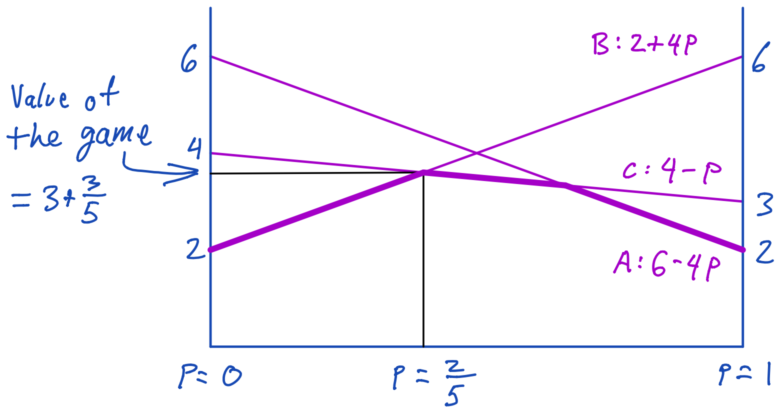
Solving 2-Row Games

In this section we give a general method for solving games with two rows (or, by symmetry, two columns). We'll develop the method with the following example:

		A	B	C
p	1	2	6	3
1−p	2	6	2	4



The game we'll analyze is shown in the 2×3 matrix above with values for the row player. We'll call the three options for the column player A, B, and C, and the two options for the row player 1 and 2.

The general mixed strategy for the row player is to choose option 1 with probability p and option 2 with probability $1 - p$.

In the diagram above, the horizontal axis represents p, and the vertical axis represents the payoff to the row player. The three lines in the diagram correspond to the three options for the column player. For example, the line labeled B, which is the graph of the function $2 + 4p$, is the expected value of the game for the row player (as a function of p) if the column player chooses column B.

The lower envelope of the three options for the column player is highlighted. This concave function represents the expected pay-off for the row player if he chooses option 1 with probablility p, assuming that the column player plays optimally knowing the value of p. Thus, it represents a lower bound on the value of the game for the row player for each value of p.

By inspection of this graph, the concave function achieves its maximum value at the point of intersection between lines B and C. This point is $p = \frac{2}{5}$, and with that choice of p the value of the game is $3 + \frac{3}{5}$. That's the lower bound on the game's value.

What's a good strategy for the column player? Consider the convex combination of B and C chosen such that the result is a horizontal line. This is $\frac{4}{5}C + \frac{1}{5}B$. With this mixed strategy, the value of the game for the row player is $3 + \frac{3}{5}$, no matter what the row player does. So this is an upper bound on the value of the game for the row player. Since the lower and upper bound are equal, we know that this is the value of the game.

The same technique can be applied to any game with two rows. The value of the game is obtained by constructing the concave function and finding where it achieves its maximum.