The Boosting Approach to Machine Learning

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Boosting

- General method for improving the accuracy of any given learning algorithm.
- Works by creating a series of challenge datasets s.t. even modest performance on these can be used to produce an overall high-accuracy predictor.
  - Works well in practice (Adaboost and its variations one of the top 10 algorithms).
  - Backed up by solid foundations.

Readings:

- The Boosting Approach to Machine Learning: An Overview. Rob Schapire, 2001
- Theory and Applications of Boosting. NIPS tutorial.

  http://www.cs.princeton.edu/~schapire/talks/nips-tutorial.pdf

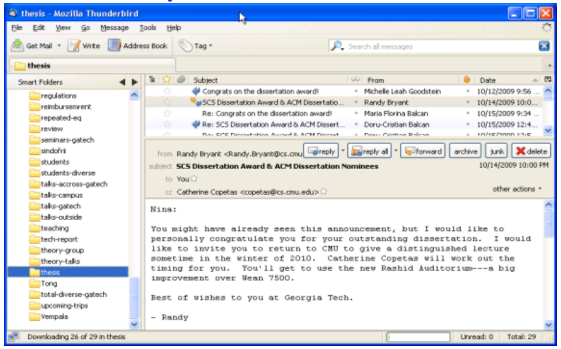
Plan for today:

- Motivation.
- A bit of history.
- Adaboost: algo, guarantees, discussion.
- Focus on supervised classification.

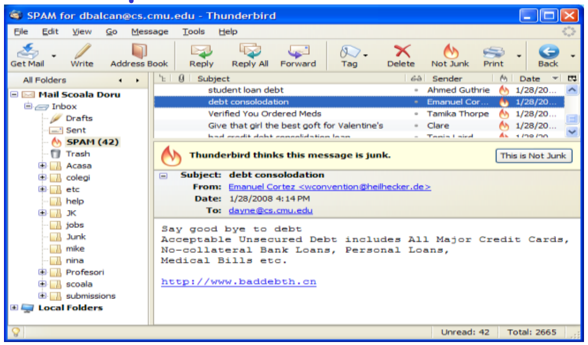
An Example: Spam Detection

- E.g., classify which emails are spam and which are important.

