Course announcements

- Lot’s of grades posted during spring break.
  - Mid-semester grades.
  - Comments for HW1.
  - Grades and comments for HW2.

- Homework 4 will be posted tonight and will be due on March 3rd.
  - It’s another shorter homework, based on material from this and the next lecture.

- Talk tomorrow: Jun-Yan Zhu, “Learning to synthesize images,” noon-1 pm, GHC 6115
Overview of today’s lecture

- Measuring light and radiometry.
- Reflectance and BRDF.
- Light sources.
Slide credits

Most of these slides were adapted from:

• Srinivasa Narasimhan (16-385, Spring 2014).
• Todd Zickler (Harvard University).
• Steven Gortler (Harvard University).
Appearance
Appearance
“Physics-based” computer vision (a.k.a “inverse optics”)

Our challenge: Invent computational representations of shape, lighting, and reflectance that are **efficient**: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena.

$I \rightarrow$ shape, illumination, reflectance
Example application: Photometric Stereo
Why study the physics (optics) of the world?

Let's see some pictures!
Light and Shadows
Reflections
Refractions
Interreflections
Scattering
More Complex Appearances
Measuring light and radiometry
Solid angle

- The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O.

Depends on:
- orientation of patch
- distance of patch
Solid angle

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- Depends on:
  - orientation of patch
  - distance of patch

One can show:

\[ d\omega = \frac{dA \cos \theta}{r^2} \]

Units: steradians [sr]
Solid angle

- The *solid angle* subtended by a small surface patch with respect to point $O$ is the area of its central projection onto the unit sphere about $O$.

- Depends on:
  - orientation of patch
  - distance of patch

One can show:

"surface foreshortening"

\[
d\omega = \frac{dA \cos \theta}{r^2}
\]

Units: steradians [sr]
Solid angle

To calculate solid angle subtended by a surface \( S \) relative to \( O \) you must add up (integrate) contributions from all tiny patches (nasty integral):

\[
\Omega = \iint_S \frac{\mathbf{r} \cdot \hat{n}}{r^3} \, dS
\]

One can show:

“surface foreshortening”

\[
d\omega = \frac{dA \cos \theta}{r^2}
\]

Units: steradians [sr]
Question

Suppose surface S is a hemisphere centered at O. What is the solid angle it subtends?
Question

- Suppose surface S is a hemisphere centered at O. What is the solid angle it subtends?

- Answer: $2\pi$ (area of sphere is $4\pi r^2$; area of unit sphere is $4\pi$; half of that is $2\pi$)
Quantifying light: flux, irradiance, and radiance

- Imagine a sensor that counts photons passing through planar patch $X$ in directions within angular wedge $W$.
- It measures *radiant flux* [watts = joules/sec]: rate of photons hitting sensor area.
- Measurement depends on sensor area $|X|$.

radiant flux $\Phi(W, X)$

* shown in 2D for clarity; imagine three dimensions
Quantifying light: flux, irradiance, and radiance

- **Irradiance:**
  A measure of incoming light that is independent of sensor area $|X|$
- Units: watts per square meter [W/m²]
Quantifying light: flux, irradiance, and radiance

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Quantifying light: flux, irradiance, and radiance

- **Irradiance**: A measure of incoming light that is independent of sensor area $|X|$
- Units: watts per square meter $[W/m^2]$
- Depends on sensor direction normal.

![Diagram of light quantification](image)

$$\Phi(W, X) \quad \text{or} \quad E_{\hat{n}}(W, x)$$

- We keep track of the normal because a planar sensor with distinct orientation would converge to a different limit.
- In the literature, notations $n$ and $W$ are often omitted, and values are implied by context.
Quantifying light: flux, irradiance, and radiance

- **Radiance**:
  A measure of incoming light that is independent of sensor area |X|, orientation n, and wedge size (solid angle) |W|
- Units: watts per steradian per square meter [W/(m^2⋅sr)]
Quantifying light: flux, irradiance, and radiance

- **Radiance:**
  A measure of incoming light that is independent of sensor area $|X|$, orientation $n$, and wedge size (solid angle) $|W|$

- Units: watts per steradian per square meter $[W/(m^2\cdot sr)]$

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction $\omega$

$$\cos \theta = \frac{\Box/2}{|X|/2}$$

“foreshortened area”
Quantifying light: flux, irradiance, and radiance

- **Radiance:**
  A measure of incoming light that is independent of sensor area $|X|$, orientation $n$, and wedge size (solid angle) $|W|$

