point

camera matrix

3D world point

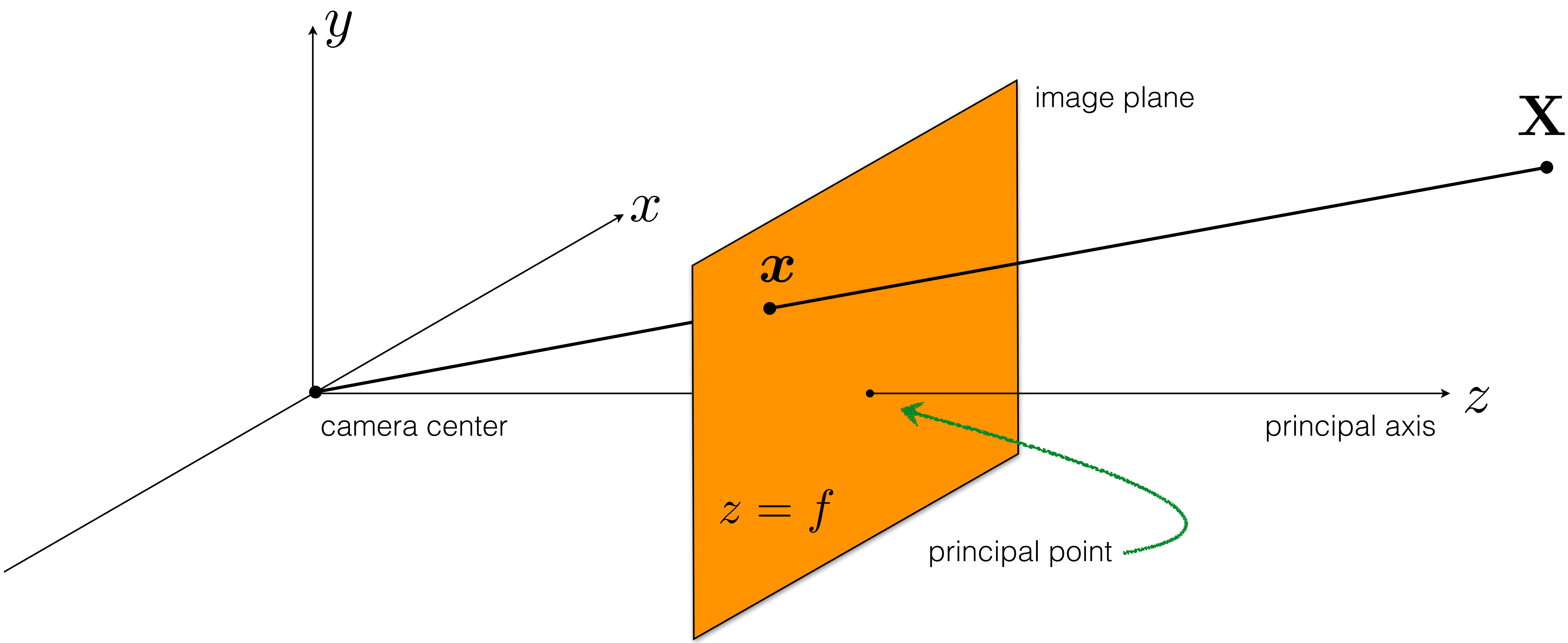
*What do you think the dimensions are?*

$$\boldsymbol{x} = \mathbf{PX}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

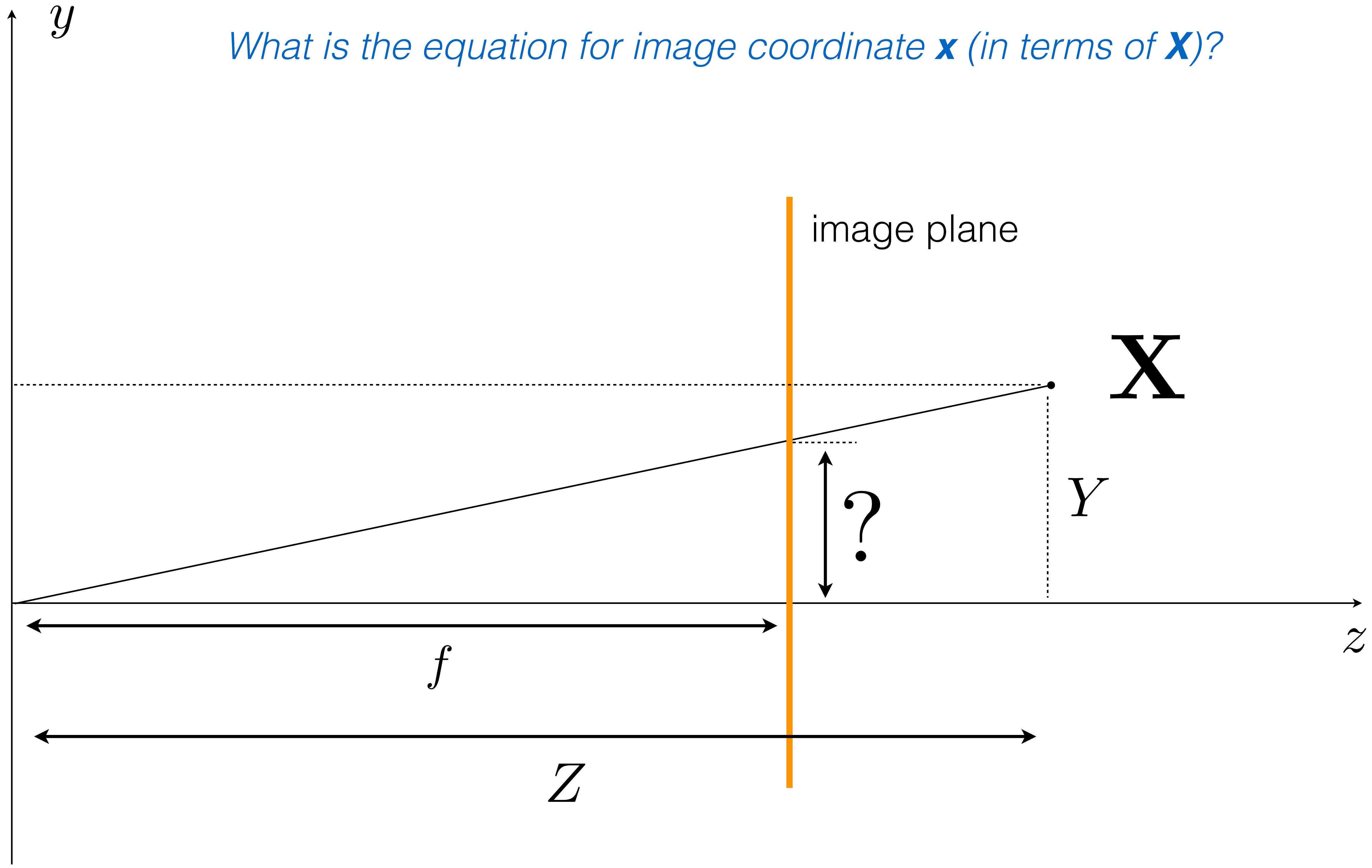
| homogeneous image 3 x 1 | Camera matrix 3 x 4 | homogeneous world point 4 x 1 |
|---|---|---|