Not spam



spam

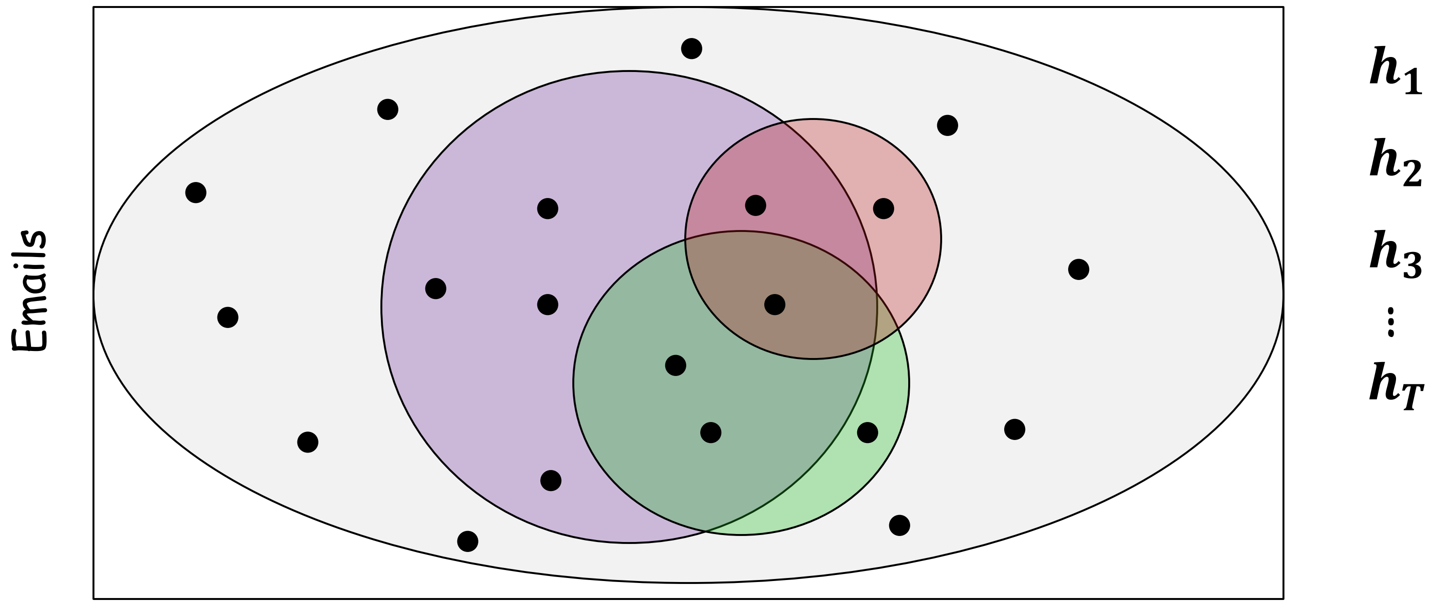


Key observation/motivation:

- Easy to find rules of thumb that are often correct.
  - E.g., "If buy now in the message, then predict spam."
  - E.g., "If say good-bye to debt in the message, then predict spam."
- Harder to find single rule that is very highly accurate.

An Example: Spam Detection

- Boosting: meta-procedure that takes in an algo for finding rules of thumb (weak learner). Produces a highly accurate rule, by calling the weak learner repeatedly on cleverly chosen datasets.



- apply weak learner to a subset of emails, obtain rule of thumb
- apply to 2nd subset of emails, obtain 2nd rule of thumb
- apply to 3rd subset of emails, obtain 3rd rule of thumb
- repeat T times; combine weak rules into a single highly accurate rule.

Boosting: Important Aspects

How to choose examples on each round?

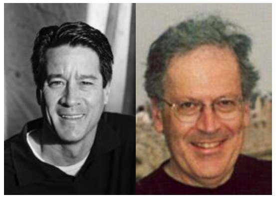
- Typically, concentrate on “hardest” examples (those most often misclassified by previous rules of thumb)

How to combine rules of thumb into single prediction rule?

- take (weighted) majority vote of rules of thumb

Historically….

Weak Learning vs Strong/PAC Learning



- [Kearns & Valiant '88]: defined weak learning: being able to predict better than random guessing (error $\leq \frac{1}{2} - \gamma$) , consistently.

- Posed an open pb: "Does there exist a boosting algo that turns a weak learner into a strong PAC learner (that can produce arbitrarily accurate hypotheses)?"

- Informally, given "weak" learning algo that can consistently find classifiers of error $\leq \frac{1}{2} - \gamma$, a boosting algo would provably construct a single classifier with error $\leq \epsilon$.

Weak Learning vs Strong/PAC Learning

Strong (PAC) Learning

- $\exists$ algo A
- $\forall\, c \in H$
- $\forall D$
- $\forall\, \epsilon > 0$
- $\forall\, \delta > 0$
- A produces h s.t.:

  $$\Pr[err(h) \geq \epsilon] \leq \delta$$

Weak Learning

h

- $\exists$ algo A
- $\exists \gamma > 0$
- $\forall\, c \in H$
- $\forall D$
- $\forall\, \epsilon > \frac{1}{2} - \gamma$
- $\forall\, \delta > 0$
- A produces h s.t.

  $$\Pr[err(h) \geq \epsilon] \leq \delta$$

- [Kearns & Valiant '88]: defined weak learning & posed an open pb of finding a boosting algo.

Surprisingly….

Weak Learning =Strong (PAC) Learning

Original Construction [Schapire '89]:

- poly-time boosting algo, exploits that we can learn a little on every distribution.

  - A modest booster obtained via calling the weak learning algorithm on 3 distributions.

    Error $= \beta < \frac{1}{2} - \gamma \rightarrow$ error $3\beta^2 - 2\beta^3$

  - Then amplifies the modest boost of accuracy by running this somehow recursively.