- Units: watts per steradian per square meter $[W/(m^2\cdot sr)]$

\[ \frac{E_n(W, x)}{|W|} \]

\[ L_n(\hat{\omega}, x) \]

\[ L(\hat{\omega}, x) \]

- Has correct units, but still depends on sensor orientation
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Quantifying light: flux, irradiance, and radiance

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Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by *any* finite sensor
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by *any* finite sensor

\[
\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA
\]
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by *any* finite sensor

\[
\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA
\]

- Constant along a ray in free space

\[
L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega})
\]
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by any finite sensor
    \[
    \Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA
    \]
  - Constant along a ray in free space
    \[L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega})\]
  - A camera measures radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to radiance.
    - “Processed” images (like PNG and JPEG) are not linear radiance measurements!!
Most light sources, like a heated metal sheet, follow Lambert’s Law.

What is the radiance $J(\hat{\omega})$ of an infinitesimal patch [W/sr^2]?

Radiant intensity [W/sr]:

$L(\hat{\omega}, x)$ of an infinitesimal patch [W/sr$\cdot$m$^2$]?
Most light sources, like a heated metal sheet, follow Lambert’s Law:

\[ J(\hat{\omega}) = J_o \langle \hat{\omega}, \hat{n} \rangle = J_o \cos \theta \]

This is known as the "Lambertian area source".

What is the radiance \( L(\hat{\omega}, x) \) of an infinitesimal patch [W/sr \( \cdot \) m\(^2\)]?

Answer: \( L(\hat{\omega}, x) = J_o / |X| \) (independent of direction)
Question

- Most light sources, like a heated metal sheet, follow Lambert’s Law

\[ J(\hat{\omega}) = J_o \langle \hat{\omega}, \hat{n} \rangle = J_o \cos \theta \]

radiant intensity [W/sr]

“Lambertian area source”

- What is the radiance \( L(\hat{\omega}, x) \) of an infinitesimal patch [W/sr \cdot m^2]?

Answer: \( L(\hat{\omega}, x) = J_o / |X| \) (independent of direction)

“Looks equally bright when viewed from any direction”
Appearance
“Physics-based” computer vision (a.k.a “inverse optics”)

I $\rightarrow$ shape, illumination, reflectance
Reflectance and BRDF
Reflectance

• Ratio of outgoing energy to incoming energy at a single point

• Want to define a ratio such that it:
  • converges as we use smaller and smaller incoming and outgoing wedges
  • does not depend on the size of the wedges (i.e. is intrinsic to the material)
Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
  - converges as we use smaller and smaller incoming and outgoing wedges
  - does not depend on the size of the wedges (i.e. is intrinsic to the material)

\[ f_{x, \hat{n}}(W_{\text{in}}, \hat{\omega}_{\text{out}}) = \frac{L_{\text{out}}(x, \hat{\omega}_{\text{out}})}{E_{\hat{n}}^{\text{in}}(W_{\text{in}}, x)} \]

- Notations x and n often implied by context and omitted; directions \( \omega \) are expressed in local coordinate system defined by normal \( \hat{n} \) (and some chosen tangent vector)
- Units: \( \text{sr}^{-1} \)
- Called Bidirectional Reflectance Distribution Function (BRDF)
BRDF: Bidirectional Reflectance Distribution Function

\[ E_{\text{surface}}^{(\theta_i, \phi_i)} \quad \text{Irradiance at Surface in direction } (\theta_i, \phi_i) \]

\[ L_{\text{surface}}^{(\theta_r, \phi_r)} \quad \text{Radiance of Surface in direction } (\theta_r, \phi_r) \]

\[ \text{BRDF} : f (\theta_i, \phi_i ; \theta_r, \phi_r) = \frac{L_{\text{surface}}^{(\theta_r, \phi_r)}}{E_{\text{surface}}^{(\theta_i, \phi_i)}} \]
Reflectance: BRDF

- Units: sr$^{-1}$
- Real-valued function defined on the double-hemisphere
- Has many useful properties
Important Properties of BRDFs

- Conservation of Energy:

\[ \forall \hat{\omega}_{in}, \int_{\Omega_{out}} f(\hat{\omega}_{in}, \hat{\omega}_{out}) \cos \theta_{out} d\hat{\omega}_{out} \leq 1 \]

