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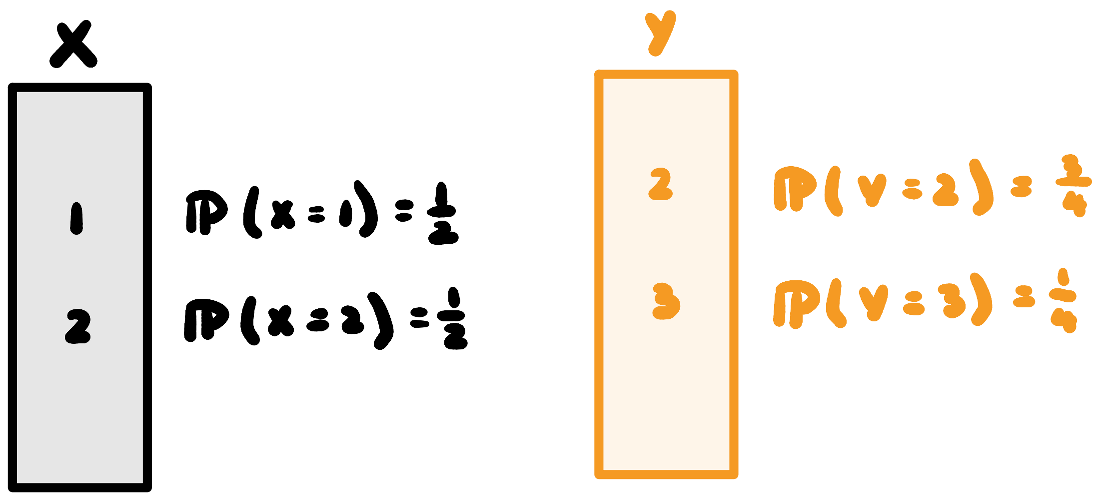

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X

X
1
2

$\mathbb{P}(X=1) = \frac{1}{2}$

$\mathbb{P}(X=2) = \frac{1}{2}$

Y

Y
2
3

$\mathbb{P}(Y=2) = \frac{3}{4}$

$\mathbb{P}(Y=3) = \frac{1}{4}$

KEY IDEA:

you can think of these r.v.'s as n-sided dies, with each side weighted by a probability.

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SMALL EXAMPLES SAVE THE DAY!

*you roll X, you roll Y and then you add the outcomes

X

1

2

$\mathbb{P}(X=1) = \frac{1}{2}$

$\mathbb{P}(X=2) = \frac{1}{2}$

Y

2

3

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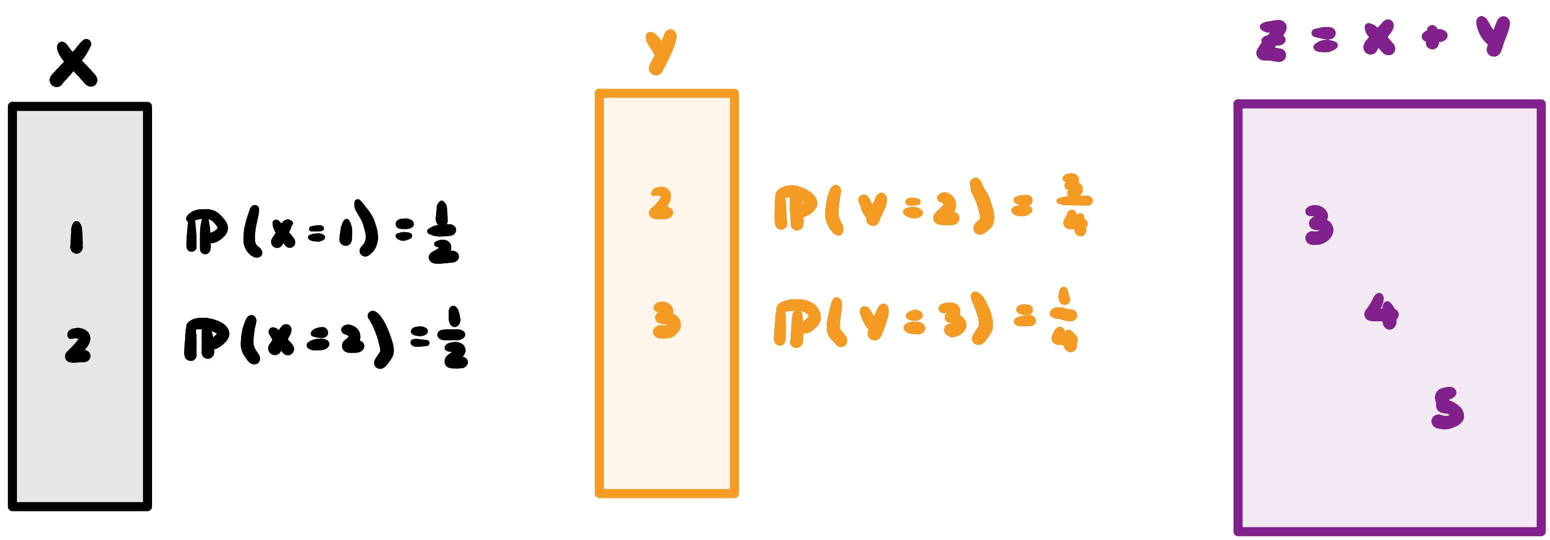
*what is the probability of each outcome?



