15780: Graduate AI (Spring 2018)

Practice Midterm 2

March 1, 2018

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1 Heuristic Search [25 points]

Consider the problem of informed search with a heuristic. For each state \( x \), let \( h^*(x) \) be the length of the cheapest path from \( x \) to a goal.

Prove or disprove the following statements:

1.1 [15 points] If \( h(x) = 2h^*(x) \) for all states \( x \), then \( A^* \) tree search with the heuristic \( h \) is optimal.

1.2 [10 points] If \( h \) is a consistent heuristic, \( A^* \) graph search with the heuristic \( h'(x) = h(x)/2 \) is optimal.
2 Learning Theory [25 points]

Determine the VC dimension of the following function classes.

2.1 [15 points] Define \( F \) to be the set of strings of length 3 composed of the symbols 0, 1, and \( * \). Each \( f \in F \) acts as a pattern matcher; i.e., when applied to a binary string \( s \), it either accepts or rejects \( s \). For example, when we apply the schema \( f = 1 \ast \ast \) to the string \( s = 101 \), it accepts, and when we apply \( f \) to \( s' = 010 \), it rejects. What is the VC dimension of \( F \)?

2.2 [10 points] The union of \( n \) intervals on the real line.
3 Integer Programming [25 points]

Consider an undirected graph $G = (V, E)$. A minimum dominating set is a smallest subset $S$ of $V$ such that every node not in $S$ is adjacent to at least one node in $S$. A minimum independent dominating set is a smallest subset $S$ of $V$ such that (1) every node not in $S$ is adjacent to at least one node in $S$ and (2) no pair of nodes in $S$ are adjacent. In your answer, you can use $N(i)$ to denote the set of neighbors of node $i$ (i.e., $N(i)$ is a set of nodes adjacent to $i$) for each node $i \in V$. Note that $i \notin N(i)$. You also can use $(i, j) \in E$ to denote the edge between node $i \in V$ and node $j \in V$.

3.1 [15 points] Formulate an integer linear program to find a minimum dominating set.

3.2 [10 points] Formulate an integer linear program to find a minimum independent dominating set.
4 Convex Optimization [25 points]

Consider a linear program of the standard form: minimize $c^T x$ such that $Ax \leq b$. Here $x \in \mathbb{R}^n$ is the vector of variables, and $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, and $b \in \mathbb{R}^m$ are constants.

Prove from the definitions that this is a convex program.