Mechanism Design I: Basic Concepts and Myerson’s Lemma

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MECHANISM DESIGN

- Game Theory: Interaction of rational, competing, strategic agents
- Mechanism Design: “Inverse Game Theory”
  - How do we design systems for rational, competing, strategic agents?
  - We’ll be interested in promoting a desired objective
  - In this class we’ll focus on auctions, but most of the tools we’ll develop are applicable more generally
OLYMPICS 2012: A CAUTIONARY TALE

- 4 groups: A, B, C, D
- 4 teams per group
- Phase 1: Round robin within each group
  - Top two from each group advance in the second phase
- Phase 2: Knockout
  - In the first match, top team from group A is matched with second best of group C. Top team in C with second best from A. Similarly for B and D.
- What does a team want?
  - Maximize probability of winning a gold medal!
- What does the Olympic committee want?
• Phase 1:
  ◦ What if teams $A_1$ and $A_2$ have destroyed teams $A_3$ and $A_4$, and in the final match are playing each other?
  ◦ No problem! the loser would play the best in $C$, so $A_1$ and $A_2$ are still incentivized to try hard!
  ◦ No problem? What if there’s a huge upset in group $C$, and the (actually) best team ends up in second place?
  ◦ Come on... What are the chances??
OLYMPICS 2012: A CAUTIONARY TALE

Video (17:30) : https://youtu.be/7mq1ioqiWEO
**HOT OFF THE PRESS!!!**

**Mandra:**

• Greek national exams: Average grade is the only criterion to go to university.
• New law: People from Mandra get a small boost.
• 2018: Huge spike in the number of people that declare Mandra as their primary residence.

**Flooding (Nov 17):**

[Map of Greece with Mandra highlighted]

[Image of flooded area with damage]

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THE APPROACH

What’s wrong with these people???

What’s wrong with these rules?
QUESTIONS

• When can we design systems that are robust to strategic manipulation?

• What does computer science bring to the table?
  ◦ How much harder is mechanism design than algorithm design?

• Tradeoffs between simplicity and optimality.

Disclaimer: This is not an economics course
ASSUMPTIONS

• We’ll be working in a setting with money.
• Agents are risk neutral:
  ◦ Value $v_i$ with probability $q_i$ for $i = 1, \ldots, n$ is the same as value $\sum_{i=1}^{n} v_i q_i$ deterministically.
• Agents have quasi-linear utilities:
  ◦ Utility for value $v$ for a price of $p$ equals $v - p$
• We’ll focus on truthfulness: reporting your true value maximizes your utility (more on this later).
• We’ll also ask for Individual Rationality: if you say the truth, expected utility (over the randomness of the mechanism) is non-negative.
  ◦ Participating is better than staying home.
AUCTIONS

We will mostly talk about auctions
AUCTIONS: EXAMPLES

- eBay
- Google AdWords
- Bing Ads
SINGLE ITEM AUCTIONS

• Single item for sale.
• \( n \) potential buyers: the bidders.
• Each bidder has a private value \( v_i \) for the item.
SEALED-BID AUCTIONS

1. Each bidder $i$ privately communicates her bid $b_i$, possibly different than $v_i$, to the auctioneer (in a sealed envelope)

2. The auctioneer decides who to allocate the item to.

3. The auctioneer decides who pays what.
SEALED-BID AUCTIONS

• Obvious answer to (2): give the item to the highest bidder

• Reasonable ways to implement (3):
  ◦ Highest bidder pays her bid, aka a first price auction.
  ◦ Highest bidder pays the minimum bid required to win, i.e. the second highest bid. This is the second price auction.
STRAWMAN

• Wait... Why charge in the first place?
• Proposal: give the item to the highest bidder and charge them nothing.
• Aka, “who can name the highest number?”
• Remember fair division?
  ◦ In retrospect, truthful algorithms that eschew payments look even more amazing!
FIRST PRICE AUCTIONS

• How do I bid??
• If I bid my true value $v_i$ I always get utility zero!
  ◦ If I lose, I get nothing and pay nothing.
  ◦ If I win, I pay $v_i$ and get value $v_i$.
• So, I "should" bid something smaller than $v_i$
• How much smaller?
Assume your value = month + day of your birthday. E.g. 10/08/1997, value = 18. How much would you bid?
FIRST PRICE AUCTIONS

• In order to argue about bidding behavior, we need to make more assumptions about the information agents have about other agents’ bids.

• Common assumption: values come from known distribution $D_i$.

• Common question: what is an equilibrium bidding strategy? That is, if everyone follows this strategy, no one deviates.

• See homework.
SECOND PRICE AUCTIONS

• Who gets the item: highest bidder.
• What do they pay: the second highest bid.
• Claim: For a bidder to set $b_i = v_i$ (weakly) maximizes her utility *no matter what everyone else is doing!*
• Definition: When a player has a strategy that is (weakly) better than all other options, regardless of what the other player does, we will refer to it as a **dominant strategy.**
SECOND PRICE AUCTIONS

• Claim: Truth-telling is a dominant strategy.