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X: 1, 2

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$\mathbb{P}(X=2) = \frac{1}{2}$

Y: 2, 3

$\mathbb{P}(Y=2) = \frac{3}{4}$

$\mathbb{P}(Y=3) = \frac{1}{4}$

$Z = X + Y$: 3, 4, 5

$\mathbb{P}(Z=3) = \frac{3}{8}$

$\mathbb{P}(Z=4) = \frac{4}{8}$

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KEY OBSERVATION:

what process did you use to derive $\mathbb{P}(Z = z)$?

does it remind you of smth?

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1 $\quad \mathbb{P}(X = 1) = \frac{1}{2}$

2 $\quad \mathbb{P}(X = 2) = \frac{1}{2}$

Y

2 $\quad \mathbb{P}(Y = 2) = \frac{3}{4}$

3 $\quad \mathbb{P}(Y = 3) = \frac{1}{4}$

$Z = X + Y$

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POLYNOMIAL MULTIPLICATION

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2	$\mathbb{P}(X = 2) = \frac{1}{2}$

Y

Y	
2	$\mathbb{P}(Y = 2) = \frac{3}{4}$
3	$\mathbb{P}(Y = 3) = \frac{1}{4}$

$Z = X + Y$

Z	
3	$\mathbb{P}(Z = 3) = \frac{3}{8}$
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$\boxed{\text{POLYNOMIAL MULTIPLICATION}}$

*let's convert X and Y into polynomials and represent Z as their multiplication.

X

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2

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$\mathbb{P}(Y = 2) = \frac{3}{4}$

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$Z = X + Y$

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4
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we need <u>degrees</u> for our polynomial

and we need <u>coefficients</u>

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degrees coefs...

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X

degrees coefs...

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2 $\quad \mathbb{P}(x = 2) = \frac{1}{2}$

$$\frac{1}{2}a^1 + \frac{1}{2}a^2 = P_x(a)$$

Y

2 $\quad \mathbb{P}(Y = 2) = \frac{3}{4}$

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$$\downarrow$$

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Y

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3    $\mathbb{P}(Y = 3) = \frac{1}{4}$

$$\downarrow$$

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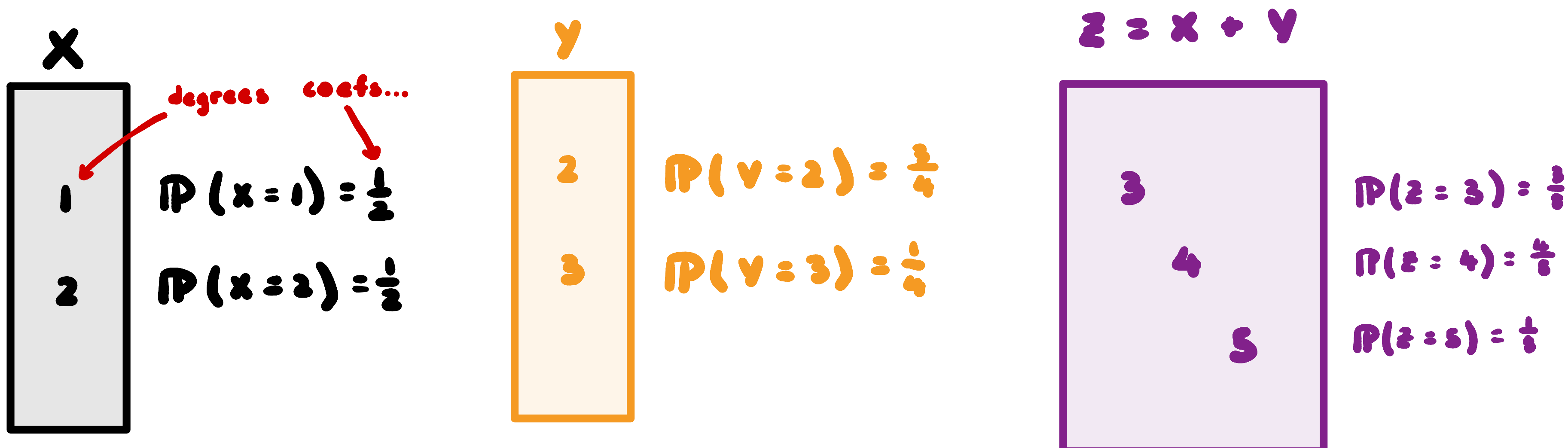

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$$\mathbb{P}(z = \ell) = c_\ell^z =$$

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$$\mathbb{P}(Z = \ell) = c_\ell^Z = \sum_{0 \le i \le \ell} c_i^X \cdot c_{\ell-i}^Y$$

from the defn of poly mult. in notes

$$\left(\frac{1}{2}a^1 + \frac{1}{2}a^2 = P_X(a)\right) \cdot \left(\frac{3}{4}a^2 + \frac{1}{4}a^3 = P_Y(a)\right) = \frac{3}{8}a^3 + \frac{4}{8}a^4 + \frac{1}{8}a^5 = P_Y(a)$$

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substitute terms

$$\left(\frac{1}{2}a^1 + \frac{1}{2}a^2 = P_x(a)\right) \cdot \left(\frac{3}{4}a^2 + \frac{1}{4}a^3 = P_y(a)\right) = \frac{3}{8}a^3 + \frac{4}{8}a^4 + \frac{1}{8}a^5 = P_y(a)$$

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2.

