Modules II

15-150
Lecture 17: October 30, 2025

Stephanie Balzer
Carnegie Mellon University

Announcement: midterm II

When and where:

- Thursday, **November 6**, **11:00am—12:20pm**.
- **PH 100**.

Scope:

- Lectures: 1—15.
- Labs: 1—8 and midterm review section of Lab 10.
- Assignments: up to including Exceptions/Regex.

What you may have on your desk:

- Writing utensils, something to drink/eat, tissues.
- 8.5” x 11” cheatsheet (back and front), handwritten or typeset.
- No cell phones, laptops, or any other smart devices.

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Be on time; next lecture starts at 12:30pm!

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- Allows us to hide implementation details from the client.
  - Representation independence: the client becomes independent of the choice of internal representation.
  - Any two implementations that satisfy specifications are indistinguishable to the client and thus equal.
- Facilitates modular reasoning (component-wise reasoning).

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SML modules facilitate abstraction:

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- Specification: **signature**.
- Implementation: **structure**.

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SML modules allow us to control the “flow of information”:

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- **Transparent ascription**: for undefined type specified in signature, **representation type** chosen by structure is **revealed**.

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SML modules allow us to control the “flow of information”:

- Structures can **hide** auxiliary, implementation-specific components, not specified by signature.
- **Transparent ascription**: for undefined type specified in signature, **representation type** chosen by structure is **revealed**.
- **Opaque ascription**: for undefined type specified in signature, **representation type** chosen by structure is **hidden**.

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- Let's resume our dictionary example!

Example: dictionary

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Let's use strings as keys for now

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Permit value type to be polymorphic

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```

Replace entry, if key already exists

Search tree representation of dictionary

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Implementation: **represent** dictionary as a binary search tree, where

`(key, value)`

are stored in nodes.

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  - Tree must be sorted.
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  - (Similarly for key uniqueness.)

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structure BST : DICT =
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Transparent ascription can be useful for debugging purposes.

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forced by signature

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Because datatype is not declared in signature, constructors (and thus pattern matching) are not available outside signature.

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explore next!

Search tree representation of dictionary

Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e’ as (k’,_), rt),
            e as (k,_)) =
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Layered pattern matching

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Replace existing entry with new one

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       | GREATER => Node(lt, e', insert(rt,e)))
```

Search tree representation of dictionary

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```
(* lookup : 'a dict -> key -> 'a option *)
fun lookup tree key =
 let
    fun lk (Empty) = NONE
    | lk (Node(left, (k,v), right)) =
      (case String.compare(key,k) of
        EQUAL => SOME(v)
        | LESS => lk left
        | GREATER => lk right)
  in
    lk tree
  end
```

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The binding for **d** will be revealed because of opaque ascription. However, because the tree datatype is not declared in the signature, a client cannot pattern match on its constructors.

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Now consider:

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Now consider: `val look = BST.lookup d`

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BST.key -> int option
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          BST.empty,(“a",1)),("b",2)),("c",3))
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What is the type of **d**?

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int BST.dict
```

Now consider: `val look = BST.lookup d`

What is the type of **look**?

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BST.key -> int option
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Now consider:

Search tree representation of dictionary

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Let's reconsider our DICT signature

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signature DICT =
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  type key = string                     (* concrete type *)
  type 'a entry = key * 'a              (* concrete type *)
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  val empty : 'a dict
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How to implement now?

- What if we needed keys other than strings?
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How to implement now?

Used `String.compare`

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signature DICT =
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  type 'a key = 'a                        (* concrete type *)
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How to implement now?

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- What if we needed keys other than strings?
- We could try to make key polymorphic too.
- Keys should become comparable!

Let’s reconsider our DICT signature

```
lookup:

insert:
```

Keys should become comparable!

Let’s reconsider our DICT signature

```
lookup: ('a *'a -> order) -> ('a,'b) dict -> ‘a ->'b option

insert: ('a *'a -> order) -> (('a,'b) dict * ('a,'b) entry)
                          -> ('a,'b) dict
```

Keys should become comparable!

Let’s reconsider our DICT signature

```
lookup: ('a *'a -> order) -> ('a,'b) dict -> ‘a ->'b option

insert: ('a *'a -> order) -> (('a,'b) dict * ('a,'b) entry)
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- Keys should become comparable!
- Require a comparison function as an argument.

