1. [8pts] This question concerns generating random numbers in Ruby. Recall that the Ruby function `rand(n)` returns a random integer between 0 and n-1, inclusive. Using the `rand` function, show how to compute the following:

1a. [2pt] A random integer between 0 and 49, including 49. \[ \text{rand}(50) \]

1b. [2pt] A random integer between 15 and 20, including 20. \[ \text{rand}(6) + 15 \]

1c. [2pt] A random string from the array `cars` below. \[ \text{cars}[\text{rand}(4)] \]

```ruby
cars = ["Chevrolet", "Honda", "Mercedes", "Toyota"]
```

1d. [2pt] A random even integer between 2 and 10, inclusive. \[ \frac{2 \times \text{rand}(5) + 2}{\text{rand}(5) + 1} \]

2. [12pts] This question concerns writing functions that involve randomness.

2a. [2pt] Suppose that a course grade can either be a “pass” or “fail”. The following function uses randomness to return a grade. The function is written such that a pass or fail grade is equally likely. Fill in the blanks. You can assume that when `rand(n)` is used, every value in the range 0 to n-1 is equally likely as a return value.

```ruby
def lazy_teacher()
    if rand(2) == 1 then
        return "fail"
    else
        return "pass"
    end
end
```
2b. [2pt] Write a different version of the function above that returns “pass” with probability 80% and “fail” with probability 20%.

```python
def generous_teacher():
    if rand(10) <= 7:
        return "fail"
    else:
        return "pass"
end
```

2c. [4pt] Consider the simple functions given below. They simulate throwing a 6-sided die and a 10-sided die respectively.

```python
def roll6():
    return rand(6) + 1
end

def roll10():
    return rand(10) + 1
end
```

Suppose that we want to simulate a game by throwing a 6-sided die and a 10-sided die by using the functions above. If the result of the 6-sided die is greater than the result of the 10-sided die we return the sum of two dice. Otherwise, we return the result of the 10-sided die. Point out the flaw in the following function that attempts to implement this simulation.

```python
def faulty():
    if roll6() > roll10():
        return roll6() + roll10()
    else:
        return roll10()
end
```

The function above is not a correct implementation because each time we use one of the roll functions we generate a new random number. In order to use the numbers we generate we need to store them in a variable.
2d. [4pts] Write a function of your own that eliminates the flaw in the function `faulty()`. Your function should call the functions `rol16()` and `rol110()`. Hint: Your function should also use assignments.

```python
def correct():
    x = roll 6()
    y = roll 10()
    if x > y then
        return x + y
    else
        return y
end # if
end # def
```

3. [14pts] This problem concerns one-dimensional binary cellular automata such as the ones you experimented with in the OLI module and homework assignments. Consider an automaton whose initial state is shown in line (i) below. The next generation is shown in line (ii).

3a. [4pts] What is the rule for this automaton? In the spaces below, fill in the dashed boxes with a 1 (for black) or a 0 (for white) to specify the rule this automaton must be using to produce line (ii) from line (i).

```
7 6 5 4 3 2 1 0
```

3b. [4pts] Using the rule numbering scheme we studied, the rule for this automaton must be a number between 0 and 255. What is the number? 92

3c. [6pts] Apply the rule to compute generations (iii) and (iv) above by coloring in selected squares.

4. [14pts] This question concerns networking and the Internet.

4a. [4pts] Suppose that computers in an institution are assigned IPv4 addresses that start with a common sequence of 8 bits that uniquely identify the institution’s network. Given that there are 32 bits in an IPv4 address, how many hosts can this institution have in its network using this addressing scheme?

```
2^24
```

4b. [2pts] In packet-switching networks such as the Internet, two network nodes establish a dedicated connection via one or more switching stations, and the communication is reliable and uninterruptable. Is this statement true or false? False

4c. [2pts] The principle known as **net neutrality** advocates no restrictions by ISPs or governments on consumers’ access to networks that participate in the Internet.
4d. [2pts] A domain name server is used to translate human-friendly computer hostnames into IP addresses.

4e. [2pts] A Web page is identified by a Uniform Resource Locator (URL), which has the following format:

\[
\text{part1://part2/page}
\]

Part 1 stands for a _______ such as HTTP.

Part 2 stands for a _______ or host address.

4f. [2pts] A communication _______ is a collection of rules that govern the ways in which computers interact with one another.