- Cool conceptually and technically, not very practical.

An explosion of subsequent work

Background (cont.)

- [Freund & Schapire '95]:
  - introduced "AdaBoost" algorithm
  - strong practical advantages over previous boosting algorithms
- experiments and applications using AdaBoost:

[Drucker & Cortes '96]
[Jackson & Craven '96]
[Freund & Schapire '96]
[Quinlan '96]
[Breiman '96]
[Maclin & Opitz '97]
[Bauer & Kohavi '97]
[Schwenk & Bengio '98]
[Schapire, Singer & Singhal '98]
[Abney, Schapire & Singer '99]
[Haruno, Shirai & Ooyama '99]
[Cohen & Singer' 99]
[Dietterich '00]
[Schapire & Singer '00]
[Collins '00]
[Escudero, Màrquez & Rigau '00]
[Iyer, Lewis, Schapire, Singer & Singhal '00]
[Onoda, Rätsch & Müller '00]
[Tieu & Viola '00]
[Walker, Rambow & Rogati '01]
[Rochery, Schapire, Rahim & Gupta '01]
[Merler, Furlanello, Larcher & Sboner '01]
⋮

- continuing development of theory and algorithms:

[Breiman '98, '99]
[Schapire, Freund, Bartlett & Lee '98]
[Grove & Schuurmans '98]
[Mason, Bartlett & Baxter '98]
[Schapire & Singer '99]
[Cohen & Singer '99]
[Freund & Mason '99]
[Domingo & Watanabe '99]
[Mason, Baxter, Bartlett & Frean '99, '00]
[Duffy & Helmbold '99, '02]
[Freund & Mason '99]
[Ridgeway, Madigan & Richardson '99]
[Kivinen & Warmuth '99]
[Friedman, Hastie & Tibshirani '00]
[Rätsch, Onoda & Müller '00]
[Rätsch, Warmuth, Mika, Onoda, Lemm & Müller '00]
[Allwein, Schapire & Singer '00]
[Friedman '01]
[Koltchinskii, Panchenko & Lozano '01]
[Collins, Schapire & Singer '02]
[Demiriz, Bennett & Shawe-Taylor '02]
[Lebanon & Lafferty '02]
⋮

Adaboost (Adaptive Boosting)

"A Decision-Theoretic Generalization of On-Line Learning and an Application to Boosting"

[Freund-Schapire, JCSS'97]

Godel Prize winner 2003

Informal Description Adaboost

- Boosting: turns a weak algo into a strong (PAC) learner.

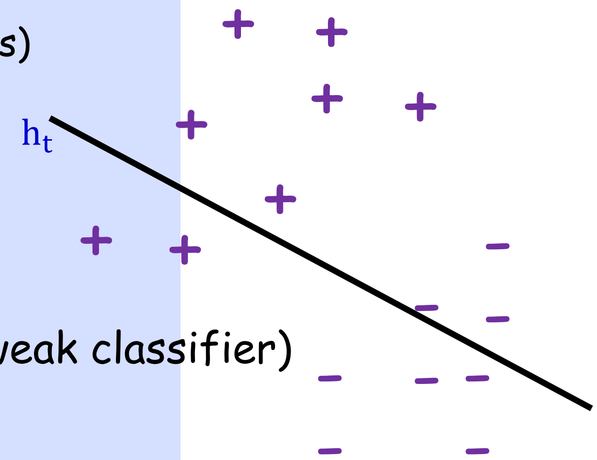
<u>Input</u>: $S=\{(x_1, y_1), \ldots,(x_m, y_m)\}$; $x_i \in X,\, y_i \in Y = \{-1,1\}$

weak learning algo A (e.g., Naïve Bayes, decision stumps)

- For $t=1,2, \ldots ,T$
  - Construct D_t on $\{x_1, \ldots, x_m\}$
  - Run A on D_t producing $h_t: X \rightarrow \{-1,1\}$ (weak classifier)

    $\epsilon_t = P_{x_i \sim D_t}(h_t(x_i) \neq y_i)$ error of h_t over D_t

- Output $H_{\text{final}}(x) = \text{sign}(\sum_{t=1} \alpha_t h_t(x))$



Roughly speaking D_{t+1} increases weight on x_i if h_t incorrect on x_i ; decreases it on x_i if h_t correct.

