15-150 Fall 2020

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LECTURE 1

Introduction to Functional Programming

Plan

This is a REMOTE class

- Lectures using Zoom *at class time* (then saved)
    - Please show up online, ***on time***
        - *If in a different time zone, watch promptly.*
    - Study, work through examples, later.
- Homeworks and exams *online*
    - Do your own work!

Logistics

- Get to know course staff
  - email, request a zoom chat, …
- TAs will announce online office hours, …
- Email me about any concerns
  - Class size is large, so please be patient
  - Can also *cc* my assistant,
    Christina Contreras (cc8k@andrew)

Diversity

This class aims to give full and fair consideration
to students from diverse backgrounds.

Diversity will be appreciated as a resource, a strength and a benefit.

Course staff aim to be respectful, and responsive to needs.

If any class meetings or deadlines conflict with religious events,
let me know in advance so we can make arrangements.

Your suggestions are encouraged and appreciated.

Please let me know ways to improve the course.

Functional programming

LISP • APL • FP • Scheme • KRC • Hope
Miranda™ • Erlang • Curry • Gofer • Mercury
Charity • Cayenne • Mondrian • Epigram
SML • Clean • Caml • Haskell

Everything else is just *dys*functional programming!

The SML language

- *functional*

  computation = expression evaluation

- *typed*

  only *well-typed* expressions are evaluated

- *polymorphic*

  well-typed expressions have a *most general type*

- *call-by-value*

  function calls evaluate their argument

Advantages

- *functional*

  easy to design and analyze

- *typed*

  common errors caught early

- *polymorphic*

  easy to re-use code

- *call-by-value*

  predictable control flow

example

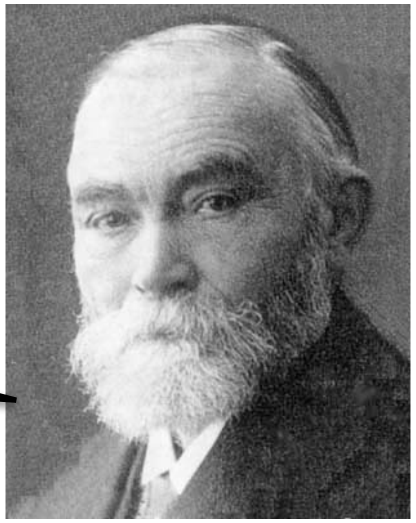
```
Standard ML of New Jersey […]
fun length [ ] = 0
l     length (x::L) = 1 + length L;

 - val length = fn - : ’a list -> int


 length [1, 2, 4, 8];
 -  val it = 4 : int

 length [true, false];
  -  val it = 2 : int

 length 42;
  -  type error!
```

Features

- **referential transparency**
  - *equivalent* code is *interchangeable*
- **mathematical foundations**
  - use math to define *equivalence*
  - use logic to prove *correctness*, *termination*, …
- **functions are values**
  - can be used as data in lists, tuples, ...
  - can be an argument or result of other functions

Referential transparency

- The *type* of an expression depends only on the *types* of its sub-expressions
- The *value* of an expression depends only on the *values* of its sub-expressions



safe substitution,
compositional reasoning

Equivalence

- Expressions of type **int** are *equivalent* if they evaluate to the same integer
- Functions of type **int -> int** are *equivalent* if they map *equivalent arguments* to *equivalent results*
- Expressions of type **int list** are *equivalent* if they evaluate to the same list of integers

Equivalence is a form of *semantic equality*

Equivalence

- 21 + 21 is *equivalent* to 42
- [2,4,6] is *equivalent* to [1+1, 2+2, 3+3]
- **fn** x => x+x is *equivalent* to **fn** y => 2*y

21 + 21 = 42

fn x => x+x = **fn** y => 2*y

(**fn** x => x+x) (21 + 21) = (**fn** y => 2*y) 42 = 84

We use = for *equivalence*

Don’t confuse with = in ML

equality in ML

- ML has a built-in = operator
- Can use with expressions of simple types like int, bool, int list, … (called *equality types*)
- Will check if expressions evaluate to same value

(2 + 2) = 4 evaluates to **true**

Equivalence

- For every type t there is a notion of *equivalence* for expressions of that type
  - We usually just use $=$
  - When necessary we use $=_t$

Our examples so far illustrate:

$=_{\text{int}}$

$=_{\text{int list}}$

$=_{\text{int -> int}}$

Compositionality

- Replacing a sub-expression of a program with an *equivalent* expression always gives an *equivalent* program