5. [14pts] This question concerns public key encryption.

5a. [2pts] Which one of the following statements is true (circle your choice)?

(a) Public key encryption is called a “symmetric” encryption scheme because keys always come in pairs: a public key and an associated private key.

(b) Public key encryption is called an “asymmetric” encryption scheme because the key Alice uses to encrypt messages to Bob is not the key Bob uses to decrypt those messages.

5b. [3pts] The RSA public key encryption algorithm is believed to be secure because _______ is believed to take time that is exponential in the length (number of digits) of the input?

\[\text{Prime Factoring}\]

5c. [3pts] What is not public in “public key encryption”? Circle one answer:

(a) The product of the two prime numbers used to generate the keys.

(b) The public key.

(c) The decryption algorithm.

(d) The two prime numbers used to generate the keys.

5d. [6pts] Alice and Bob want to use public key encryption to send a secret message. Assume Alice has private key \(a\) and public key \(A\), and Bob has private key \(b\) and public key \(B\). They both know a function
\(\text{encrypt}(\text{plaintext}, \text{key})\) that can be used to encrypt a message, and a corresponding function \(\text{decrypt}(\text{ciphertext}, \text{key})\) to decrypt it.

Write an expression involving the \(\text{encrypt}\) function and the appropriate key to show how Alice would securely send the message "victory" to Bob.

\[
\text{encrypt}(\text{"victory"}, \text{B})
\]

Write an expression involving the \(\text{decrypt}\) function to show how Bob would decrypt the message he received from Alice. You can assume that Alice's encrypted message is stored in the variable \(M\).

\[
\text{decrypt}(M, \text{b})
\]

6. [20pts] This question concerns AI and recursion.

Consider a two-player game where there are two choices for each move, A or B. We can represent the game tree in Ruby using a nested array. Before the first move, the tree is just the root node:

\[
[
]
\]

We can extend the tree to the next level by replacing each terminal node by a list of two new nodes, generated by appending an symbol :a or :b to the original node. Initially the root is the only terminal node. After the first move, we have one nonterminal node (the root) with two terminal nodes, [:a] and [:b], so the tree (also shown at right) becomes:

\[
[
[:a], [:b]
]
\]

Notice that tree[0] is the left terminal node, and tree[1] is the right one.

We can repeat the tree generation process described above to create the tree for a two move game as shown below and at right:

\[
[
[[[:a, :a]], [:a, :b]], [[:b, :a], [:b, :b]]
]
\]

As the tree shows, there are four possible games consisting of two moves. The left subtree, i.e., the first nonterminal node below the root, is tree[0], and the first terminal node is tree[0][0], which is [:a,:a]. The last terminal node is tree[1][1], which is [:b,:b].

We can in turn use this tree to generate the tree for three-move games:

\[
[
[[[[[:a, :a, :a]], [:a, :a, :b]], [[:a, :b, :a], [:a, :b, :b]]],
[[[:b, :a, :a]], [:b, :a, :b]], [[:b, :b, :a], [:b, :b, :b]]
]
\]
The above tree has eight terminal nodes, corresponding to the eight possible games with three moves. The first terminal node is [:a;a,a] and the last is [:b; b; b]. Terminal nodes are lists composed of the symbols :a and :b; they do not contain any lists inside them.

6a. [6pts] Write a function `terminal?(node)` that takes as input a list representing some node in a game tree for this game, and returns true if `node` is a terminal node, and false otherwise. Your function should return true for inputs such as [:b] or [:a; b] or [:b; a; a] and false for non-terminal inputs such as [[:a],[:b]]. Note that the empty game tree [] is also a valid terminal node. You can assume that `node` will always be either a valid terminal node or a valid non-terminal node, not some random junk.

```python
def terminal?(node):
    if node == [] or node[0] == :a or node[0] == :b
        return True
    else:
        return False

6b. [7pts] Write a recursive function that takes a non-empty game tree as input and returns the last (rightmost) terminal node. Your function should call the `terminal?` function you wrote above.