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lookup: ('a *'a -> order) -> ('a,'b) dict -> ‘a ->'b option

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```

- Keys should become comparable!
- Require a comparison function as an argument.
- Restricts polymorphism of keys!

Let's update our BST structure accordingly

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```
structure BST : DICT =
struct
  type 'a key = ‘a

  type ('a,'b) entry = 'a key * 'b

  datatype ('a,'b) dict = Empty | Node of
             ('a,'b) dict * ('a,'b) entry * ('a,'b) dict

  val empty = Empty

  fun insert cmp d k =

  fun lookup cmp (d, k) =
end
```

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As specified by signature

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Again, binary search tree as representation type.

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Again, binary search tree as representation type.

This time, with polymorphic key.

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As before.

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Bodies of `insert` and `lookup` now use `cmp` instead of `String.compare`.

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Does this do the trick?

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Does this do the trick? Well, not quite.

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- Does this do the trick? Well, not quite.
- What if a client provides different `cmp` functions to `insert` than to `lookup`, for example?

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For example, a client creates the following tree using `insert` and `Int.compare`:

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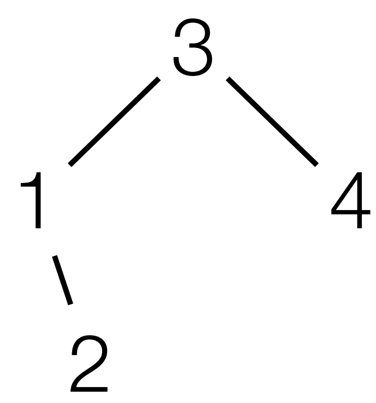

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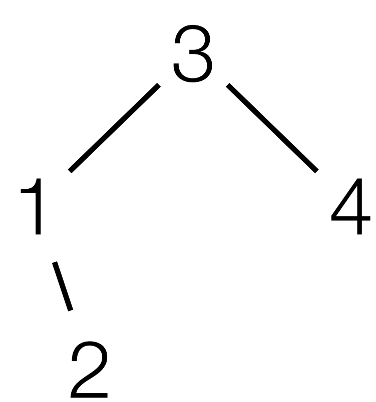

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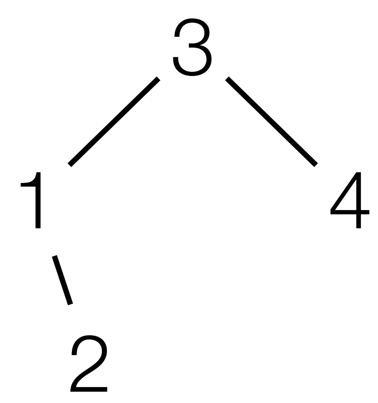

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- Can we enforce the invariant, that all operations use the same comparison function by typing?

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- Does this do the trick? Well, not quite.
- What if a client provides different `cmp` functions to `insert` than to `lookup`, for example?
- Can we enforce the invariant, that all operations use the same comparison function by typing?
- Yes, but we need type classes for this!

Type classes

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A signature specifying a type and associated operations.

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  - No expectation that specification is exhaustive.

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Example:

```
signature ORDERED =
sig
  type t       (* parameter *)
  val compare : t * t -> order
end
```

Type classes

Type class

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  - No expectation that specification is exhaustive.

Example:

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signature ORDERED =
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  type t      (* parameter *)
  val compare : t * t -> order
end
```

Signature **`ORDERED`** specifies an “ordered type class” to consist of a type **`t`** along with a comparison function **`compare`** for **`t`**.