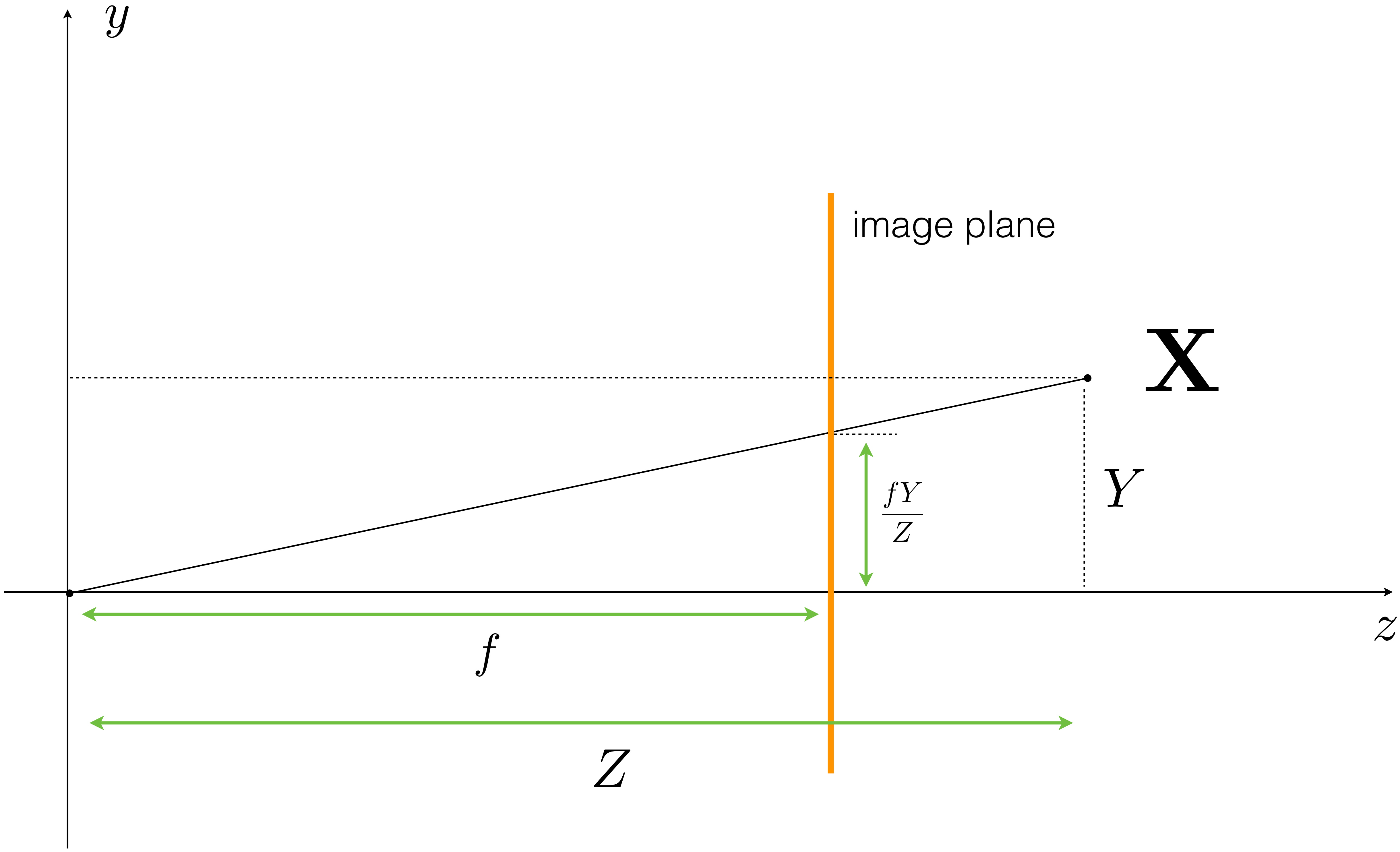
# The pinhole camera




*What is the equation for image coordinate* ***x*** *(in terms of* ***X****)?*

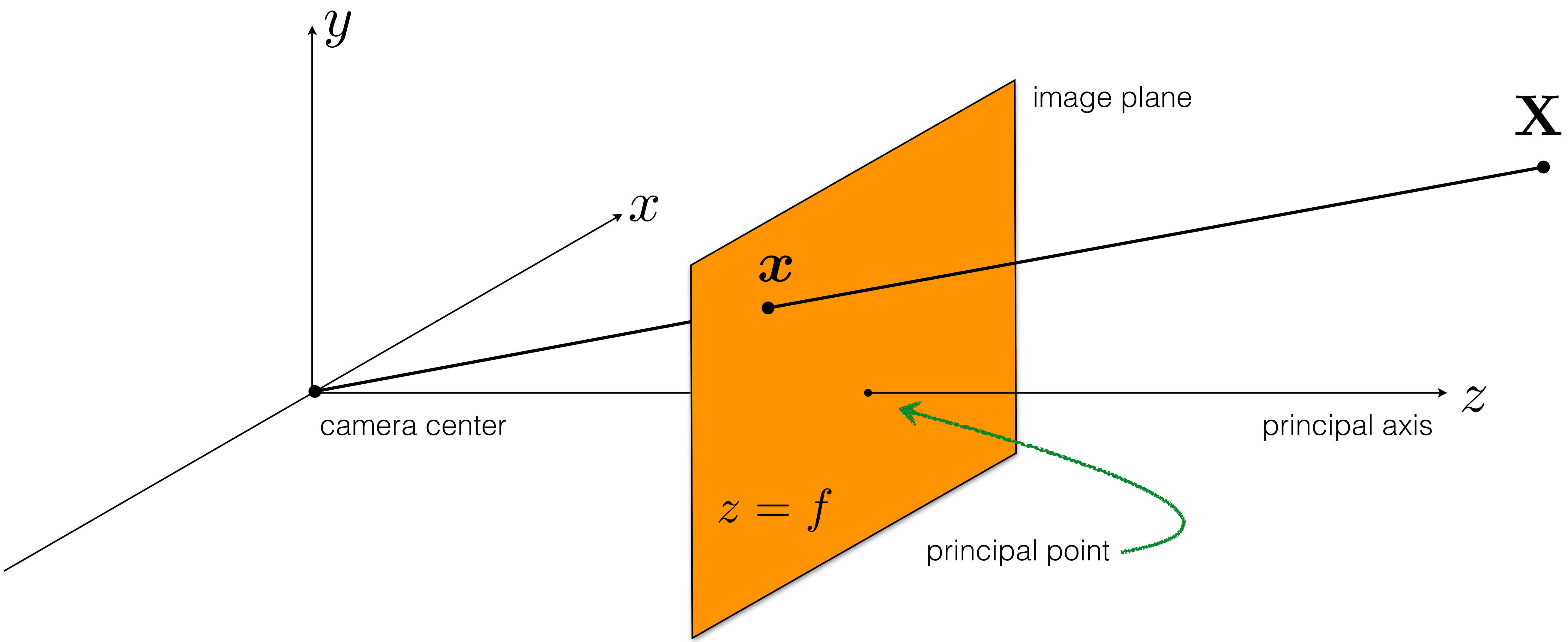
*What is the equation for image coordinate **x** (in terms of **X**)?*






$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

# Pinhole camera geometry




*What is the camera matrix* **P** *for a pinhole camera model?*

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

Relationship from similar triangles…

$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

generic camera model

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

*What does the pinhole camera model look like?*

$$\mathbf{P} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}$$

Relationship from similar triangles…

$$[X \quad Y \quad Z]^\top \mapsto [fX/Z \quad fY/Z]^\top$$

generic camera model

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

*What does the pinhole camera model look like?*

$$\mathbf{P} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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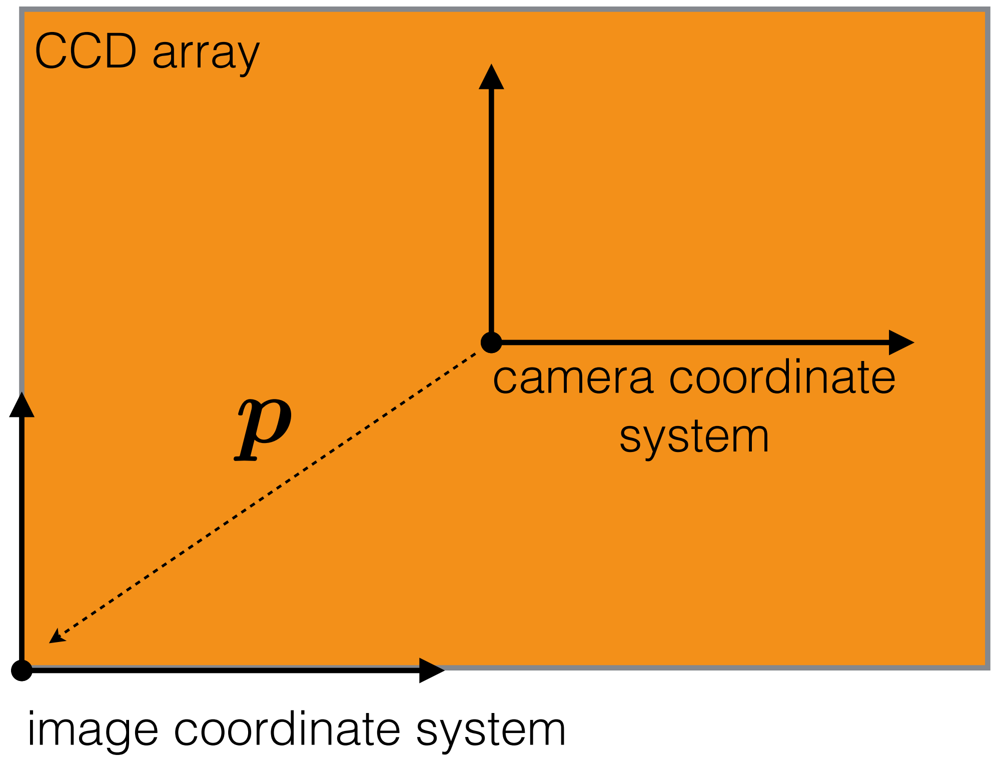
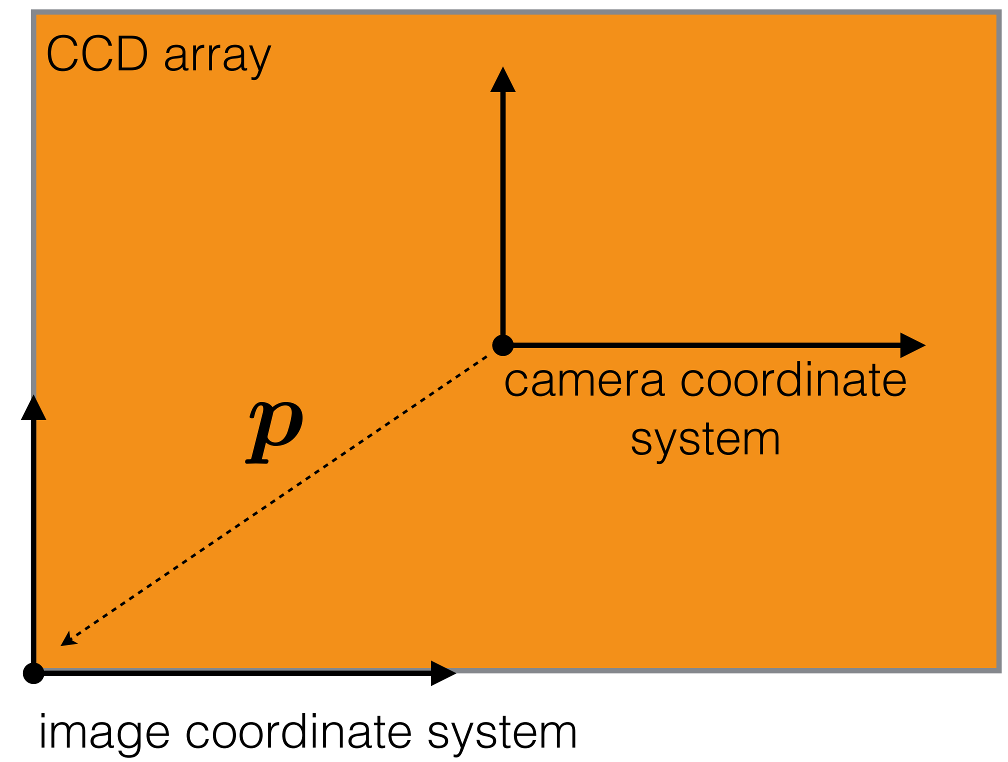
Camera origin and image origin might be different






