Rigid Body Collisions

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Collision and Contact

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Outline

- Detect Collisions
- Compute Collision Type
- Depending on Collision Type...
  - Apply Impulse Force
  - Compute Resting Contact Forces
Outline

• Detect Collisions
  • Compute Collision Type
  • Depending on Collision Type...
    • Apply Impulse Force
    • Compute Resting Contact Forces
Problem

- Positions **NOT OK**
Collision Detection

Assume we have some spatial collision detection algorithm. (This can be solved in less than $O(n^2)$ time.)
Simulations with Collisions

We want rigid bodies to behave as solid objects, and not interpenetrate. By applying constraint forces between contacting bodies, we prevent interpenetration from occurring. We need to:

a) Detect interpenetration
b) Determine contact points
c) Compute constraint forces

\[ Y(t_0) \]
We want rigid bodies to behave as solid objects, and not interpenetrate. By applying constraint forces between contacting bodies, we prevent interpenetration from occurring. We need to:

- Detect interpenetration
- Determine contact points
- Compute constraint forces

Simulations with Collisions

\[ Y(t_0 + \Delta t) \]

\[ Y(t_0) \]
Simulations with Collisions

Y(\(t_0\) + \(\Delta t\))

Y(\(t_0\) + 2\(\Delta t\))

Y(\(t_0\))
An Illegal State $Y$

$Y(t_0)$

$Y(t_0 + \Delta t)$

$Y(t_0 + 2\Delta t)$

$Y(t_0 + 3\Delta t)$

illegal state
Simulations with Collisions

An Illegal State

Backing up to the Collision Time

\[ Y(t) \]

\[ Y(t_0) \]

\[ Y(t_0 + \Delta t) \]

\[ Y(t_0 + 2\Delta t) \]

\[ Y(t_0 + 3\Delta t) \]

\[ p_a \]

\[ n \cdot p_a < 0 \]

\[ p_a \]

\[ n \cdot p_a < 0 \]

Colliding Contact

\[ p_a \]

\[ n \cdot p_a < 0 \]
Outline

- Detect Collisions

- **Compute Collision Type**
  - Depending on Collision Type...
    - Apply Impulse Force
    - Compute Resting Contact Forces
Geometric Contact

- Vertex-Face
- Edge-Edge
Physical Contact

- Impulse Collision ("bounce")
- Resting Contact
Physical Contact

\[ p_a(t) = \text{contact point on body A} \]
\[ p_b(t) = \text{contact point on body B} \]

\[ p_a(t_0) = p_b(t_0) \text{ but in general } \dot{p}_a(t_0) \neq \dot{p}_b(t_0) \]
Physical Contact

\((\dot{p}_a(t_0) - \dot{p}_b(t_0)) \cdot n < 0\)

Impulse collision.

\((\dot{p}_a(t_0) - \dot{p}_b(t_0)) \cdot n = 0\)

Resting contact.

\((\dot{p}_a(t_0) - \dot{p}_b(t_0)) \cdot n > 0\)

No collision.
Outline

- Detect Collisions
- Compute Collision Type
- Depending on Collision Type...
  - Apply Impulse Force
  - Compute Resting Contact Forces
Problem

- Positions **OK**
- Velocities **NOT OK**
Simulations with Collisions

An Illegal State $Y(t_0)$

$Y(t_0 + \Delta t)$

$Y(t_0 + 2\Delta t)$

$Y(t_0 + 3\Delta t)$

Backing up to the Collision Time

Colliding Contact

$\hat{n}$

$\dot{p}_a$

$\hat{n} \cdot \dot{p}_a < 0$

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Collision Process

Δt

no force

no force
Collision Process

A Soft Collision

A Harder Collision

A Very Hard Collision

\[ \Delta t \]

no force

no force
A Soft Collision

force

velocity

$\Delta t$
A Harder Collision

force

velocity

$\Delta t$
A Very Hard Collision

force

velocity

$\Delta t$
A Rigid Body Collision

impulsive force

\[ f_{imp} = \infty \]

\[ \Delta t = 0 \]

velocity

\[ \text{velocity} \]
Colliding Contact

impulse (alters velocity)
Mathematically...
Computing Impulses

\[ \dot{p}_a^- = ? \]

\[ \dot{p}_a^+ = ? \]
Coefficient of Restitution

\[ \hat{n} \cdot \dot{p}_a^+ = -\varepsilon (\hat{n} \cdot \dot{p}_a^-) \]
Computing j

\begin{align*}
v^+_a(t_0) &= v^-_a(t_0) + \frac{j\hat{n}(t_0)}{M_a} \\
\omega^+_a(t_0) &= \omega^-_a(t_0) + I^{-1}_a \left( r_a \times j\hat{n}(t_0) \right) \\
\dot{p}^+_a(t_0) &= v^+_a(t_0) + \omega^+_a(t_0) \times r_a
\end{align*}

