Appearance and reflectance
Course announcements

- Apologies for cancelling last Wednesday’s lecture.
- Homework 3 has been posted and is due on March 9th.
  - Any questions about the homework?
  - How many of you have looked at/started/finished homework 3?
- Office hours for Yannis’ this week: Wednesday 3-5 pm.
- Results from poll for adjusting Yannis’ regular office hours: They stay the same.
- Many talks this week:
  2. Manolis Savva, "Human-centric Understanding of 3D Environments," Wednesday March 7, 2:00 PM, NSH 3305.
  3. David Fouhey, "Recovering a Functional and Three Dimensional Understanding of Images," Thursday March 8, 4:00 PM, NSH 3305.
- How many of you went to Pulkit Agrawal’s talk last week?
Overview of today’s lecture

- Appearance phenomena.
- Measuring light and radiometry.
- Reflectance and BRDF.
Slide credits

Most of these slides were adapted from:

• Srinivasa Narasimhan (16-385, Spring 2014).
• Todd Zickler (Harvard University).
• Steven Gortler (Harvard University).
Course overview

1. Image processing.  ← Lectures 1 – 7
   See also 18-793: Image and Video Processing

2. Geometry-based vision.  ← Lectures 7 – 12
   See also 16-822: Geometry-based Methods in Vision

3. Physics-based vision.  ← We are starting this part now

4. Learning-based vision.

5. Dealing with motion.
Appearance
Appearance
“Physics-based” computer vision (a.k.a “inverse optics”)

Our challenge: Invent computational representations of shape, lighting, and reflectance that are **efficient**: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena.
Example application: Photometric Stereo
Why study the physics (optics) of the world?

Let's see some pictures!
Light and Shadows
Reflections
Refractions
Interreflections
Mies Courtyard House with Curved Elements
Scattering
More Complex Appearances
Measuring light and radiometry
The solid angle subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O.

- Depends on:
  - orientation of patch
  - distance of patch
Solid angle

The solid angle subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O.

- Orientation of patch
- Distance of patch

One can show:

\[ d\omega = \frac{dA \cos \theta}{r^2} \]

Units: steradians [sr]
Solid angle

- The solid angle subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O.

- Depends on:
  - orientation of patch
  - distance of patch

- One can show: "surface foreshortening"

\[
d\omega = \frac{dA \cos \theta}{r^2}
\]

- Units: steradians [sr]
Solid angle

To calculate solid angle subtended by a surface $S$ relative to $O$ you must add up (integrate) contributions from all tiny patches (nasty integral)

\[ \Omega = \iiint_{S} \frac{\vec{r} \cdot \hat{n} \ dS}{|\vec{r}|^3} \]

One can show: "surface foreshortening"

\[ d\omega = \frac{dA \cos \theta}{r^2} \]

Units: steradians [sr]
Question

- Suppose surface $S$ is a hemisphere centered at $O$. What is the solid angle it subtends?
Question

- Suppose surface S is a hemisphere centered at O. What is the solid angle it subtends?

  Answer: $2\pi$ (area of sphere is $4\pi r^2$; area of unit sphere is $4\pi$; half of that is $2\pi$)
Quantifying light: flux, irradiance, and radiance

- Imagine a sensor that counts photons passing through planar patch X in directions within angular wedge W.
- It measures *radiant flux* [watts = joules/sec].
- Measurement depends on sensor area |X|.

\[ \Phi(W, X) \]

* shown in 2D for clarity; imagine three dimensions
Quantifying light: flux, irradiance, and radiance

- **Irradiance:**
  A measure of incoming light that is independent of sensor area $|X|$
- Units: watts per square meter $[W/m^2]$
Quantifying light: flux, irradiance, and radiance

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Quantifying light: flux, irradiance, and radiance

- **Irradiance**: A measure of incoming light that is independent of sensor area $|X|$
- Units: watts per square meter $[W/m^2]$.
- Depends on sensor direction normal.

\[
\Phi(W, X) = \frac{E_\hat{n}(W, x)}{|X|}
\]

- We keep track of the normal because a planar sensor with distinct orientation would converge to a different limit.
- In the literature, notations $n$ and $W$ are often omitted, and values are implied by context.
Quantifying light: flux, irradiance, and radiance

- **Radiance**: A measure of incoming light that is independent of sensor area $|X|$, orientation $n$, and wedge size (solid angle) $|W|$
- Units: watts per steradian per square meter \([W/(m^2\cdot sr)]\)

\[
\frac{E_n(W, x)}{|W|} \quad \text{and} \quad L_n(\hat{\omega}, x)
\]