$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Accounts for different origins

Can be decomposed into two matrices

$$\mathbf{P} = \left[\begin{array}{ccc} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

(3 x 3) (3 x 4)

$$\mathbf{P} = \mathbf{K}[\mathbf{I}|\mathbf{0}]$$

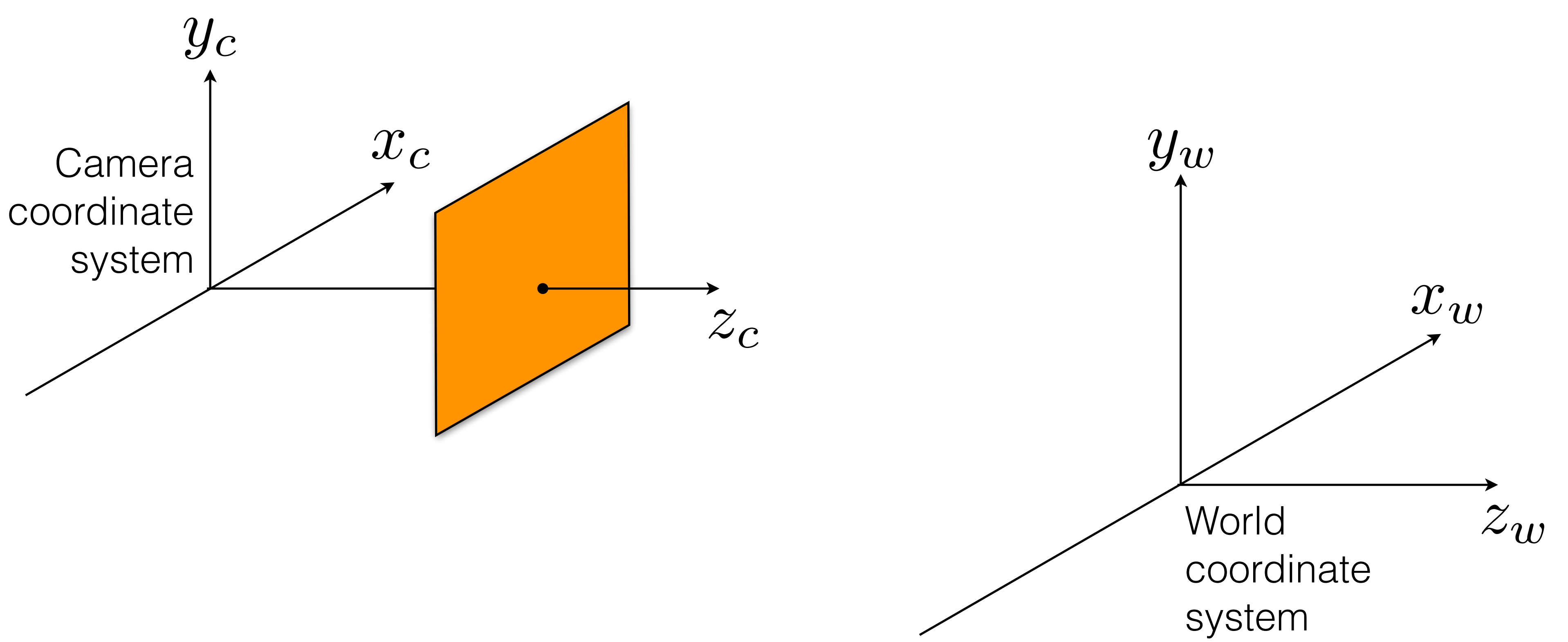
$$\mathbf{K} = \left[\begin{array}{ccc} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{array}\right]$$

