15-451/651 Algorithm Design & Analysis, Spring 2024

Extra Review Problems

Polynomials

1. **(Short answer / multiple choice)**

   (a) Given some set of d unique numbers $r_1, r_2, ..., r_d$, what is a d-degree polynomial $P(x)$ with these as roots?

   (b) Perform the convolution of the two vectors $\langle 3, 4, -1 \rangle$ and $\langle 2, 5, 0, 1 \rangle$.

   (c) If Alice has a polynomial of degree d that she would like to send to Bob over a channel that can replace at most k symbols with the value $*$, how many points should she evaluate p at when sending these evaluations to Bob?

   (d) If Alice has a polynomial of degree d that she would like to send to Bob over a channel that can change the values in k locations, how many points should she evaluate p at when sending these evaluations to Bob?

2. **(Evenly Spaced Ones)**

   Given a binary string S of length n. We wish to determine whether there exists three evenly spaced ones within S. For example, 11100000, 110110010 both have three evenly spaced 1s, while 1011 does not.

   (a) Derive a brute-force algorithm solving this problem with $O(n^2)$ complexity.

   (b) Derive an algorithm with $O(n \log n)$ complexity that uses polynomial multiplication and convolutions.