Adaboost (Adaptive Boosting)

- Weak learning algorithm *A*.
- For t=1,2, … ,T
  - **Construct D_t on $\{x_1, \ldots, x_m\}$**
  - Run *A* on D_t producing h_t

<u>Constructing D_t</u>

- D_1 uniform on $\{x_1, \ldots, x_m\}$ [i.e., $D_1(i) = \frac{1}{m}$]
- Given D_t and h_t set

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t}\, e^{\{-\alpha_t\}} \quad \text{if } y_i = h_t(x_i)$$

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t}\, e^{\{\alpha_t\}} \quad \text{if } y_i \neq h_t(x_i)$$

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t}\, e^{\{-\alpha_t y_i h_t(x_i)\}}$$

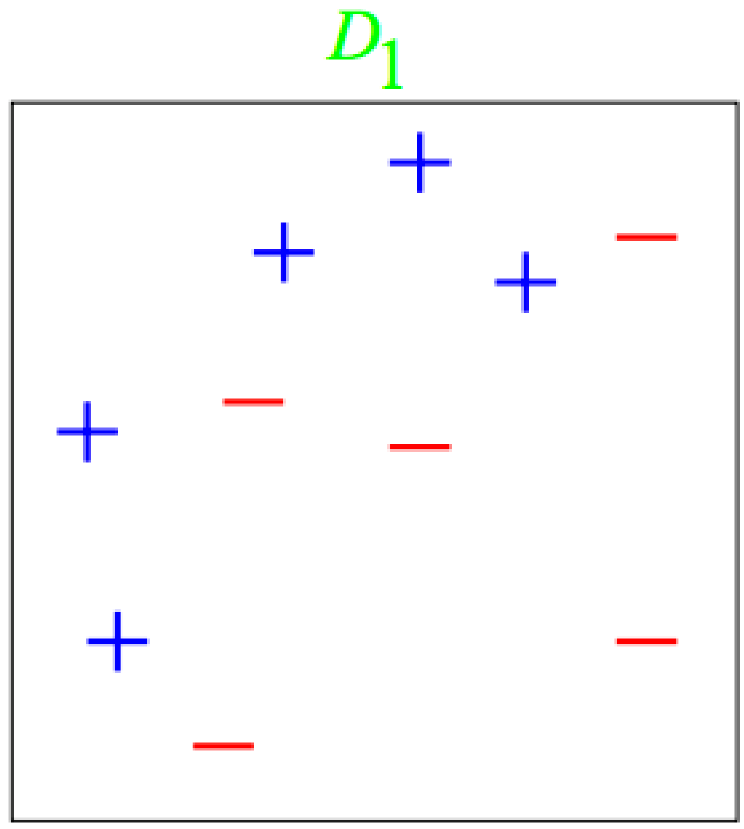
$$\alpha_t = \frac{1}{2}\ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right) > 0$$

D_{t+1} puts half of weight on examples x_i where h_t is incorrect & half on examples where h_t is correct