calibration matrix

Assumes that the camera and world share the same coordinate system

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

*What if they are different?*
*How do we align them?*



Assumes that the camera and world share the same coordinate system

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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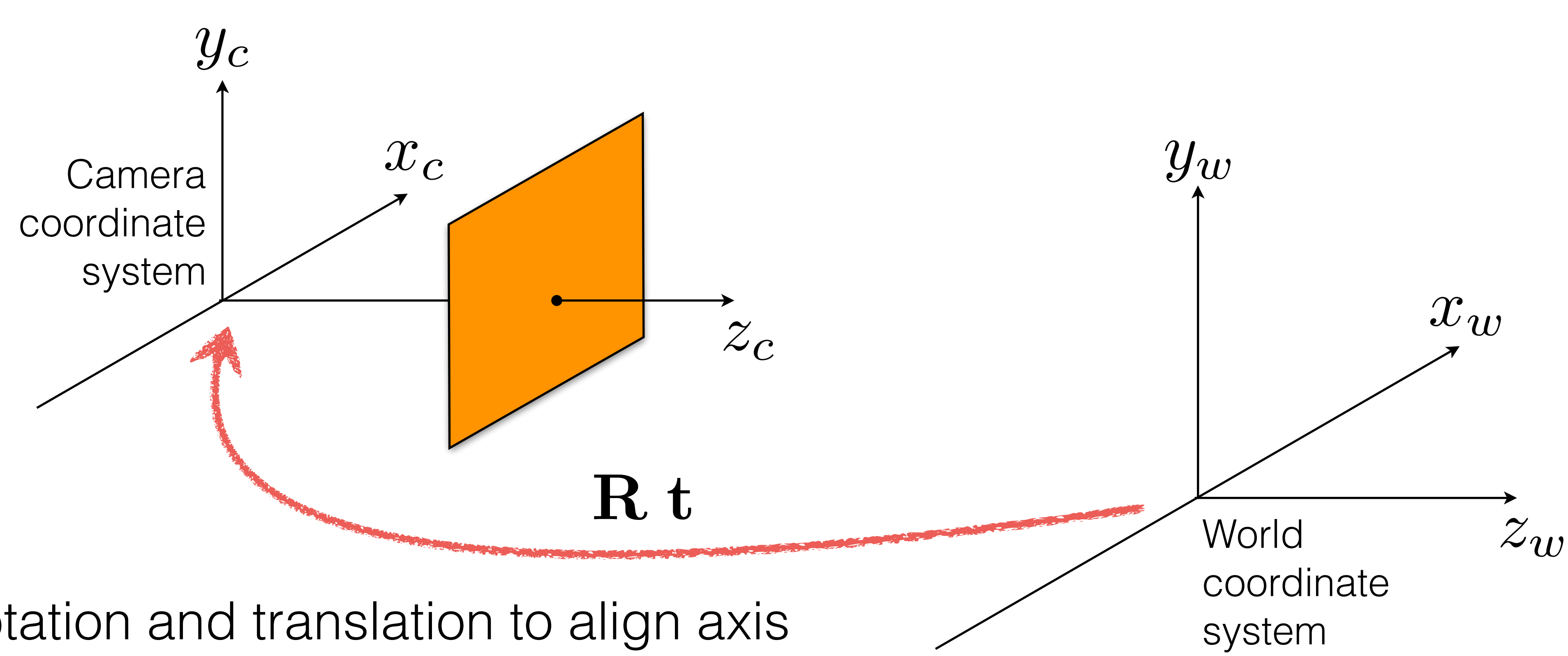
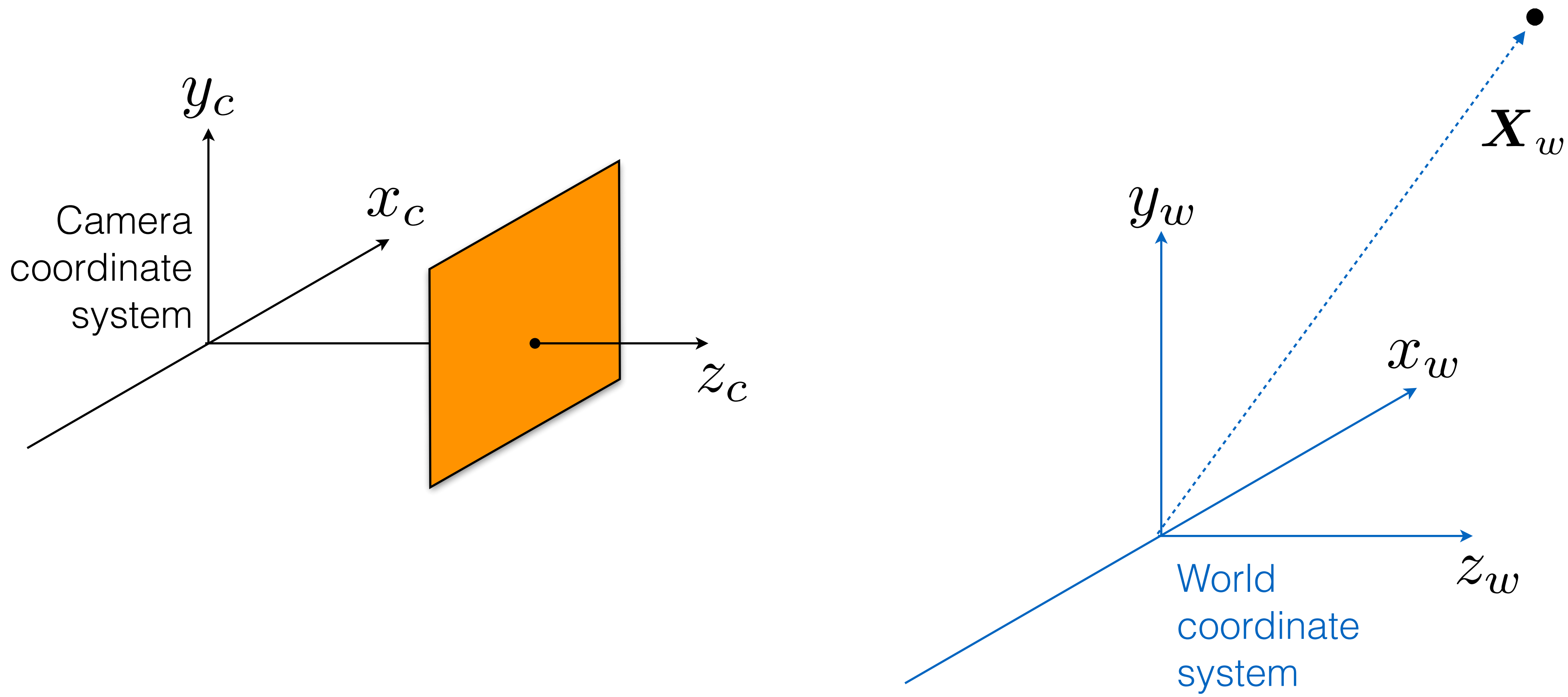
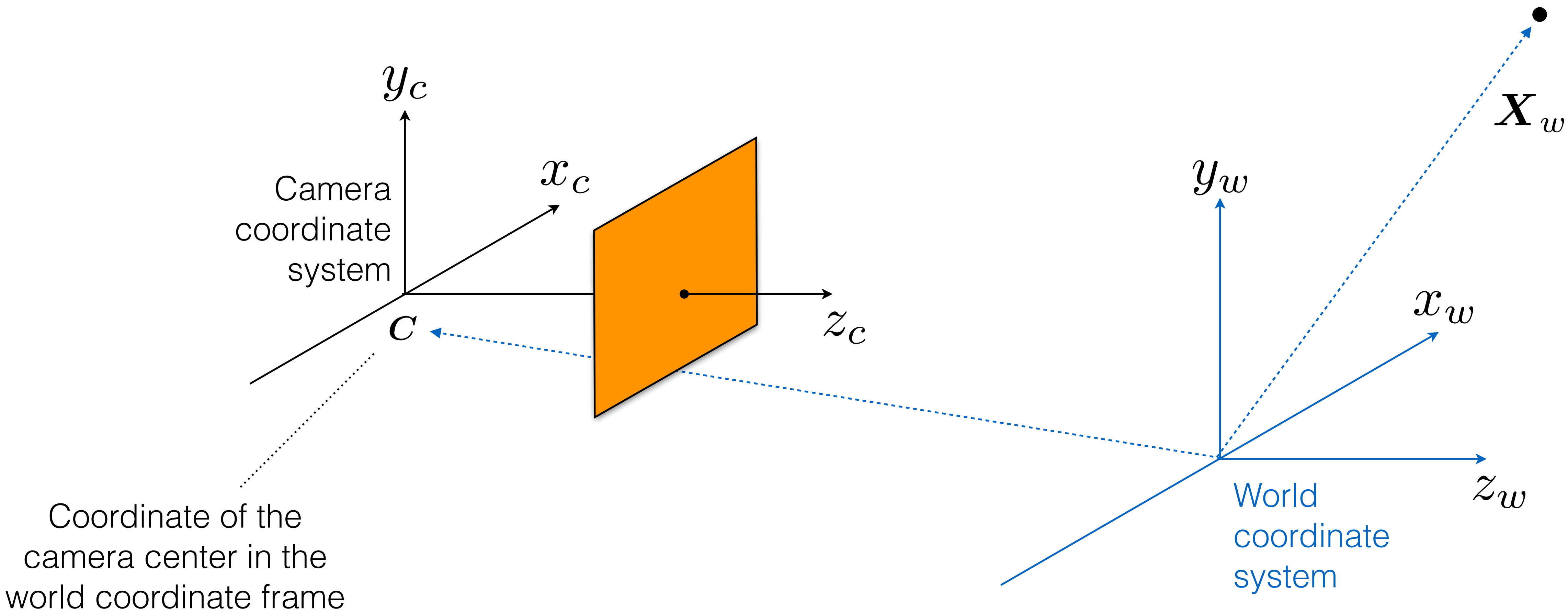


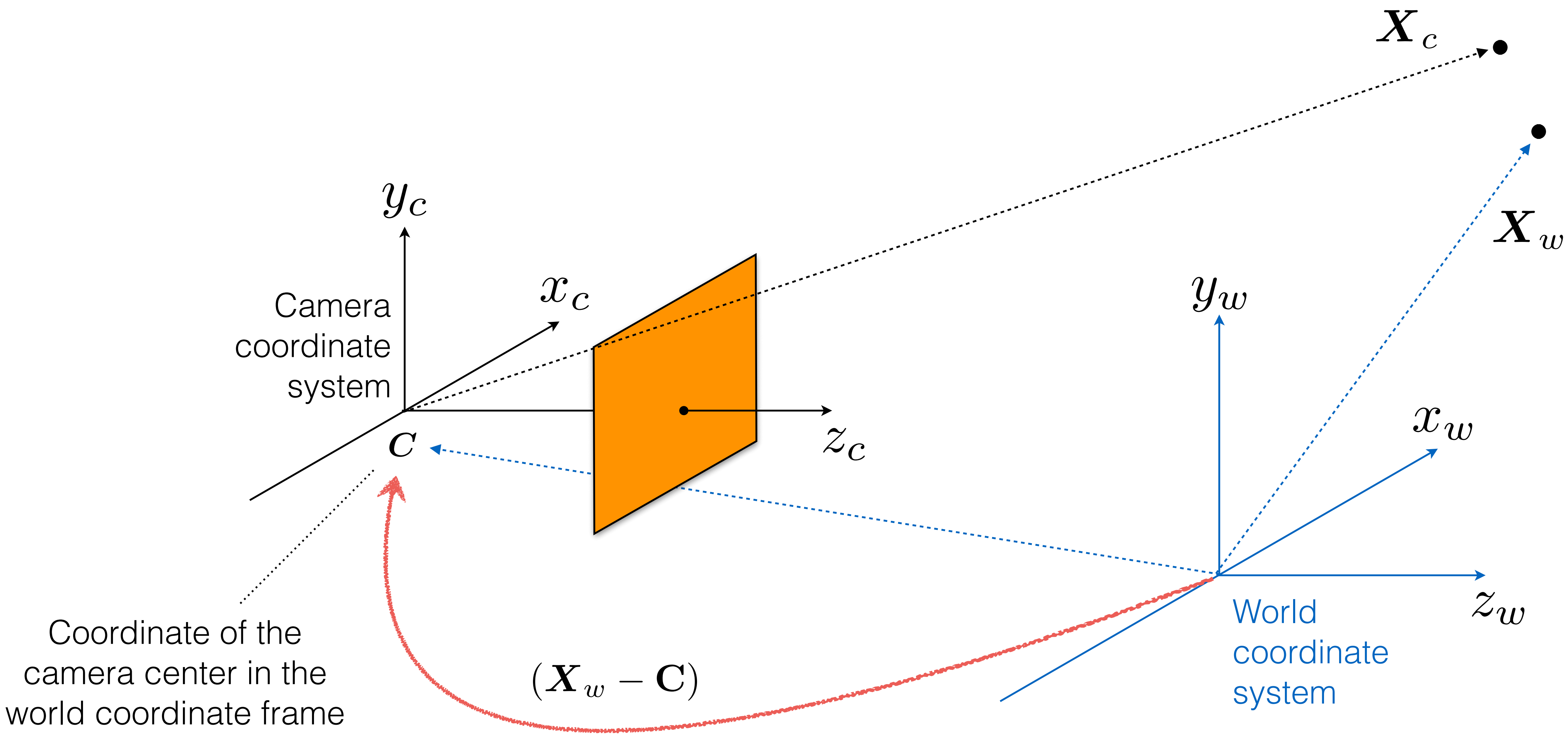

3R rotation and translation to align axis



$y_c$
$x_c$
Camera coordinate system
$z_c$
$\boldsymbol{X}_w$
$y_w$
$x_w$
$z_w$
World coordinate system