Why smaller than or equal?
Property: “Helmholtz reciprocity”

- Helmholtz Reciprocity: (follows from 2\textsuperscript{nd} Law of Thermodynamics)

BRDF does not change when source and viewing directions are swapped.

\[ f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}) = f_r(\vec{\omega}_{\text{out}}, \vec{\omega}_{\text{in}}) \]
Common assumption: Isotropy

Bi-directional Reflectance Distribution Function (BRDF)

Can be written as a function of 3 variables: \( f(\theta_i, \theta_r, \phi_i - \phi_r) \)
Reflectance: BRDF

- Units: sr\(^{-1}\)
- Real-valued function defined on the double-hemisphere
- Has many useful properties
- Allows computing output radiance (and thus pixel value) for any configuration of lights and viewpoint

\[
L_{\text{out}}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L_{\text{in}}(\hat{\omega}_{\text{in}}) \cos \theta_{\text{in}} d\hat{\omega}_{\text{in}}
\]

Why is there a cosine in the reflectance equation?
Derivation of the Reflectance Equation

From the definition of BRDF:

\[ L_{\text{surface}}(\theta_r, \phi_r) = E_{\text{surface}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \]
Derivation of the Scene Radiance Equation

From the definition of BRDF:

\[
L_{\text{surface}} (\theta_r, \phi_r) = E_{\text{surface}} (\theta_i, \phi_i) f (\theta_i, \phi_i; \theta_r, \phi_r)
\]

Write Surface Irradiance in terms of Source Radiance:

\[
L_{\text{surface}} (\theta_r, \phi_r) = L_{\text{src}} (\theta_i, \phi_i) f (\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i d\omega_i
\]

Integrate over entire hemisphere of possible source directions:

\[
L_{\text{surface}} (\theta_r, \phi_r) = \int \int L_{\text{src}} (\theta_i, \phi_i) f (\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \, d\omega_i
\]

Convert from solid angle to theta-phi representation:

\[
L_{\text{surface}} (\theta_r, \phi_r) = \int \int L_{\text{src}} (\theta_i, \phi_i) f (\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \sin \theta_i \, d\theta_i d\phi_i
\]
Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]

\[ S = \int_0^\pi \int_0^{2\pi} \sin \theta \, d\theta \, d\phi = 4\pi \]
BRDF

\[ f_r(\bar{\omega}_{in}, \cdot) \]

\[ f_r(\bar{\omega}_{in}, \bar{\omega}_{out}) \]

Bi-directional Reflectance Distribution Function (BRDF)
BRDF

Lambertian (diffuse) BRDF: energy equally distributed in all directions

What does the BRDF equal in this case?

\[ f_r(\vec{\omega}_{in}, \cdot) \]

Bi-directional Reflectance Distribution Function (BRDF)
Diffuse Reflection and Lambertian BRDF

- Surface appears equally bright from ALL directions! (independent of $\mathbf{v}$)

- Lambertian BRDF is simply a constant: $f(\theta_i, \phi_i ; \theta_r, \phi_r ) = \frac{\rho_d}{\pi}$

- Most commonly used BRDF in Vision and Graphics!
BRDF

Specular BRDF: all energy concentrated in mirror direction

What does the BRDF equal in this case?

$fr(\vec{ω}_{in}, \cdot)$

$fr(\vec{ω}_{in}, \vec{ω}_{out})$

Bi-directional Reflectance Distribution Function (BRDF)
Specular Reflection and Mirror BRDF

- Valid for very smooth surfaces.
- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).
- Mirror BRDF is simply a double-delta function:

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \delta(\theta_i - \theta_v) \delta(\phi_i + \pi - \phi_v)$$
Example Surfaces

Body Reflection:
- Diffuse Reflection
- Matte Appearance
- Non-Homogeneous Medium
  Clay, paper, etc

Surface Reflection:
- Specular Reflection
- Glossy Appearance
  Highlights
  Dominant for Metals

Many materials exhibit both Reflections:
BRDF

Glossy BRDF: more energy concentrated in mirror direction than elsewhere

\[ f_r(\vec{\omega}_{in}, \cdot) \]

\[ f_r(\vec{\omega}_{in}, \vec{\omega}_{out}) \]

Bi-directional Reflectance Distribution Function (BRDF)
Trick for dielectrics (non-metals)