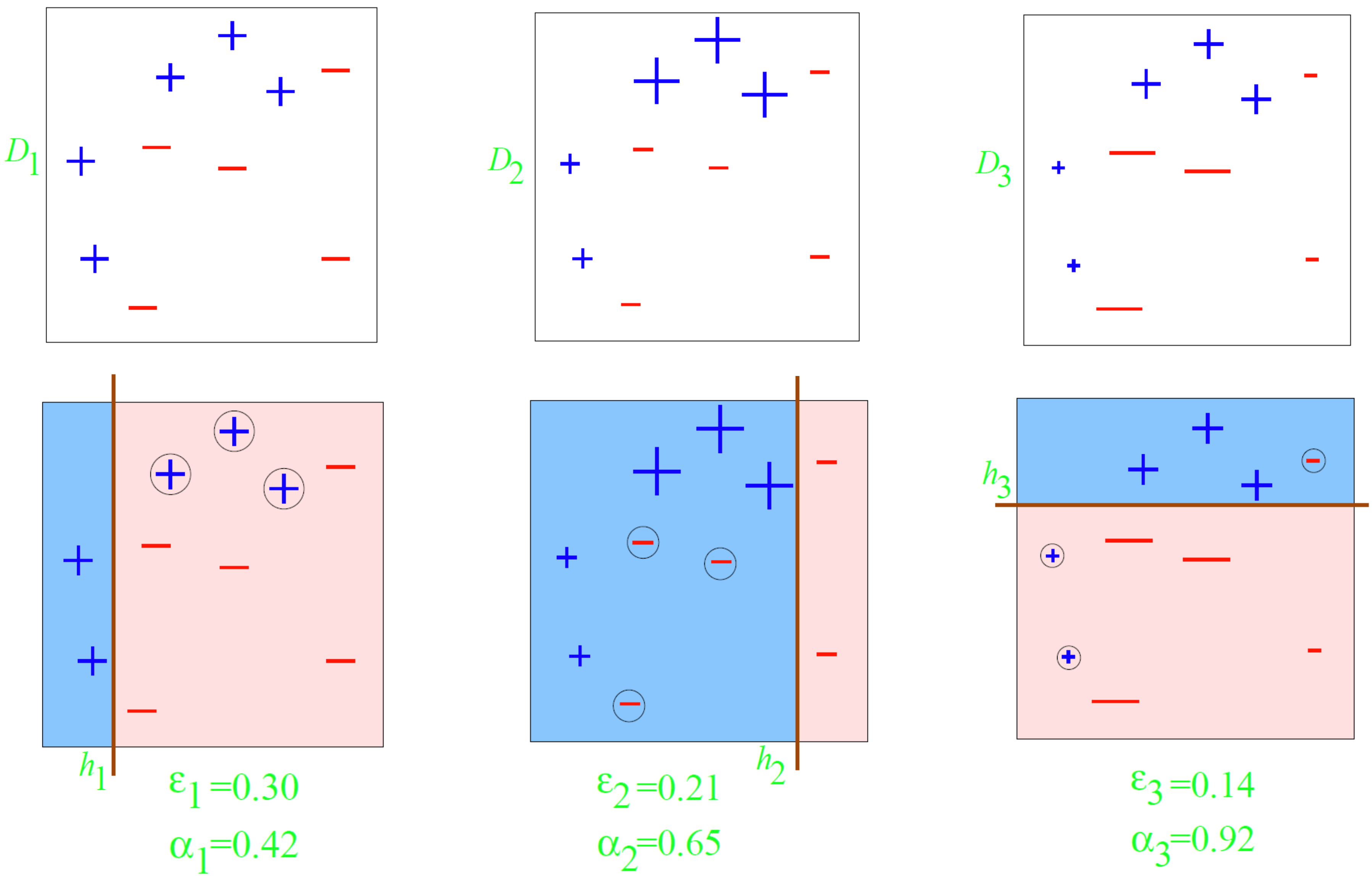
<u>Final hyp</u>: $H_{final}(x) = \mathrm{sign}(\sum_t \alpha_t h_t(x))$

Adaboost: A toy example

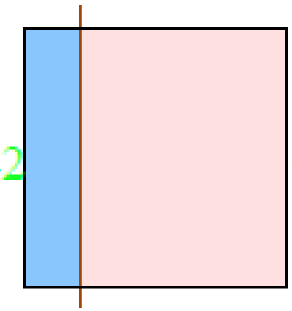
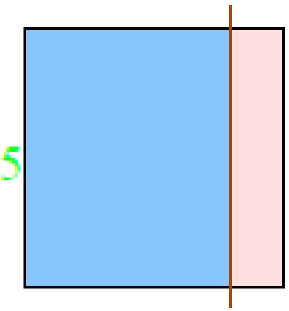
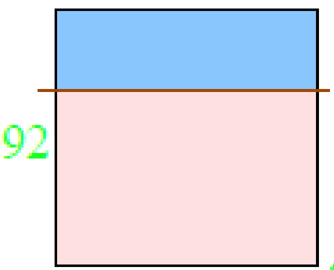
Weak classifiers: vertical or horizontal half-planes (a.k.a. decision stumps)

