15-251: Great Theoretical Ideas In Computer Science

Recitation 9

Quiz

Note that there is a quiz at the beginning of lecture tomorrow, which will cover graph theory and automata.

Graph Theory

1. Show that every graph G has a (simple) path of length δ, where δ is the minimum degree over all the vertices of G. A path is called simple if it does not contain any repeated vertices, and the length of a path is the number of edges in the path.

   Show that every graph has a cycle of length at least $\delta + 1$.

2. Prove that in a planar bipartite graph G with n vertices, there are at most $2n - 4$ edges.

DFAs

3. Which of the following statements about regular languages are true? For every language claimed to be regular, contruct a DFA. For every language claimed to be non-regular, give a proof.

   (a) The empty set is a regular language.

   (b) The set of all strings over an alphabet Σ is a regular language.

   (c) If L is a regular language, then $L^c = \{w \mid w \notin L\}$ is a regular language.

   (d) If L_1 is a regular language, and Ł$_2 \subset L_1$, then L_2 is regular.

   (e) If L_1 is a regular language, and Ł$_1 \subset L_2$, then L_2 is regular.

   (f) If $\Sigma = \{a, b, c\}$, then the language $L = \{w \mid$the number of occurrences of $ab =$ the number of occurrences of $ba\}$ is regular.

   (g) Fix the alphabet $\Sigma = \{(,)\}$ (the symbols are the two parentheses). Show that the language $L' = \{w \in \Sigma^* \mid$ the parentheses in w match$\}$ is regular.

4. For all of the below, the alphabet is $\Sigma = \{0, 1\}$. For each one, construct a DFA that accepts the language.

   (a) $\{w \in \Sigma^* \mid w$ begins with a 1 and ends with a 0$\}$

   (b) $\{w \in \Sigma^* \mid w$ contains at least three 1s$\}$

   (c) $\{w \in \Sigma^* \mid w$ contains the substring "0101"$\}$

   (d) $\{w \in \Sigma^* \mid$the length of w is at least 5$\}$

   (e) $\{w \in \Sigma^* \mid w$ is any string except "11" or "111"$\}$

   (f) $\{w \in \Sigma^* \mid w$ contains at least two 0's and at most one 1$\}$