

# 15-251: Great Theoretical Ideas In Computer Science

---

## Recitation 3

### Exam 1

Remember that there is a one-hour exam next week, 9/28, during recitation. It will cover all material done in Lectures 1-8. Please look over lectures, recitations and HWs solutions we have posted. A practice test will be posted soon.

### Partitions

1. Prove that the number of partitions of  $n$  with  $k$  parts is the same as the number of partitions of  $n$  whose largest term is exactly  $k$ .

**Example:** Letting  $n = 4$  and  $k = 2$ , there are two partitions with two parts  $3 + 1, 2 + 2$  and two partitions where the largest term is two  $2 + 2, 2 + 1 + 1$ .

2. Prove that the number of partitions of  $n$  with only odd terms is the same as the number of partitions of  $n$  with distinct terms.

**Example:** Letting  $n = 6$ ,  $\{(1 + 1 + 1 + 1 + 1 + 1), (3 + 1 + 1 + 1), (3 + 3), (5 + 1)\}$  is the set of partitions which have only odd terms;  $\{(6), (5 + 1), (4 + 2), (3 + 2 + 1)\}$  is the set of partitions which have distinct terms.

### Generating Functions

1. What is the power series for  $\frac{1}{1-11x+30x^2}$ ?
2. Let  $v_n = \langle 1, 2, 4, 8, 16, \dots \rangle$ . Determine the generating function for this sequence.
3. There are  $p \geq 3$  pirates. Let  $c_n$  be the number of ways to distribute  $n$  pieces of gold to the pirates such that pirate 1 gets an even number of pieces, pirate 2 gets at least 5 pieces, and pirate 3 gets at most 3 pieces, and all other pirates can be given any number of gold pieces. Determine the generating function  $C(z) = \sum_{i=0}^{\infty} c_i z^i$ . Use this to derive a closed form for  $c_n$ .
4. Let  $a$  be the sequence  $\langle a_0, a_1, a_2, \dots \rangle$ , and  $b$  be the sequence  $\langle b_0, b_1, b_2, \dots \rangle$ . If  $A(x)$  and  $B(x)$  are the generating functions for  $a$  and  $b$ , find a generating function for the sequence  $c = \langle a_0, b_0, a_1, b_1, a_2, b_2, \dots \rangle$ .