```python
def last_terminal(tree):
    if terminal?(tree):
        return tree
    else:
        return last_non_terminal(tree[1])

6c. [7pts] Write a recursive function that takes a game tree as input and returns the number of games in the tree, i.e., the number of terminal nodes. Do not try to calculate the number of nodes based on the depth or any other consideration; simply recurse on the tree and count the terminal nodes that are actually there. Your function should call the `terminal?` function you wrote above.

```python
def number_of_games(tree):
    if terminal?(tree):
        return 1
    else:
        return number_of_games(tree[0]) + number_of_games(tree[1])
7. [18pts] This question concerns concurrency.

Suppose that two programs P1 and P2 have access to the same variable x in memory, and they can use actions \texttt{read}(x) and \texttt{dec}(x, c), which represent, respectively, reading from the location x and decreasing the value in location x by the value c. The program steps for P1 and P2 are given below.

\begin{verbatim}
# Program P1
P1A: a = read(x)
P1B: if a > 3 then dec(x, 4)
     else do nothing

# Program P2
P2A: b = read(x)
P2B: if b > 1 then dec(x, 2)
     else do nothing
\end{verbatim}

7a. [4pts] Suppose that x initially contains the value 4, and P1 and P2 are executed sequentially such that the program steps are executed in the following order: P1A, P1B, P2A, P2B. What will the memory location x contain at the end of the execution?

After step P1A, the value of x is: 4
After step P1B, the value of x is: 0
After step P2A, the value of x is: 0
After step P2B, the value of x is: 0

7b. [4pts] Suppose that the programs P1 and P2 are executed by two different processors that have access to a shared memory containing the variable x. This means that the steps of the two programs may be interleaved. For example, the first 3 steps in an execution can be P1A, P2A, P2B. Now, assume that the memory location x initially contains the value 4 as in the previous question. Give a 4-step execution sequence that starts with the step P2A such that the resulting value of x is negative.

\begin{verbatim}
P2A, P1A, P1B, P2B
\end{verbatim}

or

\begin{verbatim}
P2A, P1A, P2B, P1B
\end{verbatim}
7c. [4pts] Now consider the following programs $P_3$ and $P_4$, which are executed concurrently. Notice that they include calls to $P_1$ and $P_2$, respectively. If we want to guarantee that $x$ never contains a negative value after the programs complete their execution, we may want to treat the calls to $P_1$ and $P_2$ as critical sections and try to ensure that only one program is executing its critical section at a time. You can assume that $x_{free_3}$ and $x_{free_4}$ are shared variables that are initialized to true and that functions $noncritical_P3()$ and $noncritical_P4()$ do not involve the location $x$. Give an execution sequence in terms of the labels in the given programs that results in a deadlock.

$$P_{3A}, P_{3B}, P_{4A}, P_{4B}, P_{3C}, P_{4C}$$

(multiple correct answers)

Program $P_3$:
while true do
  $P_{3A}$: call $noncritical_P3()$
  $P_{3B}$: $x_{free_3} = false$
  $P_{3C}$: while $x_{free_4} == false$
    do nothing
  end
  $P_{3D}$: call Program $P_1$
  $P_{3E}$: $x_{free_3} = true$
end

Program $P_4$:
while true do
  $P_{4A}$: call $noncritical_P4()$
  $P_{4B}$: $x_{free_4} = false$
  $P_{4C}$: while $x_{free_3} == false$
    do nothing
  end
  $P_{4D}$: call Program $P_2$
  $P_{4E}$: $x_{free_4} = true$
end

Any interleaving that sets both $x_{free_3}$ and $x_{free_4}$ to false before any of $P_{3C}$ or $P_{4C}$ is executed would lead to deadlock.
7d. (6pts) Pipelining is used in computers to speed up computer execution. In a 4-staged pipeline the steps for executing an instruction are as follows:

F. Fetch instruction from memory
D. Decode the instruction and fetch operands
E. Execute the instruction
W. Write the result into registers or memory

Suppose that each of the above steps takes 1 time unit to perform, and we want to execute 5 instructions in a pipelined fashion. Draw a diagram to illustrate the pipelined execution of the five instructions, i1 through i5. There are multiple ways to graphically depict pipelining; we used two different ways in the lecture notes. You are free to pick whatever convention you like, provided that it correctly expresses the pipelining process. For example, you might choose to have time run vertically, or you might prefer that it run horizontally. Use the grid below to make your diagram. You can label the rows and columns, in any way you need to show how your diagram makes sense. What you write in the grid squares is also up to you. And we’ve given you way more space than you need, so don’t expect to use every row and column. This is actually an easy problem, so don’t panic!