1 ANGULAR SORTING (w/o angles)

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list of points
$[(x_1, y_1), (x_2\ y_2) \cdots$

how do you determine whether one point is "bigger"?
(now to compare 2 points?)

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currently: here is our comparator

COMPARE (A, B):
1. calculate $\angle AQ = \alpha$
2. calculate $\angle BQ = \beta$
3. return $\alpha \geq \beta$

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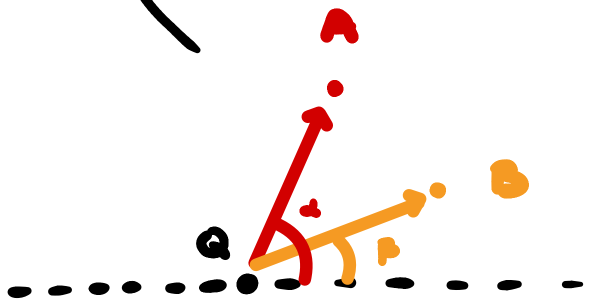

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what is another
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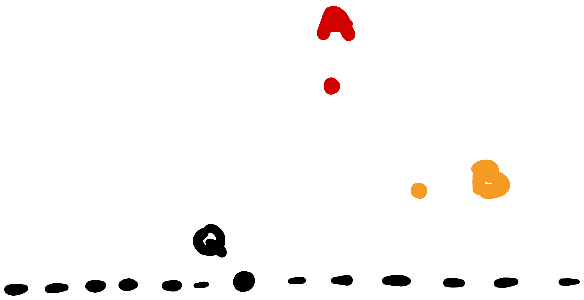

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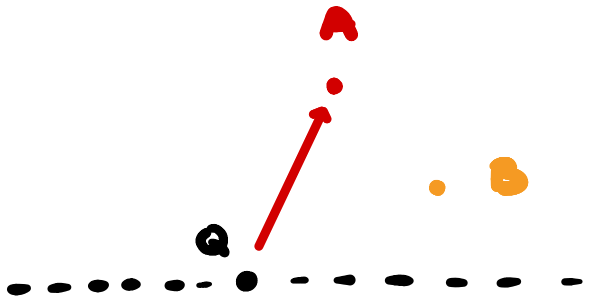
is B left or right of $\overrightarrow{QA}$



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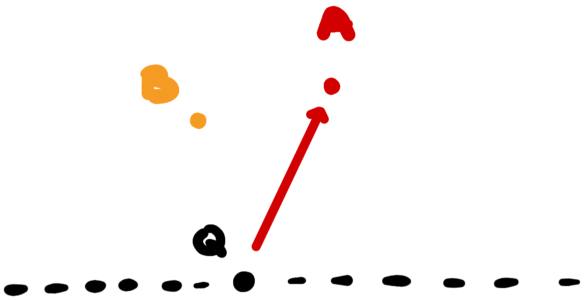

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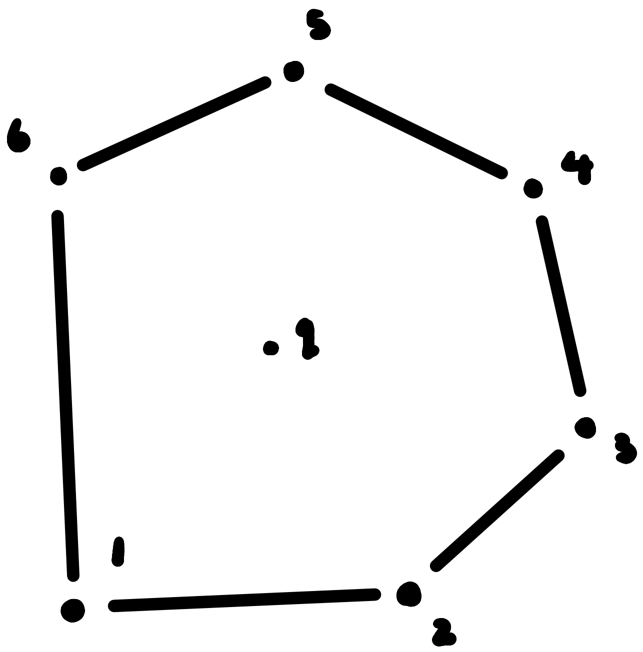
2 POINT IN POLYGON

2. **(Point-in-convex-polygon)** Given a convex polygon P represented by points $P[1], P[2], \ldots, P[n]$ in counter-clockwise order, and a point q

   (a) Determine whether q is in the polygon in time $O(n)$.