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction $\omega$
Quantifying light: flux, irradiance, and radiance

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- Units: watts per steradian per square meter $[W/(m^2\cdot sr)]$

\[
E_n(W, x) = \frac{L_n(\hat{\omega}, x)}{|W|} \quad \text{has correct units, but still depends on sensor orientation}
\]

\[
L(\hat{\omega}, x) = \lim_{W \rightarrow \hat{\omega}} \frac{E_n(W, x)}{|W|} \quad \text{"foreshortened in the direction of travel"}
\]

- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction $\omega$
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by any finite sensor
Quantifying light: flux, irradiance, and radiance

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\[
\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA
\]
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by *any* finite sensor
    \[
    \Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta \, d\omega \, dA
    \]
  - Constant along a ray in free space
    \[
    L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega})
    \]
The Fundamental Assumption in Vision

Lighting

Surface

No Change in

Surface Radiance

Camera
Quantifying light: flux, irradiance, and radiance

Attractive properties of radiance:

- Allows computing the radiant flux measured by any finite sensor

\[ \Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA \]

- Constant along a ray in free space

\[ L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega}) \]

- A camera measures radiance (after a one-time radiometric calibration; more on this later). So RAW pixel values are proportional to radiance.
Question

Most light sources, like a heated metal sheet, follow Lambert’s Law.

\[
J(\hat{\omega}) = J_0 \langle \hat{\omega}, \hat{n} \rangle = J_0 \cos \theta
\]

radiant intensity [W/sr]

“What Lambertian area source”

What is the radiance \( L(\hat{\omega}, x) \) of an infinitesimal patch [W/sr \cdot m^2]?
Question

Most light sources, like a heated metal sheet, follow Lambert’s Law

\[ J(\hat{\omega}) = J_0 \langle \hat{\omega}, \hat{n} \rangle = J_0 \cos \theta \]

radiant intensity [W/sr]

“Lambertian area source”

What is the radiance \( L(\hat{\omega}, x) \) of an infinitesimal patch [W/sr \cdot m^2]? 

Answer: \( L(\hat{\omega}, x) = J_0 / |X| \) (independent of direction)
Most light sources, like a heated metal sheet, follow Lambert’s Law:

$$J(\hat{\omega}) = J_o \langle \hat{\omega}, \hat{n} \rangle = J_o \cos \theta$$

“Lambertian area source”

What is the radiance $L(\hat{\omega}, x)$ of an infinitesimal patch [W/sr·m²]?

Answer: $L(\hat{\omega}, x) = J_o / |X|$ (independent of direction)

“Looks equally bright when viewed from any direction”
Appearance
“Physics-based” computer vision (a.k.a. “inverse optics”)
Reflectance and BRDF
Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
  - converges as we use smaller and smaller incoming and outgoing wedges
  - does not depend on the size of the wedges (i.e. is intrinsic to the material)
Reflectance

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- Want to define a ratio such that it:
  - converges as we use smaller and smaller incoming and outgoing wedges
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\[ f_{x,n}(W_{\text{in}}, \hat{\omega}_{\text{out}}) = \frac{L_{\text{out}}(x, \hat{\omega}_{\text{out}})}{E_{\hat{n}}(W_{\text{in}}, x)} \]

- Notations \(x\) and \(n\) often implied by context and omitted; directions \(\omega\) are expressed in local coordinate system defined by normal \(n\) (and some chosen tangent vector)
- Units: \(\text{sr}^{-1}\)
- Called Bidirectional Reflectance Distribution Function (BRDF)
BRDF: Bidirectional Reflectance Distribution Function

Irradiance at Surface in direction $(\theta_i, \phi_i)$

Radiance of Surface in direction $(\theta_r, \phi_r)$

$$
E_{\text{surface}}(\theta_i, \phi_i) \quad (\theta_i, \phi_i)
$$

$$
L_{\text{surface}}(\theta_r, \phi_r) \quad (\theta_r, \phi_r)
$$

$$
\text{BRDF} : f(\theta_i, \phi_i ; \theta_r, \phi_r) = \frac{L_{\text{surface}}(\theta_r, \phi_r)}{E_{\text{surface}}(\theta_i, \phi_i)}
$$
Reflectance: BRDF

- Units: $\text{sr}^{-1}$

- Real-valued function defined on the double-hemisphere

- Allows computing output radiance (and thus pixel value) for any configuration of lights and viewpoint

\[ L^\text{out}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L^\text{in}(\hat{\omega}_{\text{in}}) \cos \theta_{\text{in}} d\hat{\omega}_{\text{in}} \]

reflectance equation
Important Properties of BRDFs

- Conservation of Energy:

\[ \forall \hat{\omega}_{in}, \int_{\Omega_{out}} f(\hat{\omega}_{in}, \hat{\omega}_{out}) \cos \theta_{out} d\hat{\omega}_{out} \leq 1 \]