• BRDF is a sum of a Lambertian diffuse component and non-Lambertian specular components

• The two components differ in terms of color and polarization, and under certain conditions, this can be exploited to separate them.

\[ f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o) \]
Trick for dielectrics (non-metals)

- BRDF is a sum of a Lambertian diffuse component and non-Lambertian specular components.
- The two components differ in terms of color and polarization, and under certain conditions, this can be exploited to separate them.

\[ f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o) \]

Often called the dichromatic BRDF:
- Diffuse term varies with wavelength, constant with polarization.
- Specular term constant with wavelength, varies with polarization.
Trick for dielectrics (non-metals)

In this example, the two components were separated using linear polarizing filters on the camera and light source.
Trick for dielectrics (non-metals)

\[ I_{\parallel} = \frac{1}{2} I_{\text{diffuse}} + I_{\text{specular}} \]

\[ I_{\perp} = \frac{1}{2} I_{\text{diffuse}} \]
Tabulated 4D BRDFs (hard to measure)

[Cylinder (1D normal variation) with stripes of the material at different orientations (1D)]

[Ngan et al., 2005]

[Gonioreflectometer]
Low-parameter (non-linear) BRDF models

- A small number of parameters define the (2D, 3D, or 4D) function
- Except for Lambertian, the BRDF is non-linear in these parameters
- Examples:

Lambertian: \( f(\omega_i, \omega_o) = \frac{a}{\pi} \)

Phong: \( f(\omega_i, \omega_o) = \frac{a}{\pi} + b \cos^c (2\langle \omega_i, n \rangle \langle \omega_o, n \rangle - \langle \omega_i, \omega_o \rangle) \)

Blinn: \( f(\omega_i, \omega_o) = \frac{a}{\pi} + b \cos^c \theta(\omega_i, \omega_o) \)

Laforutine: \( f(\omega_i, \omega_o) = \frac{a}{\pi} + b (-\omega_i^\top A \omega_o)^k \)

Ward: \( f(\omega_i, \omega_o) = \frac{a}{\pi} + \frac{b}{4\pi c^2 \sqrt{\langle n, \omega_i \rangle \langle n, \omega_o \rangle}} \exp \left( -\tan^2 \theta(\omega_i, \omega_o) \right) \)

Where do these constants come from?

\( \alpha \) is called the *albedo*
Reflectance Models

Reflection: An Electromagnetic Phenomenon

Two approaches to derive Reflectance Models:

- Physical Optics (Wave Optics)
- Geometrical Optics (Ray Optics)

Geometrical models are approximations to physical models
But they are easier to use!
Reflectance that Require Wave Optics
Light sources
“Physics-based” computer vision (a.k.a. “inverse optics”)

\[
\text{shape, illumination, reflectance}
\]
Lighting models: Plenoptic function

- Radiance as a function of position and direction
- Radiance as a function of position, direction, and time
- Spectral radiance as a function of position, direction, time and wavelength

\[ L(x, \omega, t, \lambda) \]

Fig. 1.3
The plenoptic function describes the information available to an observer at any point in space and time. Shown here are two schematic eyes—which one should consider to have punctate pupils—gathering pencils of light rays. A real observer cannot see the light rays coming from behind, but the plenoptic function does include these rays.

[Adelson and Bergen, 1991]
Lighting models: far-field (or directional) approximation

- Assume that, over the observed region of interest, all source of incoming flux are relatively far away.

\[ L(x, \omega, t, \lambda) \rightarrow L(\omega, t, \lambda) \]

\[ L(x, \omega) \rightarrow L(\omega) \]

radiance only depends on direction; not location

ignores close inter-reflections

[Debevec, 1998]
Application: augmented reality

$L(\omega)$

light probe

[Debevec, 1998]
Application: augmented reality

(a) Background photograph

(b) Camera calibration grid and light probe

(g) Final result with differential rendering

[Debevec, 1998]
Application: augmented reality

http://gl.ict.usc.edu/LightStages/
Application: augmented reality

[https://developers.google.com/ar/]

[https://developer.apple.com/arkit/]

Introducing ARKit

iOS 11 introduces ARKit, a new framework that allows you to easily create unparalleled augmented reality experiences for iPhone and iPad. By blending digital objects and information with the environment around you, ARKit takes apps beyond the screen, freeing them to interact with the real world in entirely new ways.