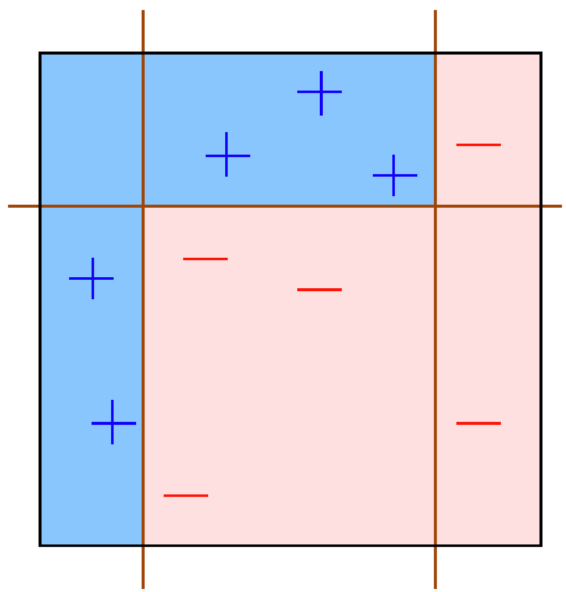


Adaboost: A toy example



Adaboost: A toy example

$H_{\text{final}} = \text{sign}\Big(0.42$  $+\ 0.65$  $+\ 0.92$  $\Big)$

$=$ 

Adaboost (Adaptive Boosting)

- Weak learning algorithm *A*.
- For t=1,2, … ,T
  - **Construct $\mathrm{D_t}$ on $\{\mathrm{x_1}, \ldots, x_{\mathrm{m}}\}$**
  - Run *A* on $\mathrm{D_t}$ producing $\mathrm{h_t}$

Constructing D_t

- $\mathrm{D_1}$ uniform on $\{\mathrm{x_1}, \ldots, \mathrm{x_m}\}$ [i.e., $\mathrm{D_1}(i) = \frac{1}{m}$]
- Given $\mathrm{D_t}$ and $\mathrm{h_t}$ set

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t}\, \mathrm{e}^{\{-\alpha_t\}} \text{ if } y_i = h_t(x_i)$$

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t}\, \mathrm{e}^{\{\alpha_t\}} \text{ if } y_i \neq h_t(x_i)$$

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t}\, \mathrm{e}^{\{-\alpha_t y_i\, h_t(x_i)\}}$$

$$\alpha_t = \frac{1}{2}\ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right) > 0$$

$\mathrm{D_{t+1}}$ puts half of weight on examples $\mathrm{x_i}$ where $\mathrm{h_t}$ is incorrect & half on examples where $\mathrm{h_t}$ is correct

Final hyp: $\mathrm{H_{final}}(x) = \mathrm{sign}(\sum_t \alpha_t h_t(x))$

Nice Features of Adaboost

- Very general: a meta-procedure, it can use any weak learning algorithm!!! (e.g., Naïve Bayes, decision stumps)
- Very fast (single pass through data each round) & simple to code, no parameters to tune.
- Shift in mindset: goal is now just to find classifiers a bit better than random guessing.
- Grounded in rich theory.
- Relevant for big data age: quickly focuses on "core difficulties", well-suited to distributed settings, where data must be communicated efficiently [Balcan-Blum-Fine-Mansour COLT'12].

