Counting Axis-Aligned Segment Intersections

You’re given \( n \) vertical line segments in the plane and \( m \) horizontal ones. The problem is to count the number of intersections. The brute-force algorithm is to try intersecting all \( n \) verticals with all \( m \) horizontals, which is \( O(nm) \). Devise a solution based on SegTrees that is \( O((n + m) \log(n + m)) \).

Hint 1: Take advantage of the fact that there are only \( 2n + m \) relevant y coordinates where anything interesting happens.

Hint 2: Sweep from left to right, processing each event as it comes. The events are: (1) a vertical segment appears, (2) a left end of a horizontal segment appears and (3) the right end of a horizontal segment appears.

First-Fit

You are packing up \( n \) items into boxes, and want to use as few boxes as possible. Each box can fit a total of 10 pounds of stuff, and the weight of the \( i^{th} \) item is \( w_i \leq 10 \). Your algorithm is this: initially, all the boxes are lined up, empty. You pick the next unpacked item (say item \( i \)), and put it in the first box that can hold the item (i.e., whose current weight is at most \( 10 - w_i \)).

1. Argue that if \( OPT \) is the optimal number of boxes into which you can pack all the items, then your algorithm uses at most \( 2 \cdot OPT + 1 \) bins.

2. How would you implement the algorithm in time \( O(n \log n) \).

VCG and Pricing Advertisements

We saw the VCG mechanism for incentive-compatible auctions in Lecture. Let’s use this for pricing online advertising slots. There are 2 ad slots that ElGogo wants to sell on a page,
the first slot has a clickthru rate of 0.5, the second has a clickthru rate of 0.3. Each bidder can get at most one slot. There are 4 bidders, with the following valuations:

- A: $10 per click (so, e.g., this bidder values the first slot at 10 * 0.5 = 5, and the second slot at 10 * 0.3 = 3.)
- B: $8 per click
- C: $7 per click
- D: $2 per click

1. What is the social-welfare maximizing allocation?

2. What are the VCG payments?

Combinatorial Auctions

VCG can be used even with complicated preferences. Suppose we have two identical hotel rooms in Las Vegas, a flight ticket $f$ from PIT to LAS, and a concert ticket $c$ in Vegas to auction off. In the following, a generic hotel room is denoted by $h$, and none of the people want two rooms.

- Buyer A: values \{h\} at $100, \{f\} at $200, \{h, f\} at $450, \{h, f, c\} at $440. (He hates the band in question so much, he gets negative value from getting $c$ along with $h, f$.) All other sets are valued at $0.
- Buyer B (doesn’t care for the concert): values \{h\} at $50, \{f\} at $400, \{h, f\} at $500, and \{h, f, c\} at $501. All other sets are valued at $0.
- Buyer C (lives in Vegas): values \{c\} (and all sets containing $c$) at $200.

What is the social-welfare maximizing allocation, and what are the VCG payments?