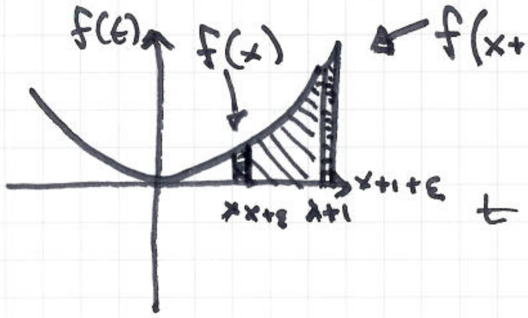
questions. answers.

① $\frac{d}{dx}\int_0^x f(t)\,dt =$ ② $f(x)$

③ $\frac{d}{dx}\int_x^{x+1} f(t)\,dt =$ ⑤ $f(x+1) - f(x)$

④ 

Circled #'s indicate order of writing on board

⑥ For example.

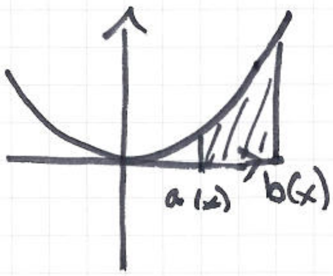
$f(t) = t^2$

solving with the left hand side:

$$\frac{d}{dx}\int_x^{x+1} t^2\,dt = \frac{d}{dx}\left\{ \frac{1}{3}t^3 \Big|_x^{x+1} \right\} = \frac{d}{dx}\frac{1}{3}\left\{ (x+1)^3 - x^3 \right\}$$

$= (x+1)^2 - x^2$ ← that's exactly what the right hand side tells us.

⑦ $\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\,dt =$ ⑨ $f(b(x))\,b'(x) - f(a(x))\,a'(x)$

⑧ 

Taylor Series Expansion about s:

$$f(x) = f(s) + f'(s)(x-s) + \frac{1}{2} f''(s)(x-s)^2 + \dots$$

example taylor series expansion of sin(x) about 0

$$\frac{d}{dx}\sin(x) = \cos(x) \qquad \sin(0) = 0$$

$$\frac{d}{dx}\cos(x) = -\sin(x) \qquad \cos(0) = 1$$

$$\sin(x) = \sin(0) + \sin'(0) + \frac{1}{2}\sin''(0)x^2 + \frac{1}{6}\sin'''(0)x^3 + \dots$$
$$= 0 + \cos(0)x + \frac{1}{2}\sin(0)x^2 - \frac{1}{6}\cos(0)x^3 + \dots$$
$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos(x) = \cos(0) + \cos'(0)x + \frac{1}{2}\cos''(0)x^2 + \frac{1}{6}\cos'''(0)x^3$$
$$= 1 - \sin(0)x - \frac{1}{2}\cos(0)x^2 + \frac{1}{6}\sin(0)x^3 - \dots$$
$$= 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots$$

Hence if we take the derivative of the taylor expansion of sin(x), we indeed get cos(x).

We can also use taylor series expansion for approximation:

$$f(x) \approx f(0) + f'(0)x \quad (1^{st} \text{ order})$$
$$f(x) \approx f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 \quad (2^{nd} \text{ order})$$

example

$$\sin(x) \approx x \qquad (!)$$

$$\sin(x) \approx x - \frac{x^3}{3!} \qquad (\text{2nd order}).$$

Multidimensional Approximation

$\cancel{\vec{x}}$

$$f_1(x_1\ x_2\ x_3\ x_4 \cdots\ x_n)$$
$$f_2(x_1\ x_2\ \cdots\ x_n).$$
$$\vdots$$
$$f_m(x_1\ x_2\ \cdots\ x_n).$$

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad \vec{f} = \begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix}$$

$$\vec{f}(\vec{x}) \approx \vec{f}(0) + \frac{d\vec{f}}{d\vec{x}}\vec{x}$$ very useful!!

where $\left[\frac{d\vec{f}}{d\vec{x}}\right]_{ij} = \frac{df_i}{dx_j}$