Proof:
• Let \( b_{-i} = (b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n) \) be the bids of all players except \( i \). Let
\[ B = \max_{j \neq i} b_j \]
• There are two possible outcomes:
  1. \( b_i < B \), \( i \) loses and gets utility \( u_i = 0 \)
  2. \( b_i \geq B \), \( i \) wins, pays \( B \) and gets utility \( u_i = v_i - B \)
• Effectively, \( i \)'s utility is picking between 0 and \( v_i - B \)
  ◦ If \( v_i < B \), \( \max\{0, v_i - B\} = 0 \), which you can get by bidding \( b_i = v_i \)
  ◦ If \( v_i \geq B \), \( \max\{0, v_i - B\} = v_i - B \), which you can get by bidding \( b_i = v_i \)
SECOND PRICE AUCTIONS

• Theorem: The second price auction, aka the Vickrey auction, is awesome!
  ◦ Dominant strategy incentive compatible (DSIC)!
  ◦ Maximizes Social surplus! That is, the item always goes to the agent with the highest value!
  ◦ Can be computed in polynomial (linear) time!
TOWARDS A MORE GENERAL RESULT

• If we have a single item and want to give it to the agent with the highest value, we can do so truthfully.

• What if we don’t want to give the item to the agent with the highest value?
SINGLE PARAMETER ENVIRONMENTS

- $n$ buyers
- Buyer $i$ has private valuation $v_i$ and submits a bid $b_i$
- An auction is a pair of two functions $(x, p)$
  - $x(b_1, \ldots, b_n) = (x_1, \ldots, x_n)$ is the allocation function.
    - $x_i =$ Probability that item goes to player $i$.
    - For single item auctions: $\sum_i x_i \leq 1$
    - Our next result will not use this fact!
  - $p(b_1, \ldots, b_n) = (p_1, \ldots, p_n)$ is the payment function.
    - $p_i =$ Price player $i$ pays.
MYERSON’S LEMMA

• Definition: An allocation rule \( x \) is implementable if there is a payment rule \( p \) such that the auction \((x, p)\) is DSIC.

• We’ve seen that the allocation rule "give the item to the highest bidder" is implementable!

• What about the allocation rule "give the item to the 3-rd highest bidder"?
MYERSON’S LEMMA

• Definition: An allocation rule $x$ is monotone if for every bidder $i$ and bids $b_{-i}$ of the other agents, the allocation $x_i(b_i, b_{-i})$ is monotone non-decreasing in $b_i$.

• Lemma (Myerson):
  ◦ An allocation is implementable iff it is monotone
  ◦ If $x$ is monotone, there exists a unique (up to a constant) payment rule $p$ that makes $(x, p)$ DSIC, given by

$$p_i(v, b_{-i}) = vx_i(v, b_{-i}) - \int_0^v x_i(z, b_{-i})dz$$
Poll

Is the allocation rule “give the item to the third highest bidder” implementable?

1. Yes
2. No
MYERSON’S LEMMA: PROOF

• IC constraint between $v$ and $v'$:
  \[
  v \ x_i(v, b_{-i}) - p_i(v, b_{-i}) \geq v \ x_i(v', b_{-i}) - p_i(v', b_{-i})
  \]
  \[
  v' \ x_i(v', b_{-i}) - p_i(v', b_{-i}) \geq v' \ x_i(v, b_{-i}) - p_i(v, b_{-i})
  \]

• $v(x_i(v, b_{-i}) - x_i(v', b_{-i})) \geq$
  \[
  p_i(v, b_{-i}) - p_i(v', b_{-i})
  \geq v'(x_i(v, b_{-i}) - x_i(v', b_{-i}))
  \]
MYERSON’S LEMMA: PROOF

• \( v(x_i(v, b_{-i}) - x_i(v', b_{-i})) \geq p_i(v, b_{-i}) - p_i(v', b_{-i}) \geq v'(x_i(v, b_{-i}) - x_i(v', b_{-i})) \)

• \( v \geq v' \) implies monotonicity of the allocation!

• Take \( v' = v - \epsilon \), and take the limit as \( \epsilon \) goes to zero.

  ◦ \( p'_i(v, b_{-i}) = vx_i'(v, b_{-i}) \)

  ◦ \( p_i(v, b_{-i}) = vx_i(v, b_{-i}) - \int_0^v x_i(z, b_{-i})dz + p_i(0, b_{-i}) + c(b_{-i}) \)

• Assuming that \( p_i(0, b_{-i}) = 0 \) (Individual rationality) we get the desired result.
MYERSON’S LEMMA PICTORIALLY

\[ x_i(v_i, b_{-i}) \]

\[ \text{value} = v \cdot x_i(v, b_{-i}) \]
MYERSON’S LEMMA PICTORIALLY

\[ x_i(v_i, b_{-i}) \]
MYERSON’S LEMMA PICTORIALLY

\[ x_i(v_i, b_{-i}) \]
SUMMARY

- Basic definitions of single parameter environments
- Second price auctions
- Myerson’s lemma