ARCodi Overview

Fundamental Concepts

ARCodi is a platform for building augmented reality apps on Android. ARCore uses three key technologies to integrate virtual content with the real world as seen through your phone’s camera:

- **Motion tracking** allows the phone to understand and track its position relative to the world.
- **Environmental understanding** allows the phone to detect the size and location of flat horizontal surfaces like the ground or a coffee table.
- **Light estimation** allows the phone to estimate the environment’s current lighting conditions.
Lighting models: far-field approximation

One can download far-field lighting environments that have been captured by others

[http://gl.ict.usc.edu/Data/HighResProbes/]

A number of apps and software exist to help you capture your own environments using a light probe

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Table of Light Probes:

<table>
<thead>
<tr>
<th>Image</th>
<th>Description</th>
<th>Interactive Preview</th>
<th>Download</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uffizi Gallery, Italy</td>
<td>Assembled from 18 14mm images taken using the Kodak DCS 620 camera</td>
<td>LDR panorama HDR panorama</td>
<td>HDR (1.3MB) EXR (1.4MB) Diffuse convolution</td>
</tr>
<tr>
<td>Grace Cathedral, San Francisco, California</td>
<td>Assembled from three 8mm fish-eye images taken using the Canon EOS-1Ds camera</td>
<td>LDR panorama HDR panorama</td>
<td>HDR (4.4MB) EXR (4.5MB) Diffuse convolution</td>
</tr>
<tr>
<td>Dining room of the Ennis Brown House, Los Angeles, California (website)</td>
<td>Assembled from six 8mm fish-eye images taken using the Canon EOS-1Ds camera</td>
<td>LDR panorama HDR panorama</td>
<td>HDR (5.4MB) EXR (5.1MB) Diffuse convolution</td>
</tr>
<tr>
<td>On a glacier in Banff National Forest, Canada</td>
<td>Assembled from three 8mm fish-eye images taken using the Canon EOS-1Ds camera</td>
<td>LDR panorama HDR panorama</td>
<td>HDR (4.3MB) EXR (4.5MB) Diffuse convolution</td>
</tr>
<tr>
<td>Piazzetta courtyard, Venice, Italy</td>
<td>Assembled from three 8mm fish-eye images taken using the Canon EOS-1Ds camera</td>
<td>LDR panorama HDR panorama</td>
<td>HDR (2.0MB) EXR (2.3MB) Diffuse convolution</td>
</tr>
<tr>
<td>Courtyard of the Doge’s palace, Venice, Italy</td>
<td>Assembled from five 8mm fish-eye images taken using the Canon EOS-1Ds camera</td>
<td>LDR panorama HDR panorama</td>
<td>HDR (2.2MB) EXR (2.9MB) Diffuse convolution</td>
</tr>
</tbody>
</table>

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Figure 6. To produce the equal-area cylindrical projection of a spherical map, one projects each point on the surface of the sphere horizontally outward onto the cylinder, and then unwraps the cylinder to obtain a rectangular “panoramic” map.
Application: inferring outdoor illumination

From a single image (left), we estimate the most likely sky appearance (middle) and insert a 3-D object (right). Illumination estimation was done entirely automatically.

[Lalonde et al., 2009]
A further simplification:
Low-frequency illumination

\[ L(\omega) = \sum_i a_i Y_i(\omega) \]

First nine basis functions are sufficient for re-creating Lambertian appearance

[Ramamoorthi and Hanrahan, 2001; Basri and Jacobs, 2003]
Low-frequency illumination

Fig. 2. On the left, a white sphere illuminated by three directional (distant point) sources of light. All the lights are parallel to the image plane, one source illuminates the sphere from above and the two others illuminate the sphere from diagonal directions. In the middle, a cross-section of the lighting function with three peaks corresponding to the three light sources. On the right, a cross-section indicating how the sphere reflects light. We will make precise the intuition that the material acts as a low-pass filtering, smoothing the light as it reflects it.

Figure 3. Plot of spherical harmonic terms in Lambertian BRDF filter.