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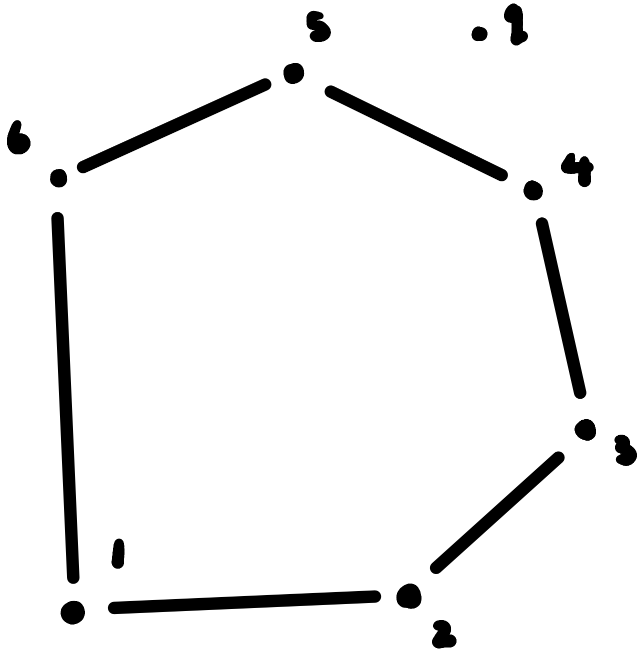

KEY STRATEGY:

use line side tests as your primative operation

CLAIM

a directed segment btw a pair of points (p_i, p_j) is on the convex hull

$\Leftrightarrow$

all other points p_k are to the left of the ray: $\overrightarrow{p_i p_j}$

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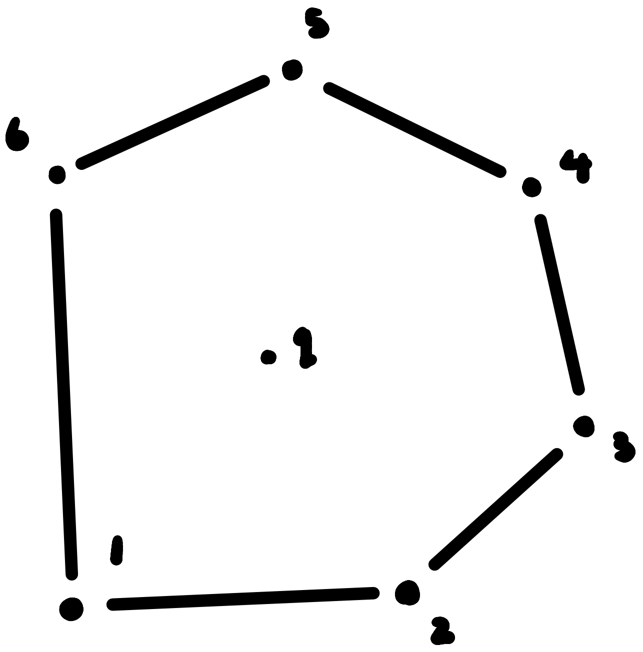

*we can iterate thru $\overrightarrow{P[i]\,P[i+1]}$ and ensure q is always left

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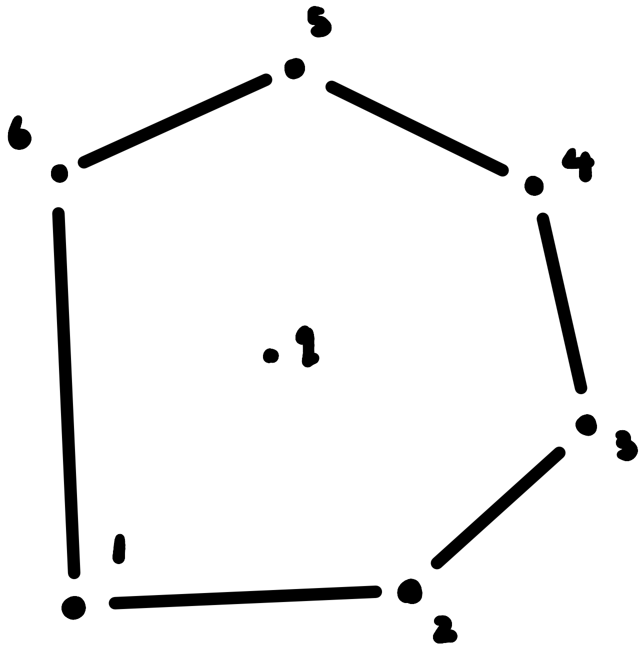
- binary search will be involved somewhere