The key to
compositional reasoning
about programs

Parallelism

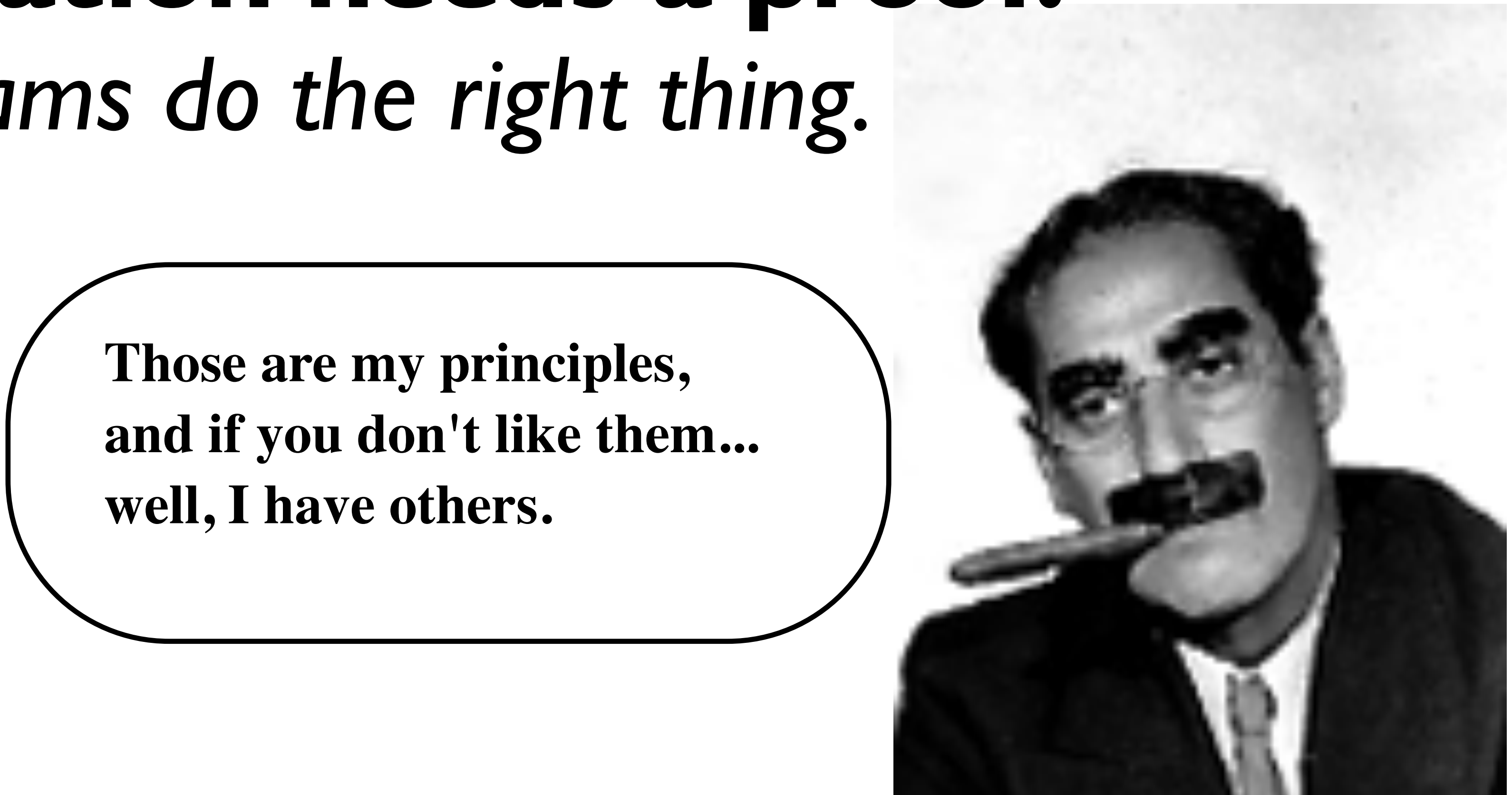
- Expression evaluation has ***no side-effects***
  - can evaluate *independent* code *in parallel*
  - evaluation order has *no effect* on *value*

- Parallel evaluation may be *faster* than sequential

Learn to *exploit* parallelism!

