15-251: Great Theoretical Ideas In Computer Science

Homework 2 (due Thursday, September 10)

Directions: *Write up carefully argued solutions to the following problems. The first task is to be complete and correct. The more subtle task is to keep it simple and succinct. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use any results proven in lecture without proof. Anything else must be argued rigorously. Unless otherwise specified, all answers are expected to be given in closed form.*

0. Warmup (0 points)

Warmup problems need not be turned in.

a. Two players decide to play a game of nim but each player may keep the chips they have removed and may decide to add chips to a pile instead of removing them. Analyze this game.

1. Revised Take-Away Game (20 points)

Two players decide to play the following game: The game starts with a pile of m coins. The first player can remove anywhere from 1 to n coins. At each step, the next player removes some non-zero number of coins, but this number must be less than or equal to the number of coins removed by her opponent in the preceding move.

a. *(5 points)* Suppose $n = 4$ and the game is following normal rules. Find all values of m such that the game is a starting P position.

b. *(10 points)* Devise a simple algorithm that takes as input a value of n and determines all P-positions for that value of n, under normal rules.

c. *(5 points)* Change the previous algorithm to make it work under misère rules.

2. Tainted Chocolate (20 points)

Alice and Bob are given a two-dimensional $m \times n$ rectanglular grid made of delicious chocolate, and they are informed that the 1×1 square at location (j, k) is poisoned. Here m, n, j, k are all integers. The two alternate taking moves: at each step, the player is allowed to break the remaining rectangle along some axis-aligned line into two integer sub-rectangles, and to eat the one that doesn't contain the poisoned location.

a. *(15 points)* Determine all N and P positions of the game under normal rules (i.e., the last player who is able to cut wins).

b. *(5 points)* Determine all N and P positions of the game under misère rules (i.e., the last player who is able to cut loses).

3. A Circle of Sixpence (20 points)

This time, instead of making a pile of m coins, you place them in a circle. In each step, a player can remove either 1,2, or 3 *consecutive* coins. (These should be consecutive with respect to the original position: e.g., if the circle was a, b, c, d, e and you remove c, then b, d cannot be removed in the same step, but a, b, e can.)

a. *(10 points)* Under misère rules, who has a winning strategy when $m = 10$?

b. *(10 points)* Under normal rules, determine all starting positions which are N positions.

4. Pawn only Chess (20 points)

Two players decide to play the following game. They have a $3 \times n$ chessboard. Initially, the first row has all the black pawns, and the third row has all the white pawns. Each player may make any pawn move, but she must make a capture move if such a move is possible. White moves first, and the first player unable to make a move loses.

a. *(10 points)* Suppose $n = 12$, who has a winning strategy?

b. *(10 points)* Suppose $n = 251$, who has a winning strategy?

Note: In case you are not familiar with the movement of pawns, check out
http://en.wikipedia.org/wiki/Pawn_(chess)

5. Keep Moving Left (20 points)

Two players play the following game on a $1 \times N$ board. Initially, there are n stones placed on different squares of this board. Each player can move a stone any number of spaces to the left (the players are sitting on the same side of the board), but may not jump over other stones or land on top of them. What are the P and N positions of this game (under normal rules)?