

15-451/651 Algorithm Design & Analysis, Spring 2024

Extra Review Problems

Online Algorithms

1. (Short answer / multiple choice)

- (a) Consider the list update problem with the following sequence of $\text{Access}(x)$ operations on an array of length 4 (1-indexed): $[2, 1, 4, 3]$
 - i. What is the cost of these accesses using the move-to-front algorithm?
 - ii. What is the optimal cost of these accesses?

2. (Online Algorithms: Splay-Coin) 15-451 is creating a new cryptocurrency: the splay-coin. To garner interest in the coin, we are allowing each coin to be exchanged for 1 bonus point to your final grade.

Naturally, you would like to acquire 100 splay-coins by the last day of the n day semester, since you did not turn in any homework. On day i you check the price p_i of splay-coins, and decide whether to purchase. Unfortunately, due to a bug in the system, you can only purchase once, and want to minimize the cost when you do so.

Since cryptocurrency values are often highly volatile, 15-451 has also implemented some price controls on our coin, including a lower bound L and upper bound U on the price p_i at any day i .

- (a) Being the clever 15-451 student that you are, you'd like to use your algorithms knowledge to help you minimize the price you pay. Come up with a $\sqrt{U/L}$ -competitive algorithm for this problem.
- (b) Now just to make sure that none of your classmates can best you, prove that no deterministic algorithm can achieve a competitive ratio better than $\sqrt{U/L}$.