Type classes

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Even though `t` is not concrete, it is not abstract.

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We expect `t` to be some already existing type, hence use the comment **parameter**.

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- Signature `ORDERED` is said to be **descriptive**.

Type classes

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- Signature `DICT` is in contrast **prescriptive**, defining an abstract type with all its operations, exhaustively.

Type classes

- Even though `t` is not concrete, it does not have to be abstract.
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Type classes

- Even though `t` is not concrete, it does not have to be abstract.
- We expect `t` to be some already existing type, hence use the comment **parameter**.
- Signature `ORDERED` is said to be **descriptive**.
- Signature `DICT` is in contrast **prescriptive**, defining an abstract type with all its operations, exhaustively.
- We tend to use transparent ascription for descriptive signatures (aka type classes), and opaque ascription for prescriptive signatures.

Perspective of types in signatures

Perspective of types in signatures

Concrete:

Signature dictates representation type, which is thus visible to client.

Abstract:

Signature hides representation type. Client code must work regardless of the representation type chosen by structure.

Parameter:

Client supplies the type, implementation must work with whatever the clients supplies.

Different ways of implementing ORDERED

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```
signature ORDERED =
sig
  type t      (* parameter *)
  val compare : t * t -> order
end
```

Different ways of implementing ORDERED

```
structure IntLt : ORDERED =
struct
  type t = int
  val compare = Int.compare
end
```

```
signature ORDERED =
sig
  type t     (* parameter *)
  val compare : t * t -> order
end
```

Different ways of implementing ORDERED

```
structure IntLt : ORDERED =
struct
  type t = int
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end

structure IntGt : ORDERED =
struct
  type t = int
  fun compare(x,y) = Int.compare(y,x)
end
```

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signature ORDERED =
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  type t      (* parameter *)
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end
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Different ways of implementing ORDERED

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structure StringLt : ORDERED =
struct
  type t = string
  val compare = String.compare
end
```

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signature ORDERED =
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Redefine DICT using type class ORDERED

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  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Redefine DICT using type class ORDERED

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```

Redefine DICT using type class ORDERED

Use type class as a parameter

```
signature DICT =
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  val insert : 'a dict * 'a entry -> 'a dict
end
```

Redefine DICT using type class ORDERED

```
signature DICT =
sig
  structure Key : ORDERED                   (* parameter *)
  type 'a entry = key * 'a                  (* concrete  *)
  type 'a dict                              (* abstract  *)
  val empty : 'a dict
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Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

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Use type class' type `t`

Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

Redefine DICT using type class ORDERED

```
signature DICT =
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  structure Key : ORDERED                  (* parameter *)
  type 'a entry = Key.t * 'a               (* concrete  *)
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  val empty : 'a dict
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  type 'a dict                          (* abstract  *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

Redefine DICT using type class ORDERED

```
signature DICT =
sig
  structure Key : ORDERED             (* parameter *)
  type 'a entry = Key.t * 'a          (* concrete  *)
  type 'a dict                        (* abstract  *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
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end
```

Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

Redefine DICT using type class ORDERED

Use type class' type `t`

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signature DICT =
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  structure Key : ORDERED               (* parameter *)
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  type 'a dict                          (* abstract  *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
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  structure Key = ORDERED                  (* parameter *)
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Let’s define dictionaries with different keys!

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Using our structures defined earlier implementing type class ORDERED, we can define dictionary structures with different keys:

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```
structure IntLtDict : DICT =
struct




end
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Let's define dictionaries with different keys!

Using our structures defined earlier implementing type class ORDERED, we can define dictionary structures with different keys:

```
structure IntLtDict : DICT =
struct
  structure Key = IntLt


end
```

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Using our structures defined earlier implementing type class ORDERED, we can define dictionary structures with different keys:

```
structure IntLtDict : DICT =
struct
  structure Key = IntLt
  (* code as before but now using Key.t instead of key
     and Key.compare instead of String.compare *)
end
```

Let's define dictionaries with different keys!