Principles

- **Expressions must be well-typed.**
  Well-typed expressions don't go wrong.
- **Every function needs a specification.**
  Well-specified programs are easier to understand.
- **Every specification needs a proof.**
  Well-proven programs do the right thing.



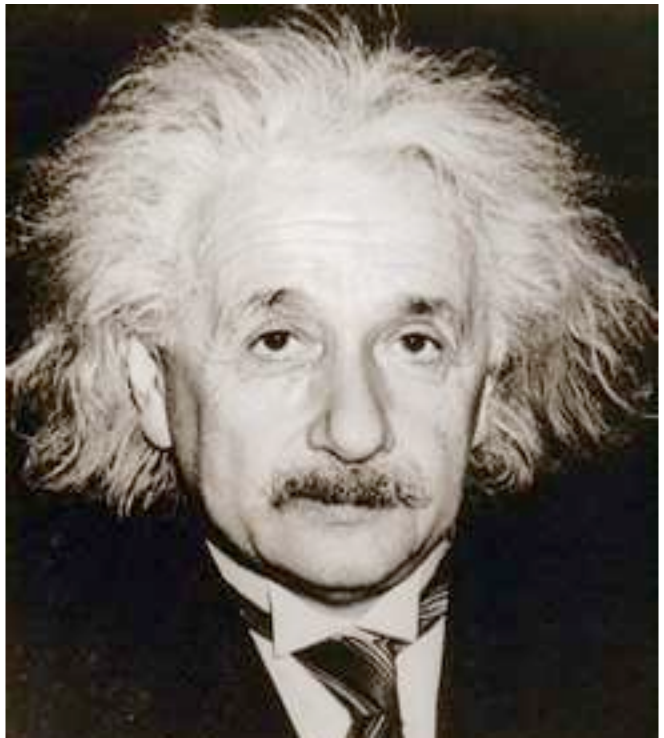
Principles

- **Large programs should be *modular*.**
  Well-interfaced code is easier to maintain.

- **Data structures algorithms.**
  Good choice of representation can lead to better code.

- **Exploit parallelism.**
  Parallel code may run faster.

- **Strive for simplicity.**
  Programs should be as simple as possible, but no simpler.



sum

```
fun sum [ ]      = 0
 |   sum (x::L) = x + sum(L)
```

A recursive function declaration
using list patterns and integer arithmetic

- sum has type int list -> int
- sum [1,2,3] evaluates to 6
- For all $n \geq 0$ and integer values $v_1, \ldots, v_n$

$$\mathrm{sum}\ [v_1, \ldots, v_n] = v_1 + \ldots + v_n$$

sum

```
fun sum [ ] = 0
 |    sum (x::L) = x + sum(L)
```

sum [1,2,3]
= 1 + sum [2,3]
= 1 + (2 + sum [3])
= 1 + (2 + (3 + sum [ ]))
= 1 + (2 + (3 + 0))
= 6

[1,2,3] = 1 :: [2,3]

equational reasoning

count

```
fun count [ ] = 0
  |   count (r::R) = (sum r) + (count R)
```

- count has type (int list) list -> int

count

```
fun count [ ] = 0
 |  count (r::R) = (sum r) + (count R)
```

- count has type (int list) list -> int
- count [[1,2,3], [1,2,3]] evaluates to 12

count

```
fun count [ ] = 0
  |   count (r::R) = (sum r) + (count R)
```

- count has type (int list) list -> int
- count [[1,2,3], [1,2,3]] evaluates to 12
- For all $n \geq 0$ and integer lists $L_1, \ldots, L_n$

  count [$L_1, \ldots, L_n$] = sum L_1 + … + sum L_n

count

Since

    sum [1,2,3] = 6

and

    count [[1,2,3], [1,2,3]]
        = sum[1,2,3] + sum [1,2,3]

it follows that

    count [[1,2,3], [1,2,3]]
        = 6 + 6
        = 12

equational reasoning

tail recursion

```
fun sum [ ] = 0
|   sum (x::L) = x + sum(L)
```

- The definition of sum is not tail-recursive
- Can define a tail recursive *helper* function sum’ that uses an integer *accumulator*

```
sum : int list -> int
sum’ : int list * int -> int
```

Q: This is a general technique. But why bother?
A: Sometimes tail recursion is more efficient.

