Introduction to Games

15-150

Principles of Functional Programming

Some Slides for Lecture 20

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Modular Framework for the following kinds of games:

- **2-player** (alternate turns)
- **deterministic** (no dice)
- **perfect information** (no hidden state)
- **zero-sum** (I win, you lose; ties ok)
- **finitely-branching** (maybe even finite)
Modular Framework for the following kinds of games:

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- finitely-branching  (maybe even finite)
- Examples: tic-tac-toe, connect4, …
Example: Nim

• Take 1, 2, or 3 pieces of chocolate
• Alternate turns
• Player who leaves an empty table loses
Game Trees

- **Nodes** represent current state of game
- **Edges** represent possible moves
- A given **level** corresponds to a given player, alternating turns

- Our players: Maxie and Minnie
A Nim Game Tree

4

Start with 4 pieces of chocolate
A Nim Game Tree

MAXIE moves first

4
A Nim Game Tree

MAXIE

3

take 1

4
A Nim Game Tree

MAXIE

Minnie moves

take 1

4

3
A Nim Game Tree

MAXIE

Minnie

2

3

4

take 1
take 1
A Nim Game Tree

MAXIE

Minnie

2 MAXIE moves

3 take 1

4 take 1
A Nim Game Tree

MAXIE

Minnie

MAXIE

1

2

3

take 1
take 1
take 1

4
A Nim Game Tree

MAXIE

Minnie

MAXIE

Minnie moves

take 1
A Nim Game Tree

MAXIE

Minnie

MAXIE

Minnie
A Nim Game Tree

MAXIE

Minnie

MAXIE

Minnie

MAXIE moves
A Nim Game Tree

Maxie wins!
Nim game tree with leaf values

Means Maxie wins, assign value +1

Means Minnie wins, assign value -1
Now compute interior node values:

- **MAXIE** means Maxie wins, assign value $+1$
- **Minnie** means Minnie wins, assign value $-1$
Now compute interior node values:

- **Maxie** wins, assign value +1
- **Minnie** wins, assign value -1

**Diagram Description:**

- Purple circle with +1 means Maxie wins.
- Green circle with -1 means Minnie wins.

**Diagram Nodes:**

- **MAXIE**
- **Minnie**
- **MAXIE**
- **Minnie**
- **MAXIE**

**Values:**

- +1, -1
Now compute interior node values:

- A purple circle means Maxie wins, assign value +1.
- A green circle means Minnie wins, assign value -1.
Maxie can win!

The other two initial Maxie moves would allow Minnie to win.
Estimators

• In practice, trees are too large to visit leaves.
• Instead:
  – expand tree to some depth,
  – use game-specific estimator to assign values (not just ±1) at bottom-most nodes explored.
• Backchain mini-max values as before.
• Repeat after each actual move.
• Issue: horizon effect.
Nim has perfect estimator

Player making move can win for sure iff

$$n \text{ mod } 4 \neq 1$$

(n is number of chocolates)
Modular Framework

- **Game**: GAME (e.g., Nim : GAME)
- **Player**: PLAYER (includes a Game)
- **Referee**: GO (glues 2 Players to play)

- Will have automated and human players.
- Will write automated players as functors that expect a Game. Code plays without knowing Game details, except implicitly via estimator.