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```
structure IntLtDict : DICT =
struct
  structure Key = IntLt
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     and Key.compare instead of String.compare *)
end

structure IntGtDict : DICT =
struct




end
```

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  structure Key = IntGt


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end
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struct
  structure Key = IntGt
  (* code as before but now using Key.t instead of key
     and Key.compare instead of String.compare *)
end
structure StringLtDict : DICT =
struct
  structure Key = StringLt
  (* code as before but now using Key.t instead of key
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end
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Let's define dictionaries with different keys!

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  structure Key = IntGt
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end

structure StringLtDict : DICT =
struct
  structure Key = StringLt
  (* code as before but now using Key.t instead of key
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```

Only differ in **Key**!

Is that it?

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Have we solved the problem of inserting with one comparison function but looking up elements with a different one?

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For example, could we accidentally insert into a dictionary using `IntLtDict.insert` but then lookup using `IntGtDict.lookup`?

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After all, `IntLtDict.Key.t` and `IntGtDict.Key.t` are both `int`.

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For example, could we accidentally insert into a dictionary using `IntLtDict.insert` but then lookup using `IntGtDict.lookup`?

After all, `IntLtDict.Key.t` and `IntGtDict.Key.t` are both `int`.

No, this is not possible! `IntGtDict.dict` and `IntLtDict.dict` are different types.

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For example, could we accidentally insert into a dictionary using `IntLtDict.insert` but then lookup using `IntGtDict.lookup`?

After all, `IntLtDict.Key.t` and `IntGtDict.Key.t` are both `int`.

No, this is not possible! `IntGtDict.dict` and `IntLtDict.dict` are different types.

ML type checker will thus prevent intermingling of dictionaries.

Is that it?

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  - YES!

Is that it?

Have we solved the problem of inserting with one comparison function but looking up elements with a different one?

YES!

Can we avoid rewriting (copying & pasting) the same code over and over when implementing dictionaries with different keys?

Is that it?

- Have we solved the problem of inserting with one comparison function but looking up elements with a different one?
  - YES!
- Can we avoid rewriting (copying & pasting) the same code over and over when implementing dictionaries with different keys?
  - YES, but we need to use a functor for this!

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  - YES, but we need to use a functor for this!
- A **functor** creates a structure, given a structure as an argument.

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YES!

Can we avoid rewriting (copying & pasting) the same code over and over when implementing dictionaries with different keys?

YES, but we need to use a functor for this!

A **functor** creates a structure, given a structure as an argument.

Let’s write a functor that creates a structure ascribing to `DICT`, given a structure ascribing to `ORDERED` as an argument.

Avoid the bloat with a functor!

Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct




end
```

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Argument structure, of type `ORDERED`

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Argument structure, of type `ORDERED`

```
signature ORDERED =
sig
  type t      (* parameter *)
  val compare : t * t -> order
end
```

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functor TreeDict (K : ORDERED) : DICT =
struct



end
```

Argument structure, of type `ORDERED`

Note: ":" denotes typing, not ascription mode.

```
signature ORDERED =
sig
  type t      (* parameter *)
  val compare : t * t -> order
end
```

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functor TreeDict (K : ORDERED) : DICT =
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signature ORDERED =
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functor TreeDict (K : ORDERED) : DICT =
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end
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functor TreeDict (K : ORDERED) : DICT =
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end
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Structured returned, transparently ascribing to `DICT`

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signature ORDERED =
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functor TreeDict (K : ORDERED) : DICT =
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Structured returned, transparently ascribing to **DICT**

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signature ORDERED =
sig
  type t     (* parameter *)
  val compare : t * t -> order
end
```

```
signature DICT =
sig
  structure Key = ORDERED                (* parameter *)
  type 'a entry = Key.t * 'a            (* concrete  *)
  type 'a dict                          (* abstract  *)
  ...
end
```

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```
functor TreeDict (K : ORDERED) : DICT =
struct



end
```

Structured returned, transparently ascribing to **DICT**

Denotes ascription mode, as usual.