Analyzing Training Error

Theorem $\epsilon_t = 1/2 - \gamma_t$ (error of h_t over D_t)

$$err_S(H_{final}) \leq \exp\left[-2 \sum_t \gamma_t^2\right]$$

So, if $\forall t, \gamma_t \geq \gamma > 0$, then $err_S(H_{final}) \leq \exp[-2\,\gamma^2 T]$

The training error drops exponentially in T!!!

To get $err_S(H_{final}) \leq \epsilon$, need only $T = O\left(\frac{1}{\gamma^2}\log\left(\frac{1}{\epsilon}\right)\right)$ rounds

Adaboost is adaptive

- Does not need to know γ or T a priori
- Can exploit $\gamma_t \gg \gamma$

Understanding the Updates & Normalization

Claim: D_{t+1} puts half of the weight on x_i where h_t was incorrect and half of the weight on x_i where h_t was correct.

Recall $D_{t+1}(i) = \frac{D_t(i)}{Z_t}\, e^{\{-\alpha_t y_i\, h_t(x_i)\}}$

Probabilities are equal!

$$\Pr_{D_{t+1}}[y_i \neq h_t(x_i)] = \sum_{i:y_i \neq h_t(x_i)} \frac{D_t(i)}{Z_t} e^{\alpha_t} = \epsilon_t \frac{1}{Z_t} e^{\alpha_t} = \frac{\epsilon_t}{Z_t}\sqrt{\frac{1-\epsilon_t}{\epsilon_t}} = \frac{\sqrt{\epsilon_t(1-\epsilon_t)}}{Z_t}$$

$$\Pr_{D_{t+1}}[y_i = h_t(x_i)] = \sum_{i:y_i = h_t(x_i)} \frac{D_t(i)}{Z_t} e^{-\alpha_t} = \frac{1-\epsilon_t}{Z_t} e^{-\alpha_t} = \frac{1-\epsilon_t}{Z_t}\sqrt{\frac{\epsilon_t}{1-\epsilon_t}} = \frac{\sqrt{(1-\epsilon_t)\epsilon_t}}{Z_t}$$

$$Z_t = \sum_{i:y_i = h_t(x_i)} D_t(i) e^{-\alpha_t y_i\, h_t(x_i)} = \sum_{i:y_i = h_t(x_i)} D_t(i) e^{-\alpha_t} + \sum_{i:y_i \neq h_t(x_i)} D_t(i) e^{\alpha_t}$$

$$= (1-\epsilon_t) e^{-\alpha_t} + \epsilon_t e^{\alpha_t} = 2\sqrt{\epsilon_t(1-\epsilon_t)}$$

Analyzing Training Error: Proof Intuition

Theorem $\epsilon_t = 1/2 - \gamma_t$ (error of h_t over D_t)

$$err_S(H_{final}) \leq \exp\left[-2\sum_t \gamma_t^2\right]$$

- On round t, we increase weight of x_i for which h_t is wrong.
- If H_{final} incorrectly classifies x_i,
  - Then x_i incorrectly classified by (wtd) majority of h_t's.
  - Which implies final prob. weight of x_i is large.

  Can show probability $\geq \frac{1}{m}\left(\frac{1}{\prod_t Z_t}\right)$

- Since sum of prob. $= 1$, can't have too many of high weight.

  Can show # incorrectly classified $\leq m\ (\prod_t Z_t)$.

  And $(\prod_t Z_t) \to 0$.

Analyzing Training Error: Proof Math

Step 1: unwrapping recurrence: $D_{T+1}(i) = \frac{1}{m}\left(\frac{\exp(-y_i f(x_i))}{\prod_t Z_t}\right)$

where $f(x_i) = \sum_t \alpha_t h_t(x_i)$. [Unthresholded weighted vote of h_i on x_i]

Step 2: $\mathrm{err}_S(H_{final}) \leq \prod_t Z_t$.