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  1. start index
  2. spacing

so what is the algorithm...?

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- try all possible combos of these... $O(n^2)$

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- the binary string!
  - coefs = 1 or 0
  - degree = index

ex. 1011

$$1x^0 + 0x^1 + 1x^2 + 1x^3$$
$$= x^0 + x^2 + x^3$$

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  - itself! ex. $1011 \rightarrow x^0 + 2x^2 + 2x^3 + x^4 + 2x^5 + x^6$

    $10101 \rightarrow x^0 + 2x^2 + 3x^4 + 2x^6 + x^8$

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(b) Derive an algorithm with $O(n \log n)$ complexity that uses polynomial multiplication and convolutions.

- the binary string!
  - coefs = 1 or 0
  - degree = index

ex. 1011

$1x^0 + 0x^1 + 1x^2 + 1x^3$

$= x^0 + x^2 + x^3$

- how does this help us? ... not so clear, let's keep exploring...
  - what should we multiply this poly by? (hint: 2-SUM)
  - itself! ex. 1011 $\longrightarrow x^0 + 2x^2 + 2x^3 + x^4 + 2x^5 + x^6$

00110 $\longrightarrow x^4 + 2x^5 + x^6$

0101 $\longrightarrow x^2 + 2x^4 + x^6$

10101 $\longrightarrow x^0 + 2x^2 + 3x^4 + 2x^6 + x^8$

10111 $\longrightarrow x^0 + 2x^2 + 2x^3 + 3x^4 + 2x^5 + 3x^6 + 2x^7 + x^8$

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* notice anything interesting about the magnitude of the coefs?

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* notice anything interesting about the magnitude of the coefs?

- NO $\rightarrow$ coefs ≤ 2
- YES $\rightarrow$ $\exists$ coefs ≥ 3

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* notice anything interesting about the magnitude of the coefs?

- NO $\rightarrow$ coefs ≤ 2
- YES $\rightarrow \exists$ coefs $\geq 3 \longrightarrow$ located at degree DOUBLE the center of ones!

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* notice anything interesting about the magnitude of the coefs?

- NO $\rightarrow$ coefs ≤ 2 * why is this happening?
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* notice anything interesting about the magnitude of the coefs?

- NO → coefs ≤ 2

  * suppose indices (i,j,k) are evenly spaced $\Rightarrow j+j = i+k = k+i$

- YES → $\exists$ coefs ≥ 3 → located at degree DOUBLE the center of ones!

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ex. 10111

$$(x^0 + x^2 + x^3 + x^4)$$

$$\cdot$$

$$(x^0 + x^2 + x^3 + x^4)$$

$$=$$

$$x^0 + 2x^2 + 2x^3 + 3x^4 + 2x^5 + 3x^6 + 2x^7 + x^8$$

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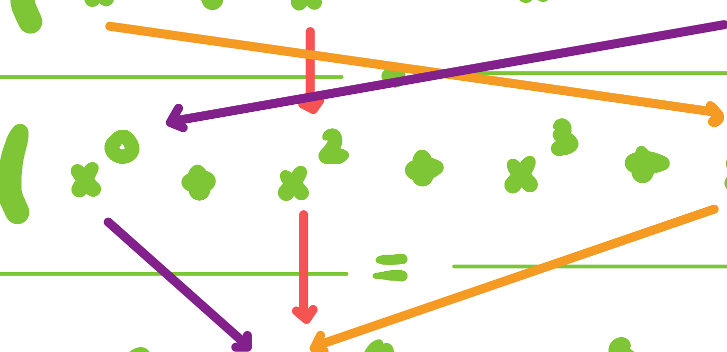
* these multiplications in the FOIL give rise to 3 (bc evenly-spaced)

ex. 10111



$$(x^0 + x^2 + x^3 + x^4)$$

$$(x^0 + x^2 + x^3 + x^4)$$

$$= x^0 + 2x^2 + 2x^3 + 3x^4 + 2x^5 + 3x^6 + 2x^7 + x^8$$

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*so, what's the algorithm?

1. make polynomial $P(x)$ for string $O(n)$
2. square it $O(n \log n)$
3. for degree $d \in P(x)$ *degrees with 1s
   1. check coefficient of $2d$ in $P(x)^2$. if ≥ 3, return TRUE
4. return FALSE

THANK YOU.

this is my last recitation ever...
y'all mean the world to me