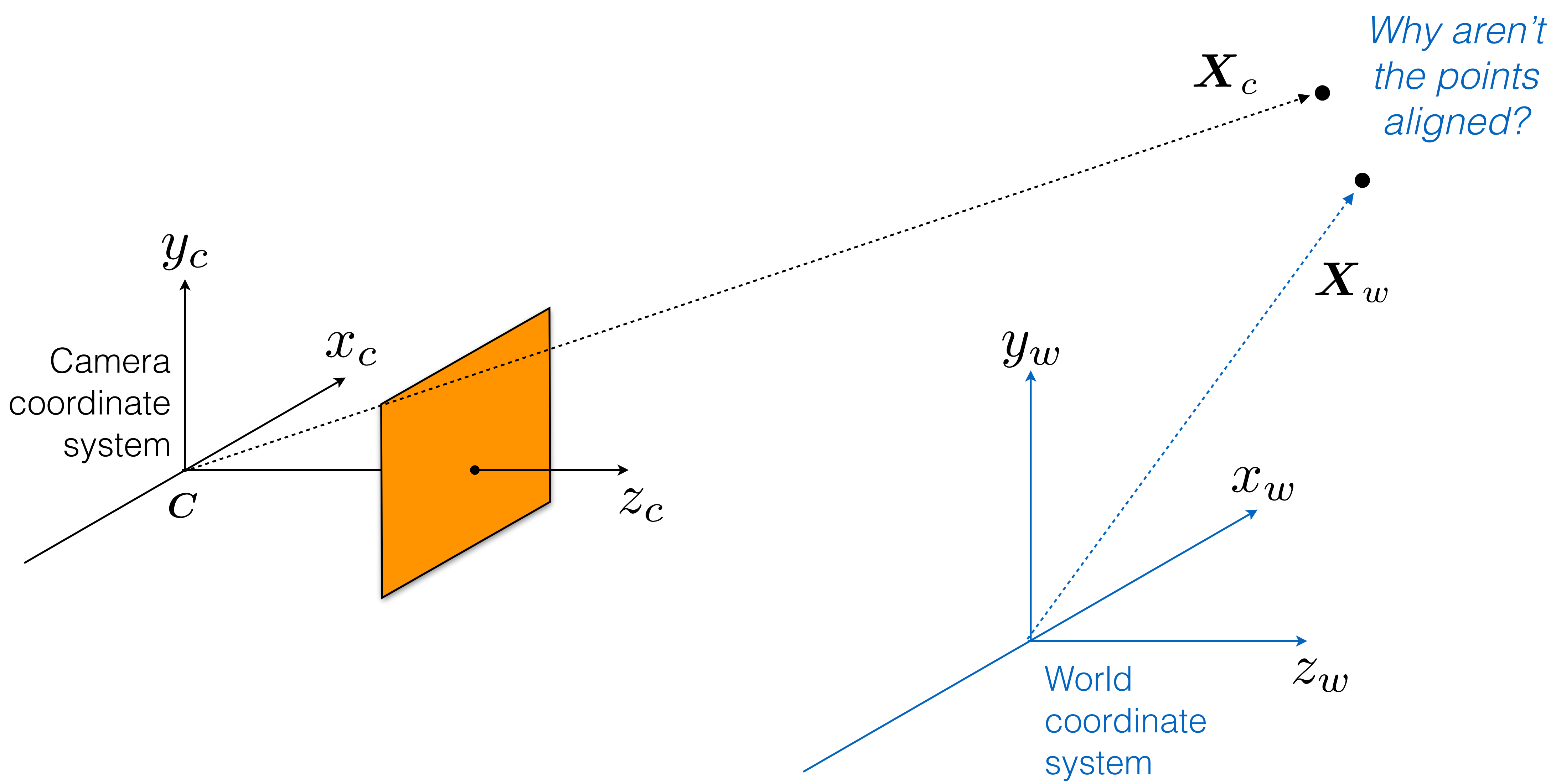


$y_c$
$x_c$
Camera coordinate system
$z_c$
$\boldsymbol{C}$
Coordinate of the camera center in the world coordinate frame
$\boldsymbol{X}_w$
$y_w$
$x_w$
$z_w$
World coordinate system




$$(\boldsymbol{X}_w - \mathbf{C})$$

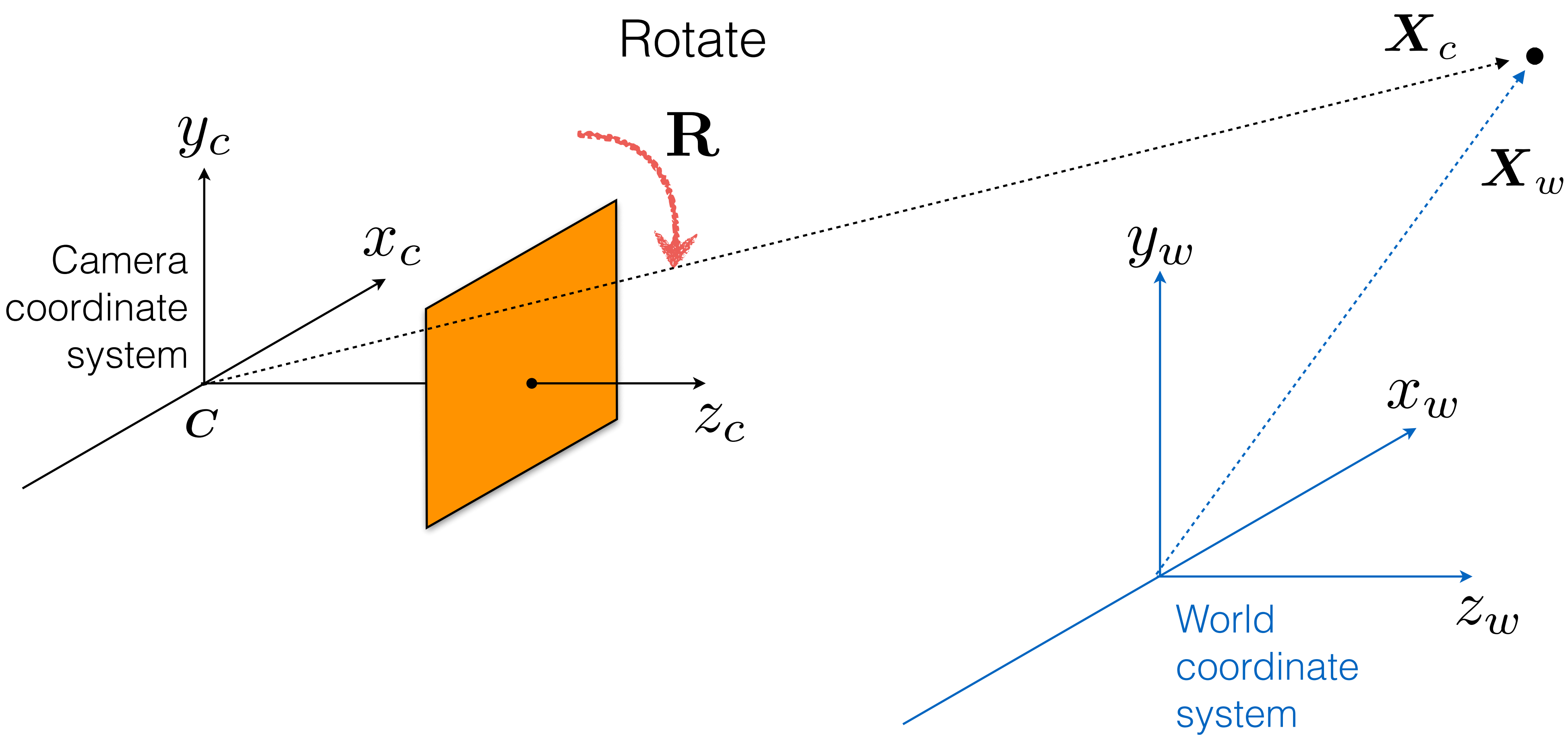
Translate




$$(\boldsymbol{X}_w - \mathbf{C})$$

Translate

# *What happens to points after alignment?*




$$\mathbf{R}(\boldsymbol{X}_w - \mathbf{C})$$

Rotate Translate

In inhomogeneous coordinates:

$$\boldsymbol{X}_c = \mathbf{R}(\boldsymbol{X}_w - \mathbf{C})$$

In homogeneous coordinates:

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & -\mathbf{RC} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

General mapping of a pinhole camera

$$\mathbf{P} = \mathbf{KR}[\mathbf{I} | -\mathbf{C}]$$

General mapping of a pinhole camera

$$\mathbf{P} = \mathbf{KR}[\mathbf{I}| - \mathbf{C}]$$

(translate first then rotate)

Another way to write the mapping

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

where

$$\mathbf{t} = -\mathbf{RC}$$

(rotate first then translate)

# Quiz

The camera matrix relates what two quantities?