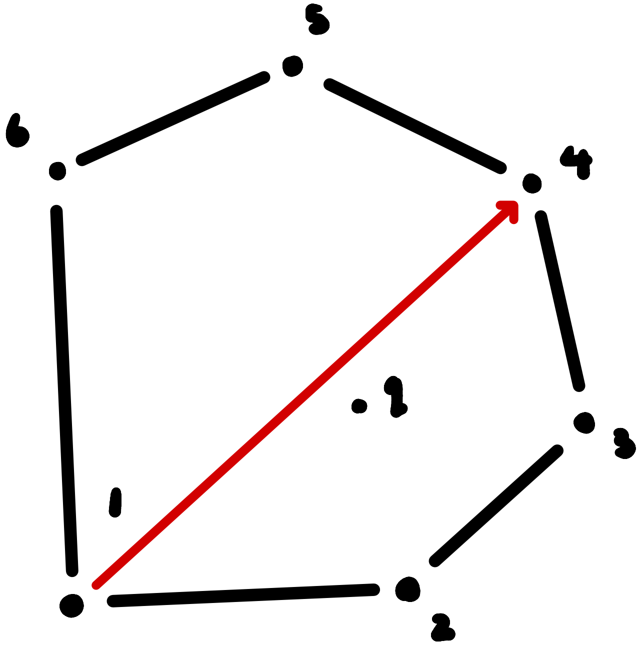
- line-side tests

- let's try to combine these
  to start...



(b) Speed up your algorithm to $\underline{O(\log n)}$

* search for closest/smallest region q is in



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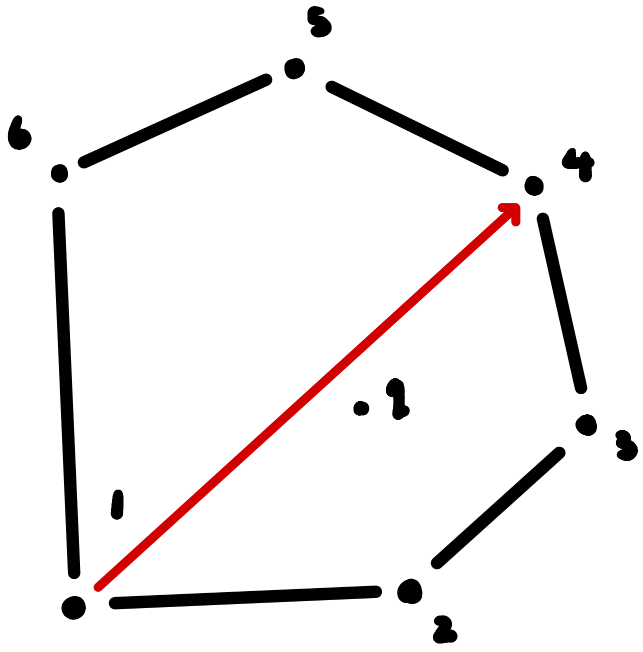
*we don't have to check....

- $\overrightarrow{P_6 P_1}$
- $\overrightarrow{P_5 P_6}$
- $\overrightarrow{P_4 P_5}$

... knowing the result of this line-side test



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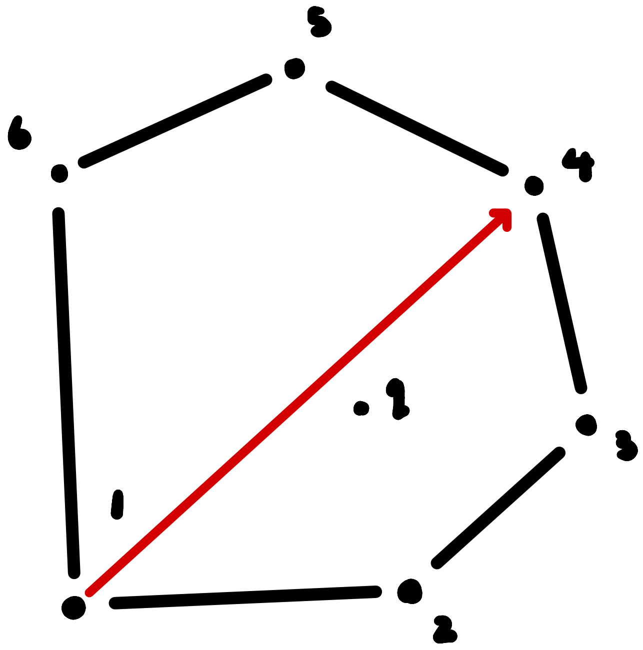