[Ramamoorthi and Hanrahan, 2001; Basri and Jacobs, 2003]
Low-frequency illumination

\[ L(\omega) = \sum_i a_i Y_i(\omega) \]

\[ \vec{\ell} = (\ell_1, \ldots, \ell_9) \]

[Ramamoorthi and Hanrahan, 2001; Basri and Jacobs, 2003]
Application: Trivial rendering

Capture light probe

Rendering a (convex) diffuse object in this environment simply requires a lookup based on the surface normal at each pixel

Low-pass filter (truncate to first nine SHs)
White-out: Snow and Overcast Skies

CAN’ T perceive the shape of the snow covered terrain!

CAN perceive shape in regions lit by the street lamp!!

WHY?
Diffuse Reflection from Uniform Sky

\[ L_{\text{surface}}^{\theta_r, \phi_r} = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} L_{\text{src}}^{\theta_i, \phi_i} f (\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \sin \theta_i \, d\theta_i \, d\phi_i \]

- Assume Lambertian Surface with Albedo = 1 (no absorption)

\[ f (\theta_i, \phi_i; \theta_r, \phi_r) = \frac{1}{\pi} \]

- Assume Sky radiance is constant

\[ L^{\text{src}} (\theta_i, \phi_i) = L^{\text{sky}} \]

- Substituting in above Equation:

\[ L^{\text{surface}} (\theta_r, \phi_r) = L^{\text{sky}} \]

Radiance of any patch is the same as Sky radiance !! (white-out condition)
Even simpler:
Directional lighting

• Assume that, over the observed region of interest, all source of incoming flux is from one direction

\[ L(x, \omega, t, \lambda) \rightarrow L(x, t, \lambda) \rightarrow s(t, \lambda)\delta(\omega = \omega_o(t)) \]
\[ L(x, \omega) \rightarrow L(\omega) \rightarrow s\delta(\omega = \omega_o) \]

• Convenient representation

\[ \hat{\ell} = (\ell_x, \ell_y, \ell_z) \]

“light direction” \[ \hat{\ell} = \frac{\ell}{||\ell||} \]

“light strength” \[ ||\hat{\ell}|| \]
Simple shading

ASSUMPTION 1:
LAMBERTIAN

\[ L^{\text{out}}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L^{\text{in}}(\hat{\omega}_{\text{in}}) \cos \theta_{\text{in}} d\hat{\omega}_{\text{in}} \]

\[ I = a \hat{n}^\top \hat{\ell} \]

ASSUMPTION 2:
DIRECTIONAL LIGHTING
"N-dot-l" shading

ASSUMPTION 1: LAMBERTIAN

\[ L^{\text{out}}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L^{\text{in}}(\hat{\omega}_{\text{in}}) \cos \theta_{\text{in}} d\hat{\omega}_{\text{in}} \]

\[ I = a \hat{n} \mathbf{\hat{\ell}} \]

ASSUMPTION 2: DIRECTIONAL LIGHTING
An ideal point light source

Think of this as a spatially-varying directional source where
1. the direction is away from $x_o$
2. the strength is proportional to $1/(\text{distance})^2$

$$L(x, \omega) = \frac{s}{||x - x_o||^2} \delta \left( \omega = \frac{x - x_o}{||x - x_o||} \right)$$
Summary of some useful lighting models

• plenoptic function (function on 5D domain)
• far-field illumination (function on 2D domain)
• low-frequency far-field illumination (nine numbers)
• directional lighting (three numbers = direction and strength)
• point source (four numbers = location and strength)
References

Basic reading:
• Szeliski, Section 2.2.
• Gortler, Chapter 21.
  This book by Steven Gortler has a great introduction to radiometry, reflectance, and their use for image formation.

Additional reading:
  These two thesis are foundational for modern computer graphics. Among other things, they include a thorough derivation (starting from wave optics and measure theory) of all radiometric quantities and associated integro-differential equations. You can also look at them if you are interested in physics-based rendering.
  A book discussing modeling and simulation of other appearance effects beyond single-bounce reflectance.
  A very thorough review of everything that has to do with modeling and measuring BRDFs.
  This paper has a great review of physics-based models for reflectance and refraction.
  This thesis introduced the largest measured dataset of 4D reflectances. It also provides detailed discussion of many topics relating to modelling reflectance.
  These two papers discuss the isotropy and other properties of common BRDFs, and how one can take advantage of them using alternative parameterizations.
  The paper introducing the dichromatic reflectance model.
• Levin et al., “Fabricating BRDFs at high spatial resolution using wave optics,” SIGGRAPH 2013.
  These three papers describe reflectance effects that can only be modeled using wave optics (and in particular diffraction).