We are studying the very important case of a PDE that "pushes" (or "advepts") a density field \( \rho \) according to a velocity field \( \mathbf{u} \).

**Case 1:** \( \rho \) is a 1D function that moves to the right.

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x} = 0
\]

**Case 2:** We suppose that \( \rho \) is 2D and \( \mathbf{u} \) defines a rotational velocity field.

\[
\mathbf{u}(x,y) = \begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}
\]

The operator \( \mathbf{u} \cdot \nabla \) is called the advection operator.

**Remark:** Another way to look at it. Suppose we cover space with infinitely many particles with the following properties:

- At density \( \rho \), \( \frac{\partial \rho}{\partial t} = 0 \)
- At \( \rho \) position \( \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x} = 0 \)
- At \( \rho \) velocity \( \frac{\partial \rho}{\partial t} = -\frac{\partial \rho}{\partial x} = -\mathbf{u} \cdot \nabla \rho \)

Does this look familiar?