sum’

```
fun sum’ ([ ], a) = a
 | sum’ (x::L, a) = sum’ (L, x+a)
```

- sum’ has type int list * int -> int
- sum’ ([1,2,3], 4) evaluates to 10
- For all integer lists L and integers a,

  sum’(L, a) = sum(L) + a

Sum

```
fun sum' ([ ], a) = a
 | sum' (x::L, a) = sum' (L, x+a)
```

```
fun Sum L = sum' (L, 0)
```

- Sum has type int list -> int
- Sum and sum are *equivalent*

For all integer lists L,

Sum L = sum L

Hence...

```
fun count [ ] = 0
  |   count (r::R) = (sum r) + (count R)

fun Count [ ] = 0
  |   Count (r::R) = (Sum r) + (Count R)
```

- Count and count are *equivalent* because Sum and sum are equivalent.

Evaluation

```
fun sum [ ] = 0
  |  sum (x::L) = x + sum(L)
```

$$\begin{aligned}
\text{sum } (1{::}[2{,}3]) &\Longrightarrow^{*} 1 + \text{sum } [2{,}3] \\
&\Longrightarrow^{*} 1 + (2 + \text{sum } [3]) \\
&\Longrightarrow^{*} 1 + (2 + (3 + \text{sum } [\ ])) \\
&\Longrightarrow^{*} 1 + (2 + (3 + 0)) \\
&\Longrightarrow^{*} 1 + (2 + 3) \\
&\Longrightarrow^{*} 1 + 5 \\
&\Longrightarrow^{*} 6
\end{aligned}$$

$\Longrightarrow^{*}$

means
“evaluates to,
in finitely many steps”

pattern of recursive calls,
order of arithmetic operations

Evaluation

count [[1,2,3], [1,2,3]]

$\Longrightarrow^*$ sum [1,2,3] + count [[1,2,3]]

$\Longrightarrow^*$ 6 + count [[1,2,3]]

$\Longrightarrow^*$ 6 + (sum [1,2,3] + count [ ])

$\Longrightarrow^*$ 6 + (6 + count [ ])

$\Longrightarrow^*$ 6 + (6 + 0)

$\Longrightarrow^*$ 6 + 6

$\Longrightarrow^*$ 12

Analysis

(details later!)

code fragment	evaluation time proportional to
sum(L), Sum(L)	length of L
count(R), Count(R)	sum of lengths of lists in R

(tail recursion **doesn’t** help here!)

These functions do *sequential evaluation*…

parallelism

+ is *associative* and *commutative*

The combination *order* doesn’t affect result, so it’s safe to evaluate in parallel

Suppose we have a function map such that

$$\text{map } f\ [x_1, \ldots, x_n] \Longrightarrow^{*} [f(x_1), \ldots, f(x_n)]$$

and we can evaluate the $f(x_i)$ in parallel…

parallel counting

```
fun parcount R = sum (map sum R)
```

parcount [[1,2,3], [4,5], [6,7,8]]

$\Longrightarrow^{*}$ sum (map sum [[1,2,3], [4,5], [6,7,8]])

$\Longrightarrow^{*}$ sum [sum [1,2,3], sum [4,5], sum [6,7,8]]

parallel evaluation of sum[1,2,3], sum[4,5] and sum[6,7,8]

$\Longrightarrow^{*}$ sum [6, 9, 21]

$\Longrightarrow^{*}$ 36

Analysis

- Let R be a list of k *rows,* and each row be a list of m integers
- *If we have enough parallel processors,* parcount R takes time proportional to k + m

computes each row sum, in parallel
then
adds the row sums

Contrast: count R takes time proportional to k*m

With m=20 and k=12,
k + m is 32, almost an 8-fold speedup over k*m = 240.

work and **span**

We will introduce techniques for analysing

- *work* (sequential runtime)
- *span* (optimal parallel runtime)

(that’s how we did those runtime calculations)

Themes

- **functional programming**
- correctness, termination, and performance
- types, specifications and proofs
- evaluation, equivalence and referential transparency
- compositional reasoning
- exploiting parallelism

Objectives

- Write well-designed ***functional programs***
- Write ***specifications***, and prove correctness
- Techniques for analyzing runtime (***sequential*** and ***parallel***)
- Choose data structures wisely and exploit ***parallelism*** to achieve ***efficiency***
- Design code using **modules** and ***abstract types***, with clear interfaces

Summary

- Don’t worry if you don’t know SML syntax
- Don’t panic about so-far-undefined terminology
  - We will cover the details in lectures
- This introduction should help you appreciate the main ideas and see where we’re going…