\downarrow

\dot{p}^+_a(t_0) = aj + b
Computing $j$

\[ \hat{n} \cdot p_a^+ = -\varepsilon (\hat{n} \cdot p_a^-) \]

\[ cj + b = d \]
Computing $j$

\[ \hat{n} \cdot (\dot{p}_a^+ - \dot{p}_b^+) = -\varepsilon (\hat{n} \cdot (\dot{p}_a^- - \dot{p}_b^-)) \]
Computing \( j \)

\[
\hat{n} \cdot (\dot{p}_a^+ - \dot{p}_b^+) = -\varepsilon (\hat{n} \cdot (\dot{p}_a^- - \dot{p}_b^-))
\]

\[c j + b = d\]
Outline

• Detect Collisions
• Compute Collision Type

• Depending on Collision Type...
  • Apply Impulse Force
  • Compute Resting Contact Forces
Resting Contact

\[ \hat{n} \cdot \dot{p}_a = 0 \]
Problem

- Positions **OK**
- Velocities **OK**
- Accelerations **NOT OK**
Resting Contact

A

\[ \hat{n} \]

\[ \dot{p}_a \]

\[ p_a \]

B

force

(alters acceleration)

e external forces

\[
\dot{Y}(t), \quad \ddot{Y}(t), \quad \gamma(t), \quad \theta, \quad \phi, \quad \psi
\]
Example
Resting Contact Forces

To avoid inter-penetration, the force strength \( f \) must prevent the vertex \( p_a \) from accelerating downwards. If \( B \) is fixed, this is written as:

\[
\mathbf{n} \cdot \mathbf{p}_a \neq 0
\]

What is \( f \)?

What is \( f \)?

Computing \( f \):

To prevent the constraint force from holding bodies together, the force must be repulsive:

\[
\mathbf{f} \cdot \mathbf{n} \neq 0
\]

\[
\mathbf{f} \cdot \mathbf{n} \neq 0
\]
Solution Outline

• Similar to constraints before, we will compute constraint forces.

• Except...
  • There will be inequalities.
  • There will be quadratic terms.
Conditions on the Constraint Force

To avoid inter-penetration, the force strength $f$ must prevent the vertex $p_a$ from accelerating downwards. If $B$ is fixed, this is written as

$$\hat{n} \cdot \ddot{p}_a \geq 0$$
Resting Contact Forces

To avoid inter-penetration, the force strength \( f \) must prevent the vertex \( p_a \) from accelerating downwards. If \( B \) is fixed, this is written as:

\[
\hat{n} \cdot \ddot{p}_a \geq 0
\]

To prevent the constraint force from holding bodies together, the force must be repulsive:

\[
af + b \geq 0
\]

Computing \( f \):

\[
\hat{n} \cdot \ddot{p}_a \geq 0 \quad \Rightarrow \quad af + b \geq 0
\]
Conditions on the Constraint Force

To prevent the constraint force from holding bodies together, the force must be repulsive:

\[ f \geq 0 \]

Does the above, along with

\[ \hat{n} \cdot \dot{p}_a \geq 0 \quad \Rightarrow \quad af + b \geq 0 \]

sufficiently constrain \( f \)?
3rd Constraint

- We require that the force at a contact point become zero if the bodies begin to separate.
Conditions on the Constraint Force

To make \( f \) be workless, we use the condition

\[
af + b = 0
\]

or

\[
af + b > 0
\]

Either

\[
af + b = 0 \quad f \geq 0
\]

or

\[
af + b > 0 \quad f = 0
\]
Conditions on the Constraint Force

To make $f$ be workless, we use the condition

$$f \cdot (af + b) = 0$$

The full set of conditions is

$$af + b \geq 0$$

$$f \geq 0$$

$$f \cdot (af + b) = 0$$
Multiple Contact Points

To make \( f \) be workless, we use the condition

The full set of conditions is

\[
af_1 + b_1 \neq 0
\]

\[
f_2 \hat{n}_2 = 0
\]

\[
f_1 \hat{n}_1 = 0
\]
Conditions on $f_1$

Non-penetration:

$$a_{11} f_1 + a_{12} f_2 + b_1 \geq 0$$

Repulsive:

$$f_1 \geq 0$$

Workless:

$$f_1 \cdot (a_{11} f_1 + a_{12} f_2 + b_1) = 0$$
Quadratic Program for $f_1$ and $f_2$

Non-penetration:

\[ a_{11} f_1 + a_{12} f_2 + b_1 \geq 0 \]
\[ a_{21} f_1 + a_{22} f_2 + b_2 \geq 0 \]

Repulsive:

\[ f_1 \geq 0 \]
\[ f_2 \geq 0 \]

Workless:

\[ f_1 \cdot (a_{11} f_1 + a_{12} f_2 + b_1) = 0 \]
\[ f_2 \cdot (a_{21} f_1 + a_{22} f_2 + b_2) = 0 \]
In the Notes – Constraint Forces

Derivations of the non-penetration constraints for contacting polyhedra.

Derivations and code for computing the $a_{ij}$ and $b_i$ coefficients.

Code for computing and applying the constraint forces $f_i \hat{n}_i$. 
Question

• What type of discrete geometric representation should we use for a deformable object?

• What sort of forces apply to deformable objects, i.e. in what ways do they resist deformation?

• How can we compute these forces?