Why smaller than or equal?
Important Properties of BRDFs

- **Helmholtz Reciprocity:** (follows from 2\textsuperscript{nd} Law of Thermodynamics)

  BRDF does not change when source and viewing directions are swapped.

  \[ f(\theta_i, \phi_i; \theta_r, \phi_r) = f(\theta_r, \phi_r; \theta_i, \phi_i) \]
Property: “Helmholtz reciprocity”

\[ f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}) = f_r(\vec{\omega}_{\text{out}}, \vec{\omega}_{\text{in}}) \]
Important Properties of BRDFs

- Rotational Symmetry (Isotropy):

BRDF does not change when surface is rotated about the normal.

Can be written as a function of 3 variables: \( f(\theta_i, \theta_r, \phi_i - \phi_r) \)
Common assumption: Isotropy

\[ f_r(\vec{\omega}_{in}, \cdot) \]

4D → 3D

\[ f_r(\vec{\omega}_{in}, \vec{\omega}_{out}) \]

Bi-directional Reflectance Distribution Function (BRDF)

[Matusik et al., 2003]
Simplification: Bivariate

4D → 3D → 2D

$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$

Bi-directional Reflectance Distribution Function (BRDF)

[Image: Original 3D, Reduced 2D]

[Text: "half-angle", "difference angle", Romeiro et al., 2008]
Reflectance: BRDF

- Units: \( \text{sr}^{-1} \)

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\[
L^\text{out}(\hat{\omega}) = \int_{\Omega^\text{in}} f(\hat{\omega}_\text{in}, \hat{\omega}_\text{out}) L^\text{in}(\hat{\omega}_\text{in}) \cos \theta^\text{in} \, d\hat{\omega}_\text{in}
\]

Why is there a cosine in the reflectance equation?
Derivation of the Scene Radiance Equation

From the definition of BRDF:

\[ L^{\text{surface}}(\theta_r, \phi_r) = E^{\text{surface}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \]
Derivation of the Scene Radiance Equation

From the definition of BRDF:

\[ L_{\text{surface}} (\theta_r, \phi_r) = E_{\text{surface}} (\theta_i, \phi_i) f (\theta_i, \phi_i ; \theta_r, \phi_r) \]

Write Surface Irradiance in terms of Source Radiance:

\[ L_{\text{surface}} (\theta_r, \phi_r) = L_{\text{src}} (\theta_i, \phi_i) f (\theta_i, \phi_i ; \theta_r, \phi_r) \cos \theta_i d\omega_i \]

Integrate over entire hemisphere of possible source directions:

\[ L_{\text{surface}} (\theta_r, \phi_r) = \int \int L_{\text{src}} (\theta_i, \phi_i) f (\theta_i, \phi_i ; \theta_r, \phi_r) \cos \theta_i \sin \theta_i d\theta_i d\phi_i \]

Convert from solid angle to theta-phi representation:
Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) \]
\[ = r^2 \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]

\[ S = \int_0^\pi \int_0^{2\pi} \sin \theta \, d\theta \, d\phi = 4\pi \]
Bi-directional Reflectance Distribution Function (BRDF)
Mechanisms of Reflection

- **Body Reflection:**
  - Diffuse Reflection
  - Matte Appearance
  - Non-Homogeneous Medium
    - Clay, paper, etc

- **Surface Reflection:**
  - Specular Reflection
  - Glossy Appearance
    - Highlights
  - Dominant for Metals

Image Intensity = Body Reflection + Surface Reflection
Example Surfaces

Body Reflection:
- Diffuse Reflection
  - Matte Appearance
  - Non-Homogeneous Medium
    - Clay, paper, etc

Surface Reflection:
- Specular Reflection
  - Glossy Appearance
  - Highlights
  - Dominant for Metals

Many materials exhibit both Reflections:
BRDF

Lambertian (diffuse) BRDF: energy equally distributed in all directions

\[ f_r(\vec{w}_{in}, \cdot) \]

Bi-directional Reflectance Distribution Function (BRDF)
Diffuse Reflection and Lambertian BRDF

source intensity \( I \)

• Surface appears equally bright from ALL directions! (independent of \( \vec{v} \))

• Lambertian BRDF is simply a constant:
  \[
  f(\theta_i, \phi_i ; \theta_r, \phi_r) = \frac{\rho_d}{\pi}
  \]

