1. (NP, coNP, PSPACE.)
   (a) (5 points.) For languages $A$ and $B$, show that $A \leq_P B$ and $B \in \text{PSPACE}$ implies $A \in \text{PSPACE}$.
   (b) (2 points.) Show that $\text{coPSPACE} = \text{PSPACE}$.
   (c) (3 points.) Show that $\text{coNP} \subseteq \text{PSPACE}$. You may use the fact (stated in class) that $3\text{SAT} \in \text{PSPACE}$.

2. (Median in log-space.) (10 points.) Consider the problem of finding the median of $n$ integers ($n$ odd). You are to show that this problem can be solved in logarithmic space.
   First, give pseudocode solving this problem, using only a constant number of “integer variables”. Explain your code through “comments”, or with a short prose description.
   Then, give some more low-level Turing Machine details — such as how things are stored on what tapes, and a little bit about how any computations are done. To be somewhat more precise, you can imagine the following more precise description of the problem: We want a TM of space complexity $O(\log n)$ having the following behavior: The input is supposed to be a string in $\{0, 1, \#\}^*$ of the form $\#y_1\#y_2\# \cdots \#y_m$, where $m$ is odd and each $y_i$ is an integer written in base-2 using exactly $2b$ bits (leading 0’s allowed), where $b$ is the number of bits in the base-2 representation of $m$. The machine rejects if the input is not in the proper form, and otherwise it prints the median value of $y_1, y_2, \ldots, y_n$ on one of its work tape and accepts.

3. (Verification definition of NL.) In class we defined NL as the set of languages $A$ for which there is a nondeterministic Turing machine deciding $A$ with space complexity $O(\log n)$. Consider the following “verification-based” definition of a class “VNL”. We say language $A$ is in VNL if there is a deterministic TM $V$ with the following properties. First, $V$ has a read-only input tape on which an input $x \in \{0, 1\}^n$ is written. Second, $V$ has a special read-once input tape on which an input $y \in \{0, 1\}^N$ is written, where $N = O(n^c)$ for some constant $c$. (A read-once tape is one where at each step the head can only stay put or move right; it cannot move left, and it cannot write.) Third, $V$ has one normal (read/write) “work” tape, initially blank. Fourth, $V$ has space complexity $O(\log n)$, in the sense that it accesses at most $O(\log n)$ work tape cells. Finally, $V$ verifies $A$ in the sense that for all $x$ it holds that
   $$x \in A \iff \exists y \ V(x, y) \text{ accepts.}$$
   Show that VNL = NL, as follows:
   (a) (5 points.) Show that VNL $\subseteq$ NL.
   (b) (5 points.) Show that NL $\subseteq$ VNL.

4. (NP vs. LINSPACE.) (10 points.) Show that $\text{NP} \neq \text{SPACE}(n)$.
   (Remark: it is unknown if $\text{NP} \subseteq \text{SPACE}(n)$ and it is unknown if $\text{SPACE}(n) \subseteq \text{NP}$.
   Hint: you have seen a problem like this before.)