the adversary game

part 1:
Claude's Education Begins

Shall we play a game?

twenty questions...

I guess a number N between 1 and 2^{20}.
you ask yes/no questions of the form "is N <= blah",
and after 20 questions you must tell me what N is.

sure!

Heh, this Claude guy sounds like an easy target...

I will not think of a single N, of course.

Each time I will try to answer in such a way that
at the end, there are at least 2 numbers satisfying
all the answers I have given.

So whatever number he says at the end,
I can say N was the other number!
hee hee.

Initially, no answers given:
N could be any number in [1,2^{20}]

OK Claude, ask away...

Is N <= 500K?

Let's see. If I say "yes", N lies in [1, 500000],
else it lies in [500001, 2^{20} = 1048576].

I have more options if I go with "no"

No.

Is N <= 800K?

Hmm. If "yes", N lies in [500001, 800000].
else N lies in [800000, 1048576].

Now I have more options if I go with "yes".

Yes.

... and so on ...

after the 20th question...

The set of numbers [561382, 583841]
all satisfy the answers I have given so far.

is it 575757?

No. Sorry, Claude, you lose,
it was 575758!

rats!

Hmm, I see what he did there.
He did not really fix a single "true" N...

He answered so that
the "set of valid N's" was at least
half of the previous set.

Since at least two valid Ns remained, I lost.

So my best bet is to halve the set of valid N's with each question!

(I messed up earlier: I did not halve each time.)

Let me call this "halving" search

Actually, "**binary**" search sounds better...

btw, can I win if I do binary search?

At the start, there are 2^{20} possible N's
Each time I will exactly halve this range.
(and I can exactly halve each time)

So after 20 questions I can get the range down to
a single value of N. And win.

Hmm. Also, 19 questions would not have been enough to find a number in [1,2^{20}].

(unless the adversary is silly, which he is not.)

This sounds like a general principle.

ok, adversary, let's play again

Initially, no answers given:
N could be any number in [1,2^{20}]

OK Claude, ask away...

Is N <= 2^{19} = 524288?

If I say "yes", N lies in [1, 2^{19}],
else it lies in [2^{19}+1, 2^{20}].

I have the same # of options (2^{19})
I can go with "no"

No. N > 2^{19}.

Is N <= 2^{19} + 2^{18}?

Hmm. If "yes", N lies in [2^{19}, 2^{19}+2^{18}].
else N lies in [2^{19}+2^{18}, 2^{20}].

Equal number of options again. I can go with either answer. Let me choose one.

Yes.

... and so on ...

ok, adversary, after these 20 questions there's a unique number N consistent with your answers.

I win.

You just got lucky...

after a few games...

this Claude kid's sharp!

OK, ok, enough with these childish games, young Claude.

How about we play a real game?

tell me...

I think of integer N in [1, 2^{20} + 1]...

But now you can ask me any twenty
yes/no questions about N.

After 20 questions you must tell me what N is.

E.g., you can ask me:
Is N in the set {7,19,63,256,1079993}
or is 2^{2^N} – 1 a prime number?
or is the 7th bit of N set?
or are (N,N+2) a Goldbach pair?
whatever yes/no questions you want.

So I expand my range of N's by just one.

And give you the vast power to ask any possible yes/no questions

Should we play?

should Claude play?

why/why not?

Where will these games end?

stay tuned for the answers...

featuring

BSD "beastie" deamon	. . .	the adversary
Claude Elwood Shannon	. . .	Claude

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