Taking Duals I. Consider this maximization linear program:

\[
\begin{align*}
\text{max} & \quad (x_1 + 3x_2 - 2x_3) \\
\text{s.t.} & \quad x_1 + x_2 + 2x_3 \leq 2 \\
& \quad 7x_1 + 2x_2 + 5x_3 \leq 6 \\
& \quad 2x_1 + x_2 - x_3 \leq 1 \\
& \quad x_1, x_2, x_3 \geq 0
\end{align*}
\]

1. Write down its dual LP. (Is it a maximization or minimization problem? What are the variables? Constraints?)

2. Now write down the dual of this dual LP. Remember, since the dual will be a minimization LP, this dual’s-dual will give a best lower bound on the dual.

Taking Duals II. Write down the dual of this minimization LP: *(Be careful, some inequalities are greater-than, some are less-than. And not all constraints have all variables.)*

\[
\begin{align*}
\text{max} & \quad (x_1 - 3x_2 + 2x_3) \\
\text{s.t.} & \quad 3x_1 + 2x_3 \geq 2 \\
& \quad 2x_2 - x_3 \leq 5 \\
& \quad x_1, x_2, x_3 \geq 0
\end{align*}
\]
C. Minimax from Duality (by Example). Let the row-player’s payoffs be given by this (non-negative) matrix

\[
\begin{array}{cc}
  & L & R \\
 L & 1 & 5 \\
 R & 3 & 2 \\
\end{array}
\]

1. If the probabilities on the two rows are \(p_1\) and \(p_2\), write down an LP for the row player’s optimal strategy:

2. Now take the dual of this LP. Show this dual is an LP computing the column player’s optimal strategy. (And hence strong duality implies the minimax theorem.)