```
signature ORDERED =
sig
  type t      (* parameter *)
  val compare : t * t -> order
end
```

```
signature DICT =
sig
  structure Key = ORDERED             (* parameter *)
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functor TreeDict (K : ORDERED) : DICT =
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end
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Avoid the bloat with a functor!

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  structure Key = K




end
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signature ORDERED =
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  structure Key = ORDERED                   (* parameter *)
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  ...
end
```

Avoid the bloat with a functor!

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functor TreeDict (K : ORDERED) : DICT =
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  structure Key = K
  type 'a entry = Key.t * 'a


end
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functor TreeDict (K : ORDERED) : DICT =
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  structure Key = K
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```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
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  (* code as before, but using Key.t and Key.compare *)
end
```

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Now, we can define our earlier dictionaries as:

Avoid the bloat with a functor!

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functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

Now, we can define our earlier dictionaries as:

```
structure IntLtDict = TreeDict(IntLt)
structure IntGtDict = TreeDict(IntGt)
structure StringLtDict = TreeDict(StringLt)
```

Avoid the bloat with a functor!

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functor TreeDict (K : ORDERED) : DICT =
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signature DICT =
sig
  structure Key = ORDERED                (* parameter *)
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  ...
end
```

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```
functor TreeDict (K : ORDERED) :> DICT =
struct
  structure Key = K
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  structure Key = ORDERED          (* parameter *)
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  ...
end
```

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```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

Let's use opaque ascription instead!

```
signature ORDERED =
sig
  type t      (* parameter *)
  val compare : t * t -> order
end
```

```
signature DICT =
sig
  structure Key = ORDERED              (* parameter *)
  type 'a entry = Key.t * 'a           (* concrete  *)
  type 'a dict                         (* abstract  *)
  ...
end
```

Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) :> DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
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  (* code as before, but using Key.t and Key.compare *)
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  structure Key = ORDERED              (* parameter *)
  type 'a entry = Key.t * 'a           (* concrete  *)
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end
```

Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) :> DICT =
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```
signature DICT =
sig
  structure Key = ORDERED               (* parameter *)
  type 'a entry = Key.t * 'a            (* concrete  *)
  type 'a dict                          (* abstract  *)
  ...
end
```

Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) :> DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

But now we hide the representation type for `Key.t`.

```
signature ORDERED =
sig
  type t     (* parameter *)
  val compare : t * t -> order
end
```

```
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sig
  structure Key = ORDERED               (* parameter *)
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end
```

Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) :> DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

But now we hide the representation type for `Key.t`.

Instead, reveal that `Key.t` is the same as input key `K.t`!

```
signature ORDERED =
sig
  type t      (* parameter *)
  val compare : t * t -> order
end
```

```
signature DICT =
sig
  structure Key = ORDERED              (* parameter *)
  type 'a entry = Key.t * 'a           (* concrete  *)
  type 'a dict                         (* abstract  *)
  ...
end
```

Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) :> DICT
                    where type Key.t = K.t =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

- But now we hide the representation type for `Key.t`.
- Instead, reveal that `Key.t` is the same as input key `K.t`!
- To rectify this, we need to add a `where` clause.

Summary

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Prescriptive signatures exhaustively specify a type's operations, typically using **opaque** ascription.

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Summary

A word on syntax:

- Functors only take a single structure as an argument.
- Multiple argument structures can be passed using nested structures or using specialized syntax.
  - More on this in labs and homework.
- Similarly, multiple where clauses are supported, using different syntactic forms.
  - More on this in labs and homework.

That's all for today. Happy 🎃👻🎃!