15-451/651 Algorithm Design & Analysis

Fall 2022, Recitation #11

Objectives

- Practice common techniques for geometric algorithms.

Recitation Problems

1. **(Angular sorting without angles)** Recall that the first step of the *Graham scan* algorithm for convex hull that we learned in lectures is to sort the points with respect to their angle from the bottom-most point. The most straightforward way to do this is of course to simply compute the angles and then sort the points. This has some drawbacks, such as having to perform floating-point computations that are susceptible to rounding errors.

   Given a set of n points with integer-valued coordinates, describe how to sort them with respect to their angle to the bottom-most point without using any floating-point computations. (Hint: use the line-side test primitive).

2. **(Point-in-convex-polygon)** Given a point q and a convex polygon P represented by points $P[1], P[2], \ldots, P[n]$ in counter-clockwise order

   (a) Determine whether q is in the polygon P in time $O(n)$.

(b) Speed up your algorithm to $O(\log n)$

3. **(Width of a Set of Points)** You're given a set $S = \{p_1, \ldots, p_n\}$ of n points in the plane. A strip of width w is the region between two parallel lines, where the distance between the two lines is w. The goals is to find the strip of minimum width that contains all the points.

   (a) Give an $O(n^2)$ algorithm for this problem.

   (b) Give an $O(n \log n)$ algorithm for this problem.

4. **(AlphaTensor)** DeepMind recently excited a lot of people by using machine learning to find an algorithm to multiply 4×4 matrices using 47 multiplications.

   Using this discovery as a structure, what is the time complexity of the resulting matrix multiplication algorithm on $n \times n$ matrices? How does this compare to Strassen's algorithm?