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$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

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$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

3D points to 2D image points

The camera matrix can be decomposed into?

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

intrinsic and extrinsic parameters

# Quiz

The camera matrix relates what two quantities?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

3D points to 2D image points

## Quiz

The camera matrix relates what two quantities?

$$\boldsymbol{x} = \mathbf{PX}$$

3D points to 2D image points

The camera matrix can be decomposed into?

# Quiz

The camera matrix relates what two quantities?

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3D points to 2D image points

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## Quiz

The camera matrix relates what two quantities?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

3D points to 2D image points

The camera matrix can be decomposed into?

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

intrinsic and extrinsic parameters

# Generalized pinhole camera model

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

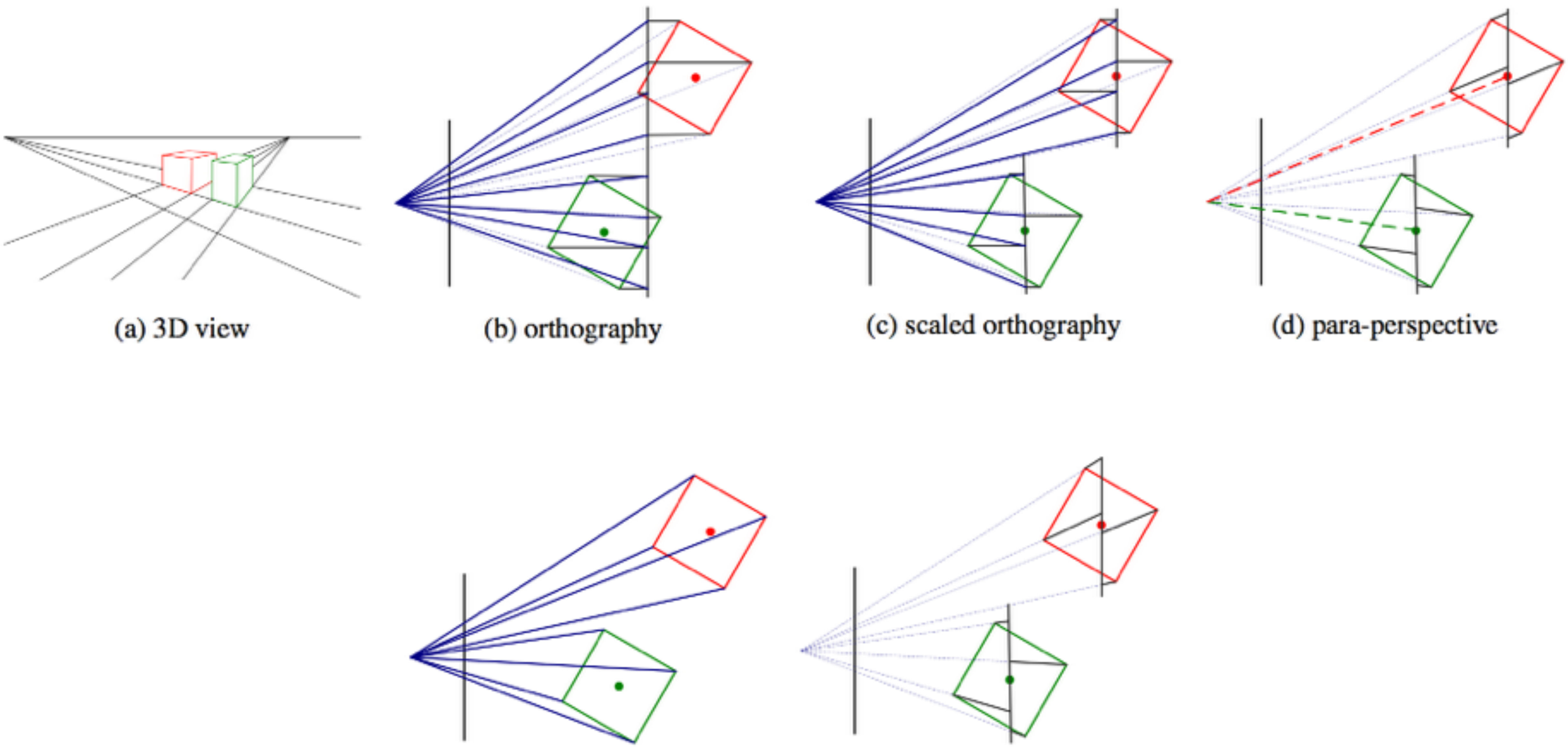
$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc:c} r_1 & r_2 & r_3 & t_1 \\ r_4 & r_5 & r_6 & t_2 \\ r_7 & r_8 & r_9 & t_3 \end{array}\right]$$

intrinsic parameters    extrinsic parameters

$$\mathbf{R} = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix} \qquad \mathbf{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$$

3D rotation    3D translation

# There are many types of camera models (projections)



(a) 3D view (b) orthography (c) scaled orthography (d) para-perspective (e) perspective (f) object-centered

# CCD camera

$$\mathbf{P} = \begin{bmatrix} \alpha_x & 0 & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(assuming that axes are aligned)

*How many degrees of freedom?*

# CCD camera

$$\mathbf{P} = \begin{bmatrix} \alpha_x & 0 & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(assuming that axes are aligned)

*How many degrees of freedom?*

10 DOF

# Finite projective camera

$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(assuming that axes are aligned)

*How many degrees of freedom?*

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$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

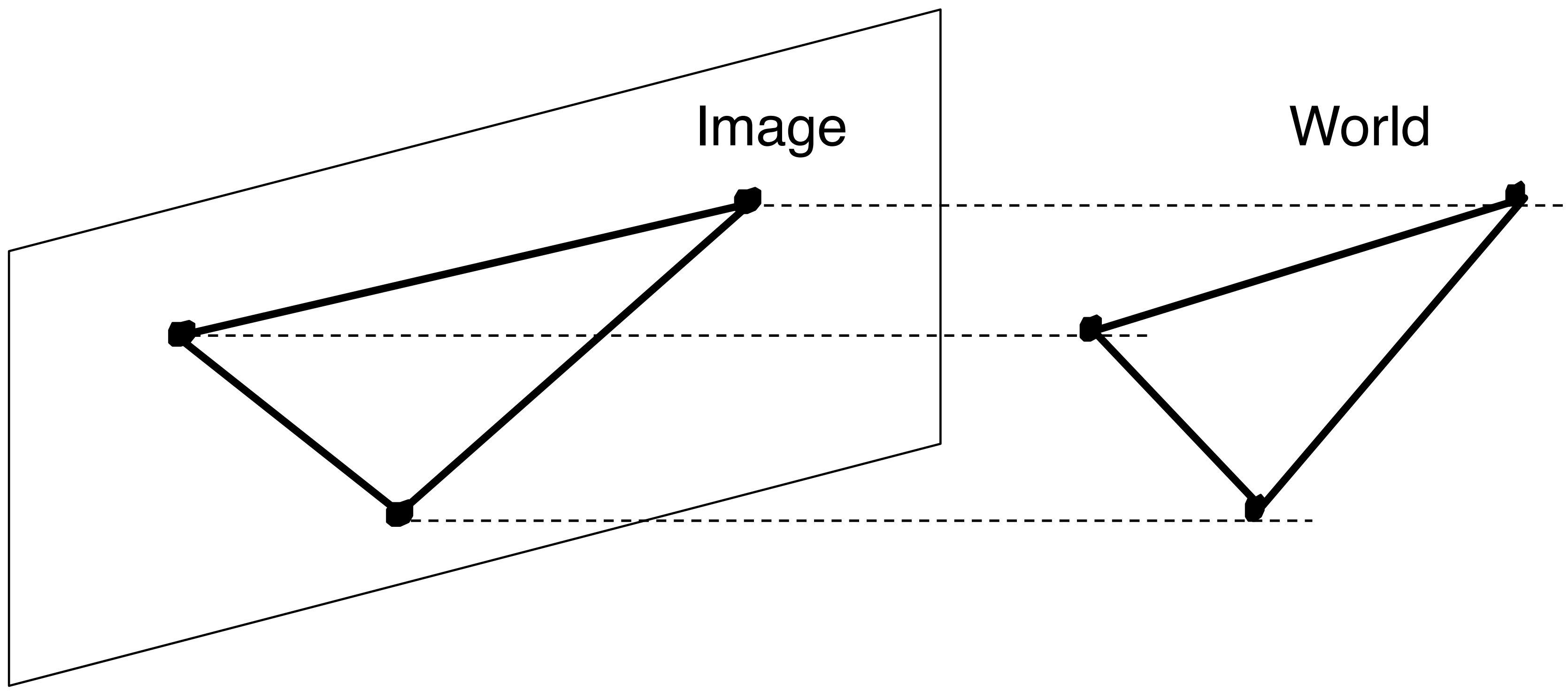
(assuming that axes are aligned)