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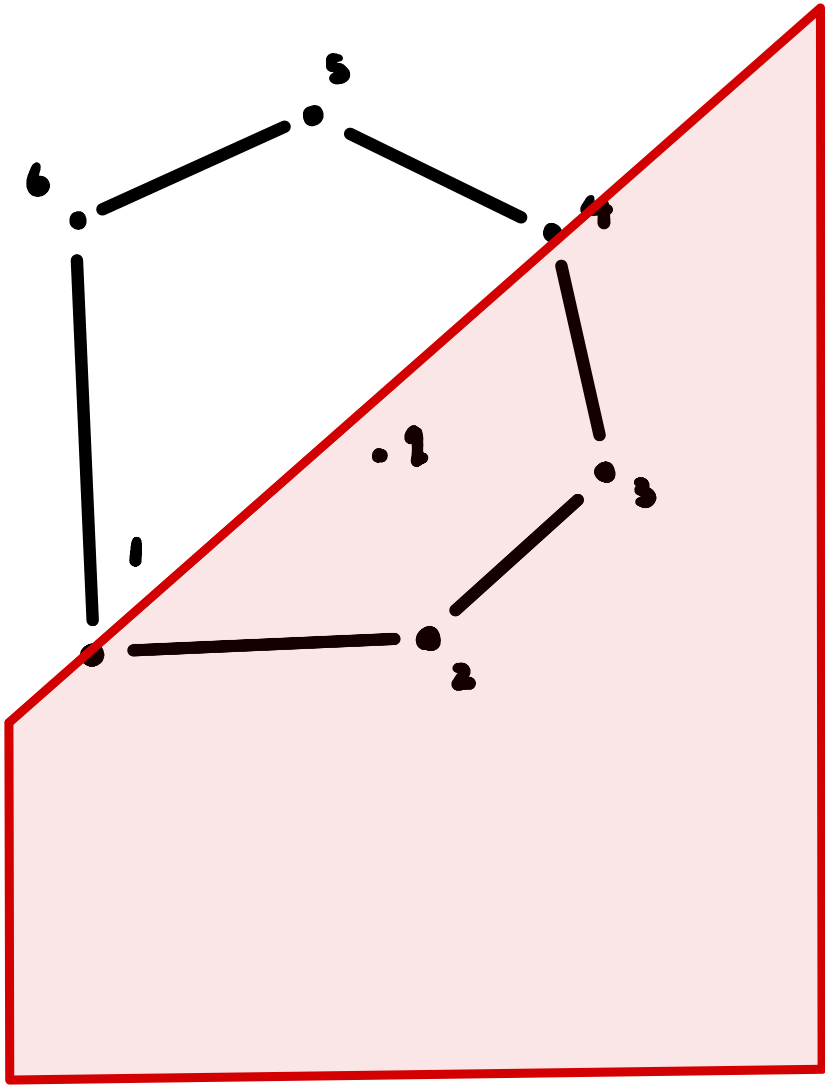
KEY IDEA:

use binary search to narrow down the polygon segments we need to check...

aka narrow the region that q is in.



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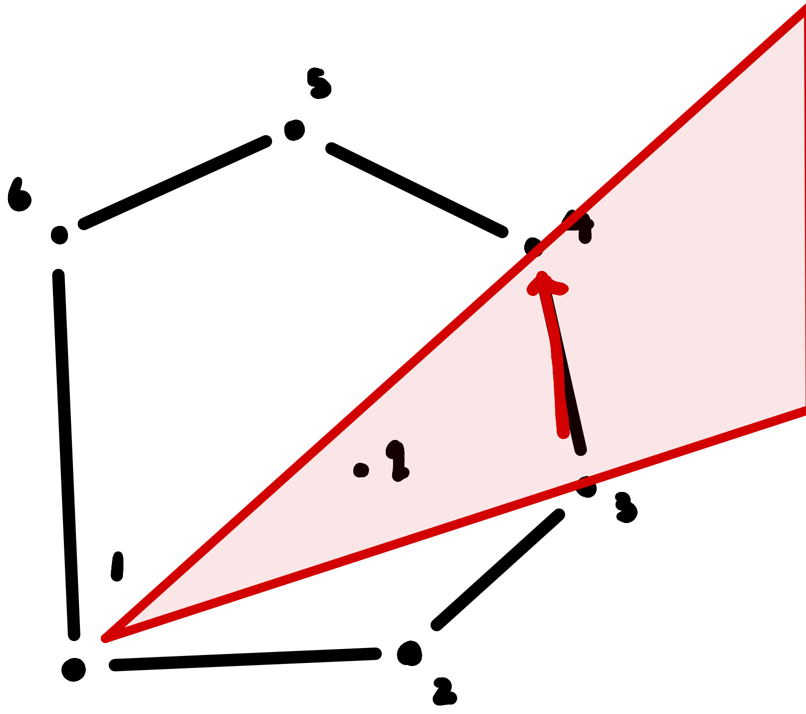
* we have the triangle where q must live thru binary search....
(narrowed down polygon edges to just 1) :)



