I. Safe Moving  An office building has one (1) safe where valuables are kept. There are
two (2) rooms in the building numbered \( \sqrt{7} \) and 43. The distance between the rooms
is 1 mile. There is a sequence of requests where an employee in some room needs to
access the safe. If the employee is in the room with the safe, it’s free. If the employee
is in the other room, it costs $1. Management is monitoring these activities, and has
the option to move the safe from time to time to a different room. The cost of moving
the safe is $p.

The requests are adversarily generated, and future requests are unknown by manage-
ment. After each request management has the option of moving the safe from one place
to another. Management’s goal is to obtain a deterministic algorithm with low total
cost. (The total cost includes the employee movement costs plus the costs incurred by
moving the safe.) The criterion of any management algorithm is the competitiveness
of the algorithm, as defined in class.

We will analyze a counter-based algorithm, where the threshold is \( 2p \). This means that
when a request is from the room without the safe, the algorithm processes it in the
current room (at a cost of 1), and increments the counter. If the counter has reached
\( 2p \) it moves the safe to the just requested room (at a cost of \( p \)), and resets the counter
to 0.

Prove that this algorithm is 3-competitive.
II. **2-SAD** As explored in a previous recitation, 3-SAT’s slightly less capable relative, 2-SAT, is solvable in linear time. Now consider the following modification: given a CNF with at most 2 literals per clause and a constant $k$, is there a satisfying assignment which satisfies at least $k$ clauses?

Turns out, this problem is NP-complete. Prove it.