*How many degrees of freedom?*

11 DOF

# Orthographic camera

(parallel projection)

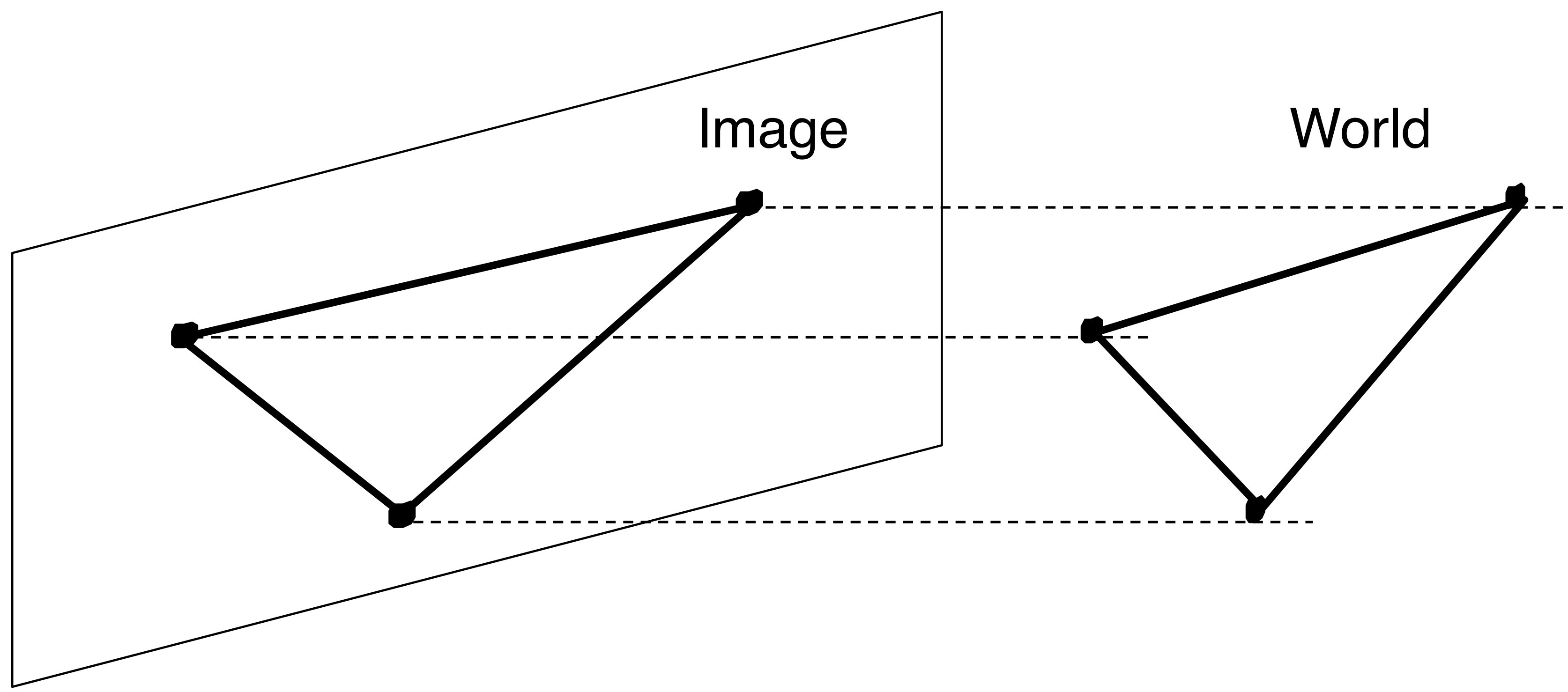



$$\mathbf{P} = \mathbf{K}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(assuming that axes are aligned)

# Orthographic camera

(parallel projection)

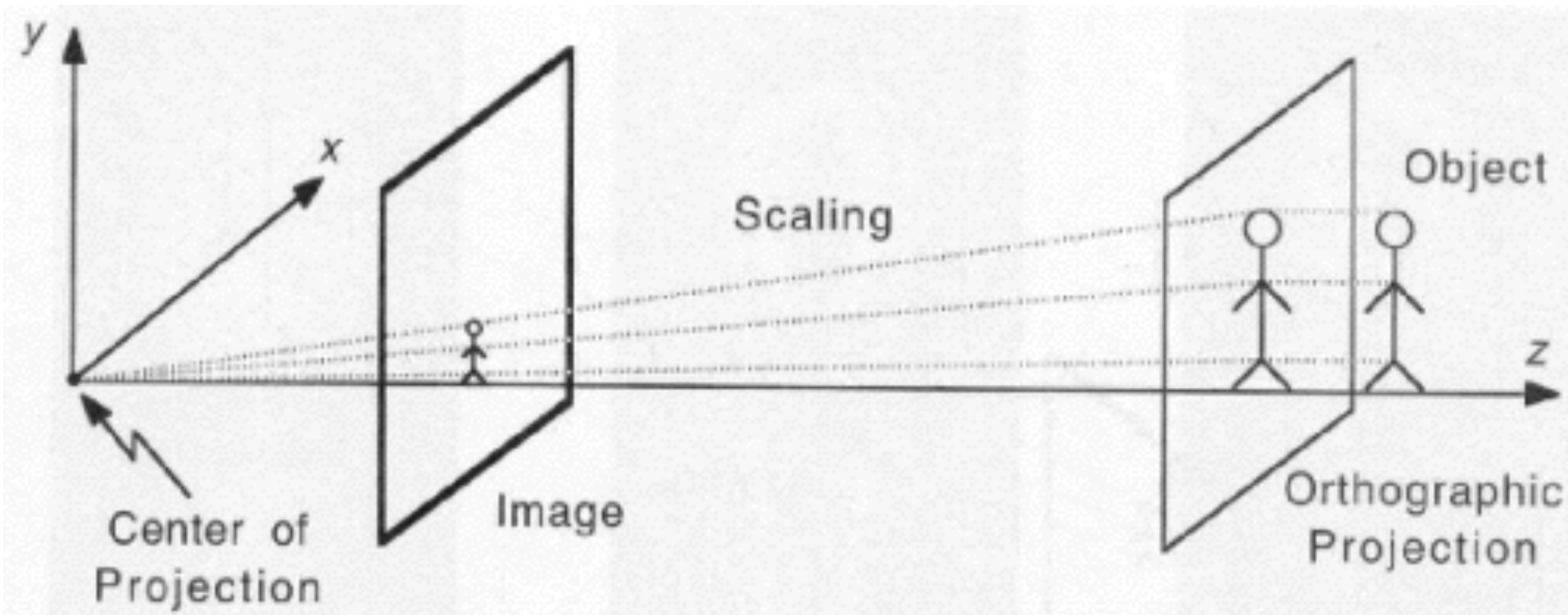



$$\mathbf{P} = \mathbf{K}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Affine camera

(assuming that axes are aligned)

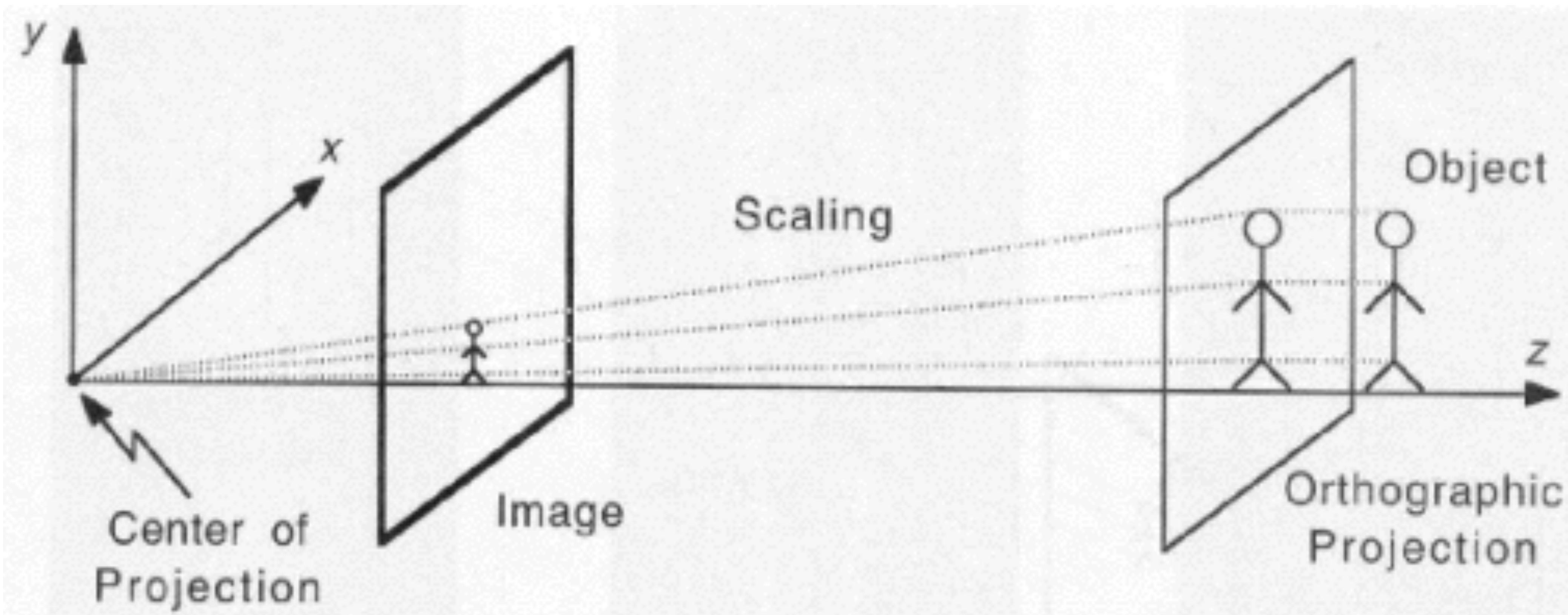
# Weak Perspective Camera




$$\mathbf{P} = \mathbf{K} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \bar{Z} \end{bmatrix}$$

