Introduction to Games

15-150
Principles of Functional Programming
Some Slides for Lecture 20
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Modular Framework for the following kinds of games:

- **2-player** (alternate turns)
- **deterministic** (no dice)
- **perfect information** (no hidden state)
- **zero-sum** (I win, you lose; ties ok)
- **finitely-branching** (maybe even finite)
Modular Framework for the following kinds of games:

• 2-player  (alternate turns)
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• finitely-branching  (maybe even finite)
• Examples:  tic-tac-toe, connect4, …
Example: Nim

- Take 1, 2, or 3 pieces of chocolate
- Alternate turns
- Player who leaves an empty table loses
Game Trees

- Nodes represent current state of game
- Edges represent possible moves
- A given level corresponds to a given player, alternating turns

- Our players: Maxie and Minnie
A Nim Game Tree

Start with 4 pieces of chocolate
A Nim Game Tree

MAXIE moves first

4
A Nim Game Tree

MAXIE

3

4

take 1
A Nim Game Tree

MAXIE

Minnie

moves

take 1

4

3
A Nim Game Tree

MAXIE

Minnie

2 → 3 → 4

take 1

take 1
A Nim Game Tree

MAXIE

Minnie

2

3

take 1

take 1

MAXIE moves

4
A Nim Game Tree

MAXIE

Minnie

MAXIE

1

2

3

take 1
take 1
take 1

4
A Nim Game Tree

MAXIE

Minnie

MAXIE

Minnie moves
A Nim Game Tree

MAXIE

Minnie

MAXIE

Minnie

0

take 1

take 1

take 1

take 1

1

take 1

take 1

2

take 1

3

take 1

4
A Nim Game Tree

MAXIE

Minnie

MAXIE

Minnie

MAXIE moves
A Nim Game Tree

Maxie wins!
A Nim Game Tree

MAXIE

Minnie

MAXIE

Minnie

Maxie wins!

Maxie wins!

Minnie wins!

Minnie wins!

Maxie wins!

Maxie wins!

Minnie wins!

Minnie wins!
Nim game tree with leaf values

- Purple circles mean Maxie wins, assign value $+1$
- Green circles mean Minnie wins, assign value $-1$
Now compute interior node values:

- Red circle: Means Maxie wins, assign value +1
- Green circle: Means Minnie wins, assign value -1
Now compute interior node values:

- Red circles mean Maxie wins, assign value +1.
- Green circles mean Minnie wins, assign value -1.
Now compute interior node values:

- Purple circle means Maxie wins, assign value +1.
- Green circle means Minnie wins, assign value -1.
Maxie can win!

The other two initial Maxie moves would allow Minnie to win.
Estimators

• In practice, trees are too large to visit leaves.
• Instead:
  – expand tree to some depth,
  – use game-specific estimator to assign values (not just ±1) at bottom-most nodes explored.
• Backchain mini-max values as before.
• Repeat after each actual move.
• Issue: horizon effect.
Nim has perfect estimator

Player making move can win for sure iff

\[ n \mod 4 \neq 1 \]

(n is number of chocolates)
Modular Framework

• **Game** : GAME (e.g., Nim : GAME)
• **Player** : PLAYER (includes a Game)
• **Referee** : GO (glues 2 Players to play)

• Will have automated and human players.
• Will write automated players as functors that expect a Game. Code plays without knowing Game details, except implicitly via estimator.