• Surface Radiance:
  \[
  L = \frac{\rho_d}{\pi} I \cos \theta_i = \frac{\rho_d}{\pi} I \vec{n} \cdot \vec{s}
  \]

• Commonly used in Vision and Graphics!
BRDF

Specular BRDF: all energy concentrated in mirror direction

\[ f_r(\vec{\omega}_{in}, \cdot) \]

\[ f_r(\vec{\omega}_{in}, \vec{\omega}_{out}) \]

Bi-directional Reflectance Distribution Function (BRDF)
Specular Reflection and Mirror BRDF

- Valid for very smooth surfaces.
- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).
- Mirror BRDF is simply a double-delta function:
  
  \[ f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \delta(\theta_i - \theta_v) \delta(\phi_i + \pi - \phi_v) \]

- Surface Radiance:
  
  \[ L = I \rho_s \delta(\theta_i - \theta_v) \delta(\phi_i + \pi - \phi_v) \]
BRDF

Glossy BRDF: more energy concentrated in mirror direction

\[ f_r(\vec{\omega}_{in}, \cdot) \]

\[ f_r(\vec{\omega}_{in}, \vec{\omega}_{out}) \]

Bi-directional Reflectance Distribution Function (BRDF)
Trick for dielectrics (non-metals)

• BRDF is a sum of a Lambertian diffuse component and non-Lambertian specular components

• The two components differ in terms of color and polarization, and under certain conditions, this can be exploited to separate them.

\[
f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o)
\]
Trick for dielectrics (non-metals)

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\[ f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o) \]

Often called the *dichromatic BRDF*:
- Diffuse term varies with wavelength, constant with polarization.
- Specular term constant with wavelength, varies with polarization.
Trick for dielectrics (non-metals)

- In this example, the two components were separated using linear polarizing filters on the camera and light source.
Trick for dielectrics (non-metals)

\[ I_{\parallel} = \frac{1}{2} I_{\text{diffuse}} + I_{\text{specular}} \]

\[ I_{\perp} = \frac{1}{2} I_{\text{diffuse}} \]
Diffuse and Specular Reflection

diffuse  specular  diffuse+specular
Tabulated 4D BRDFs (hard to measure)

[Ngan et al., 2005]
Low-parameter (non-linear) BRDF models

- A small number of parameters define the (2D, 3D, or 4D) function.
- Except for Lambertian, the BRDF is non-linear in these parameters.
- Examples:

Lambertian: \( f(\omega_i, \omega_o) = \frac{a}{\pi} \)  

Phong: \( f(\omega_i, \omega_o) = \frac{a}{\pi} + b \cos^c (2\langle \omega_i, n \rangle \langle \omega_o, n \rangle - \langle \omega_i, \omega_o \rangle) \)

Blinn: \( f(\omega_i, \omega_o) = \frac{a}{\pi} + b \cos^c \theta(\omega_i, \omega_o) \)

Lafortune: \( f(\omega_i, \omega_o) = \frac{a}{\pi} + b(-\omega_i^\top A \omega_o)^k \)

Ward: \( f(\omega_i, \omega_o) = \frac{a}{\pi} + \frac{b}{4\pi c^2 \sqrt{\langle n, \omega_i \rangle \langle n, \omega_o \rangle}} \exp \left( -\frac{\tan^2 b(\omega_i, \omega_o)}{c^2} \right) \)

\( \alpha \) is called the albedo.
Recent progress: “Active appearance capture”

Reciprocity
[ICCV 2001; ECCV 2002; CVPR 2003; CVPR 2006; Sen et al., 2005; Hawkins et al., 2005, SIGGRAPH 2010]

Isotropy

Separability
[Sato & Ikeuchi, 1994; Schluns and Wittig, 1993; ...; Barsky and Petrou, 2001; CVPR 2005; CVPR 2006; IJCV 2008; ...]

Spatial regularity
[Lensch et al., 2001; Hertzmann & Seitz, 2003; EGSR 2005; PAMI 2006; Lawrence et al., 2006; Weistroffer et al., 2007; CVPR 2008; Garg et al. 2009; ...]

Tangent-plane symmetry
[SIGGRAPH Asia 2008]

...
Reflectance Models

Reflection: An Electromagnetic Phenomenon

Two approaches to derive Reflectance Models:

– Physical Optics (Wave Optics)
– Geometrical Optics (Ray Optics)

Geometrical models are approximations to physical models
But they are easier to use!
Reflectance that Require Wave Optics
References

Basic reading:
• Szeliski, Section 2.2.
• Gortler, Chapter 21.