(assuming that axes are aligned)

# Weak Perspective Camera

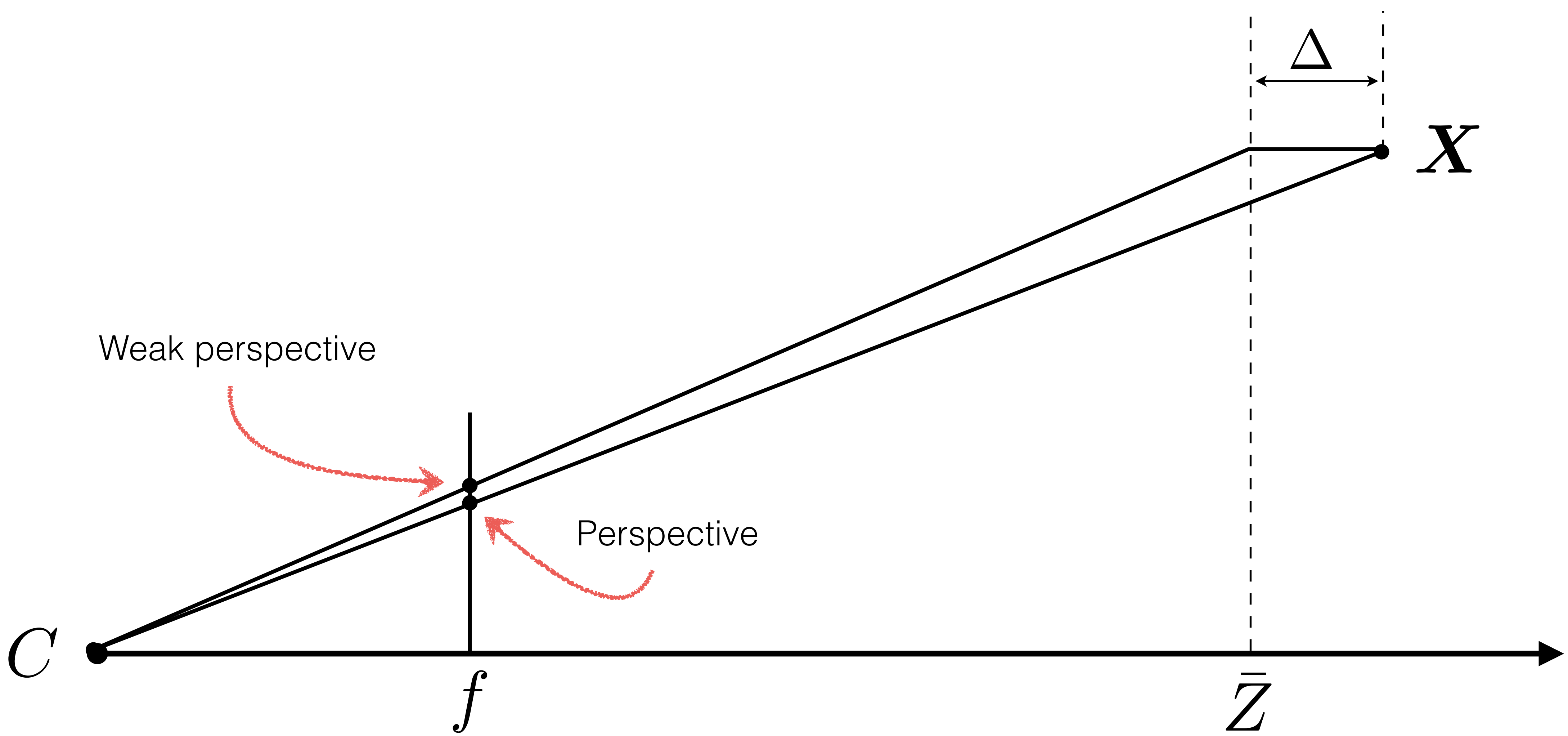


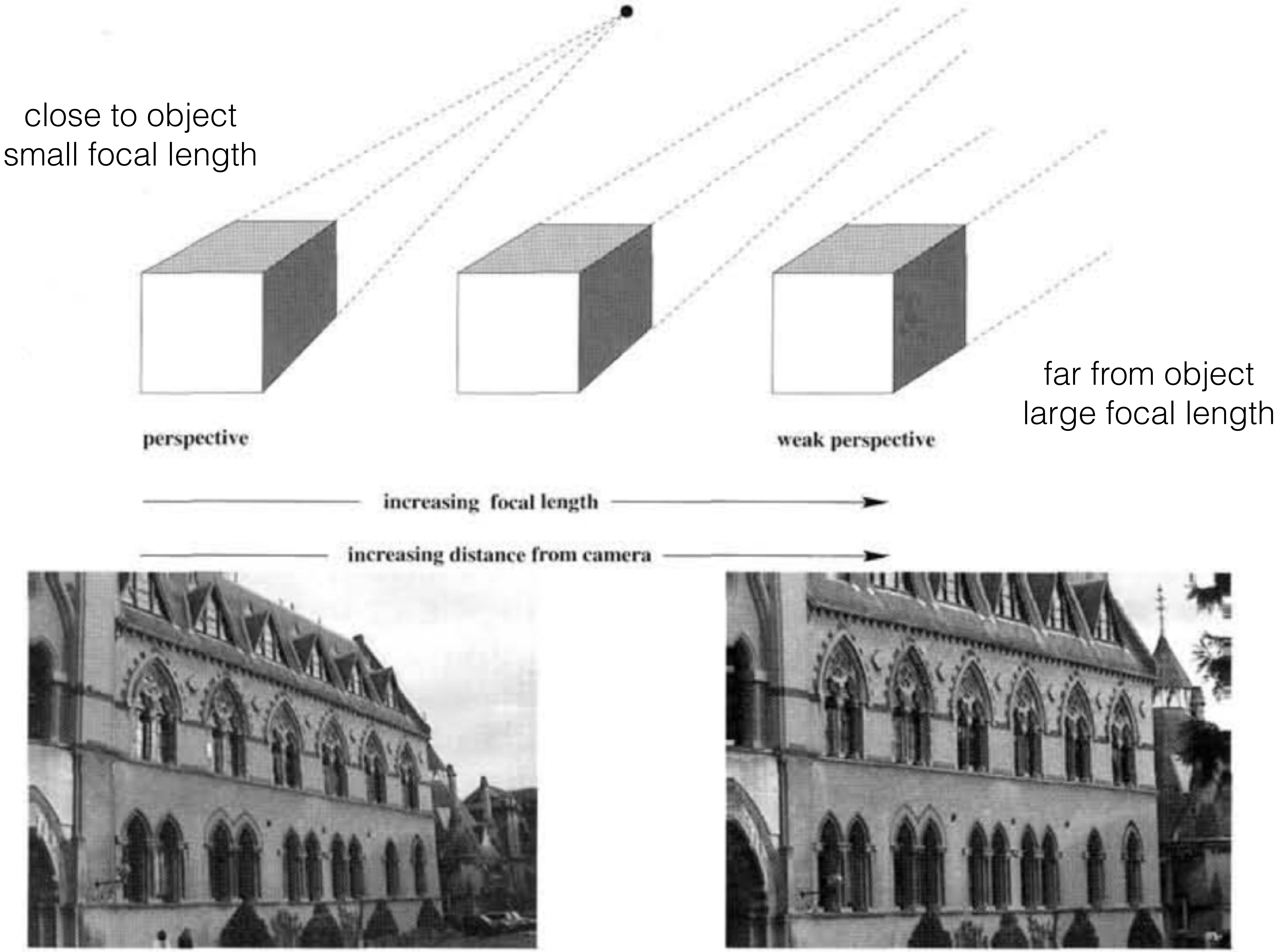

$$\mathbf{P} = \mathbf{K} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \bar{Z} \end{bmatrix}$$

Affine camera

(assuming that axes are aligned)

# Perspective vs Weak Perspective Projection




close to object
small focal length
far from object
large focal length
perspective
weak perspective
increasing focal length
increasing distance from camera