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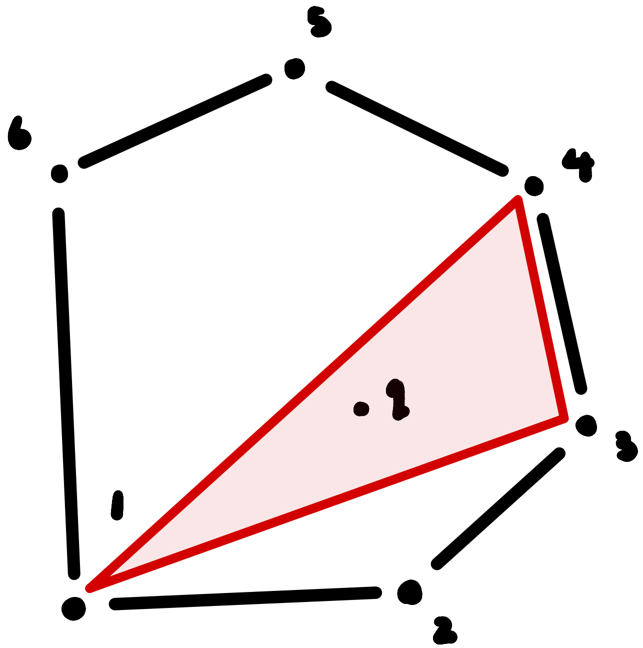

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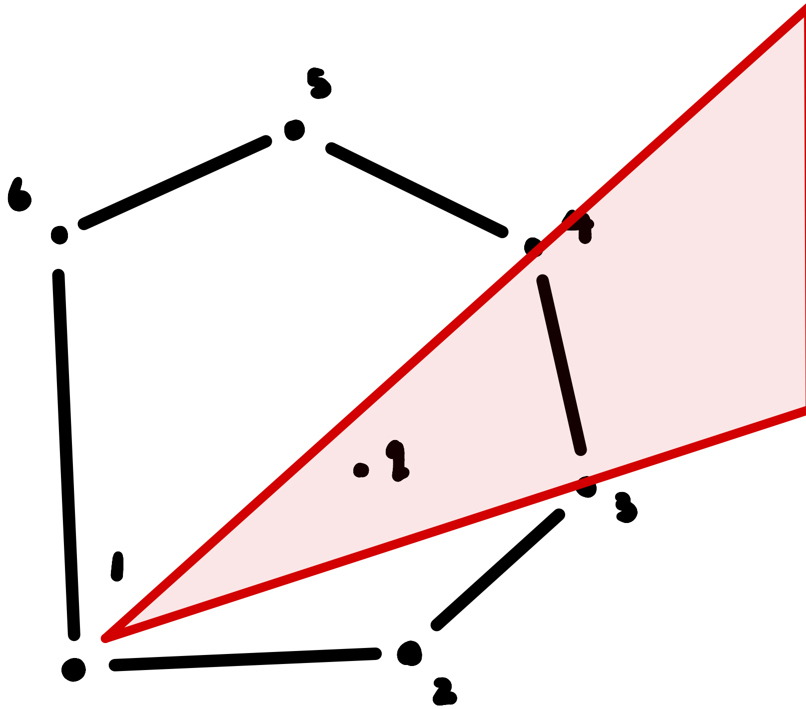

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0. the formal alg does $\overrightarrow{P_1P_2}$, $\overrightarrow{P_6P_1}$ LST to avoid edge cases
1. binary search for the triangle slice that q is in
2. check whether inside or out on that slice

<u>NOTE</u>: start point doesn't matter