Solving Non-linear equations:

How can we approximate $\vec{f}^{-1}(\vec{q})$?

Recall: $\vec{f}(\vec{x}) \approx \vec{f}(\vec{x}_0) + \frac{d\vec{f}}{d\vec{x}}\Big|_{\vec{x}=x_0}(\vec{x}-\vec{x}_0)$

$$\therefore \vec{f}(\vec{x}_1) \approx \vec{f}(x_0) + \frac{d\vec{f}}{d\vec{x}}(\vec{x}_1 - \vec{x}_0) = \vec{q}$$

implies... $\vec{x}_1 \approx \left(\frac{d\vec{f}}{d\vec{x}}\right)^{-1}\left(\vec{q} - \vec{f}(\vec{x}_0)\right) + \vec{x}_0$ (A)

Example: for what values of x & y do the following equations hold
$$\begin{cases} xy - y^2 + x^2 = \frac{1}{2} \\ x^2 + y^2 = \frac{1}{2} \end{cases}$$

$$\vec{f}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} xy - y^2 + x^2 \\ x^2 + y^2 \end{bmatrix}$$

$$\Longrightarrow \frac{d\vec{f}}{d\vec{x}} = \begin{bmatrix} y_0 + 2x_0 & x_0 - 2y_0 \\ 2x_0 & 2y_0 \end{bmatrix}$$

Using equation (A) we can come up with an approximating sequence:

$$x_0 \to x_1 \to x_2 \to \cdots$$

and we hope that $\lim_{n\to\infty} \|\vec{q} - \vec{f}(\vec{x}_n)\| = 0$