

## 15-451/651 Algorithm Design & Analysis, Fall 2025

### Recitation #7

#### Objectives

- Understand the faster flow algorithms: Edmonds-Karp and Dinic's.
- Understand and apply Minimum-cost flows.

#### Recitation Problems

1. **(Super fast matching)** In lecture, we saw that Dinic's algorithm runs in  $O(n^2 m)$  time on any graph, but on some graphs it runs even faster! For instance, in a *unit-capacity* network, where every edge has capacity one, we proved that Dinic's algorithm runs in time  $O(m\sqrt{m})$ . In this problem we will take this one step further.

Suppose our graph has one additional restriction (we still keep the unit-capacity restriction): The net flow across every vertex (except  $s$  and  $t$ ) can be at most one. This is equivalent to saying that every vertex other than  $s$  and  $t$  has either indegree one or outdegree one (but not necessarily both). Such a network is called a "unit network".

- (a) Prove that in a unit network, the number of blocking flows required to find a max flow is at most  $O(\sqrt{n})$ . (Hint: use a similar argument to the one in lecture for unit-capacity graphs)
  - (b) Prove that we can solve the Bipartite Matching problem in  $O(m\sqrt{n})$  time.
2. **(Oral homework scheduling)** You've been hired to help the 451 TAs schedule their oral sessions. There are  $n$  TAs, and TA  $i$  has  $s_i$  oral sessions that they need to book a room for. There are  $m$  available room bookings (meaning time and location)  $\{1, 2, \dots, m\}$ , and each TA  $i$  has a list  $L_i \subseteq \{1, 2, \dots, m\}$  of which room bookings would be suitable for them.

Since there is a shortage of room bookings, however, the department has started to sell the bookings for money! The  $j^{\text{th}}$  room booking costs  $c_j$  dollars. Your job is to find a way to schedule all of the oral sessions such that each TA is able to schedule for *all* of their bookings, and such that there are *no* overlaps, for the minimum amount of money, or report that it is not possible.