Step 3: $\prod_t Z_t = \prod_t 2\sqrt{\epsilon_t(1-\epsilon_t)} = \prod_t \sqrt{1-4\gamma_t^2} \leq e^{-2\sum_t \gamma_t^2}$

Analyzing Training Error: Proof Math

<u>Step 1</u>: unwrapping recurrence: $D_{T+1}(i) = \frac{1}{m}\left(\frac{\exp(-y_i f(x_i))}{\prod_t Z_t}\right)$

where $f(x_i) = \sum_t \alpha_t h_t(x_i)$.

Recall $D_1(i) = \frac{1}{m}$ and $D_{t+1}(i) = D_t(i)\,\frac{\exp(-y_i \alpha_t h_t(x_i))}{Z_t}$

$$D_{T+1}(i) = \frac{\exp(-y_i \alpha_T h_T(x_i))}{Z_T} \times D_T(i)$$

$$= \frac{\exp(-y_i \alpha_T h_T(x_i))}{Z_T} \times \frac{\exp(-y_i \alpha_{T-1} h_{T-1}(x_i))}{Z_{T-1}} \times D_{T-1}(i)$$

… … .

$$= \frac{\exp(-y_i \alpha_T h_T(x_i))}{Z_T} \times \cdots \times \frac{\exp(-y_i \alpha_1 h_1(x_i))}{Z_1}\,\frac{1}{m}$$

$$= \frac{1}{m}\,\frac{\exp(-y_i(\alpha_1 h_1(x_i) + \cdots + \alpha_T h_T(x_T)))}{Z_1 \cdots Z_T} \quad = \frac{1}{m}\,\frac{\exp(-y_i f(x_i))}{\prod_t Z_t}$$

Analyzing Training Error: Proof Math

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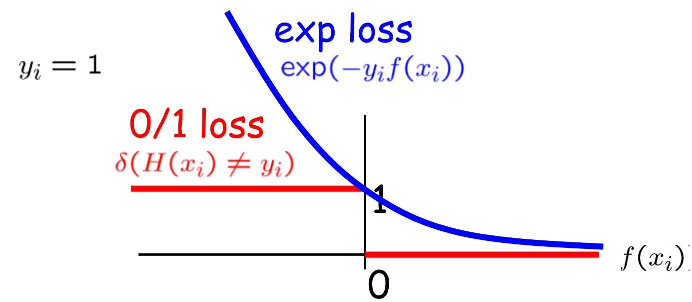
Step 2: $\mathrm{err}_S(H_{final}) \leq \prod_t Z_t$.

$$\mathrm{err}_S(H_{final}) = \frac{1}{m}\sum_i 1_{y_i \neq H_{final}(x_i)}$$

$$= \frac{1}{m}\sum_i 1_{y_i f(x_i) \leq 0}$$

$$\leq \frac{1}{m}\sum_i \exp(-y_i f(x_i))$$

$$= \sum_i D_{T+1}(i) \prod_t Z_t \;\; = \prod_t Z_t.$$



Analyzing Training Error: Proof Math

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Step 2: $\mathrm{err}_S(H_{final}) \leq \prod_t Z_t$.

Step 3: $\prod_t Z_t = \prod_t 2\sqrt{\epsilon_t(1-\epsilon_t)} = \prod_t \sqrt{1-4\gamma_t^2} \leq e^{-2\sum_t \gamma_t^2}$

Note: recall $Z_t = (1-\epsilon_t)e^{-\alpha_t} + \epsilon_t e^{\alpha_t} = 2\sqrt{\epsilon_t(1-\epsilon_t)}$

α_t minimizer of $\alpha \to (1-\epsilon_t)e^{-\alpha} + \epsilon_t e^{\alpha}$

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Generalization Guarantees

Theorem $err_S(H_{final}) \leq \exp\left[-2\sum_t \gamma_t^2\right]$ where $\epsilon_t = 1/2 - \gamma_t$

How about generalization guarantees?

Original analysis [Freund&Schapire'97]

- H space of weak hypotheses; d=VCdim(H)

  H_{final} is a weighted vote, so the hypothesis class is:

  G={all fns of the form $\text{sign}(\sum_{t=1}^{T} \alpha_t h_t(x))$ }

Theorem [Freund&Schapire'97]

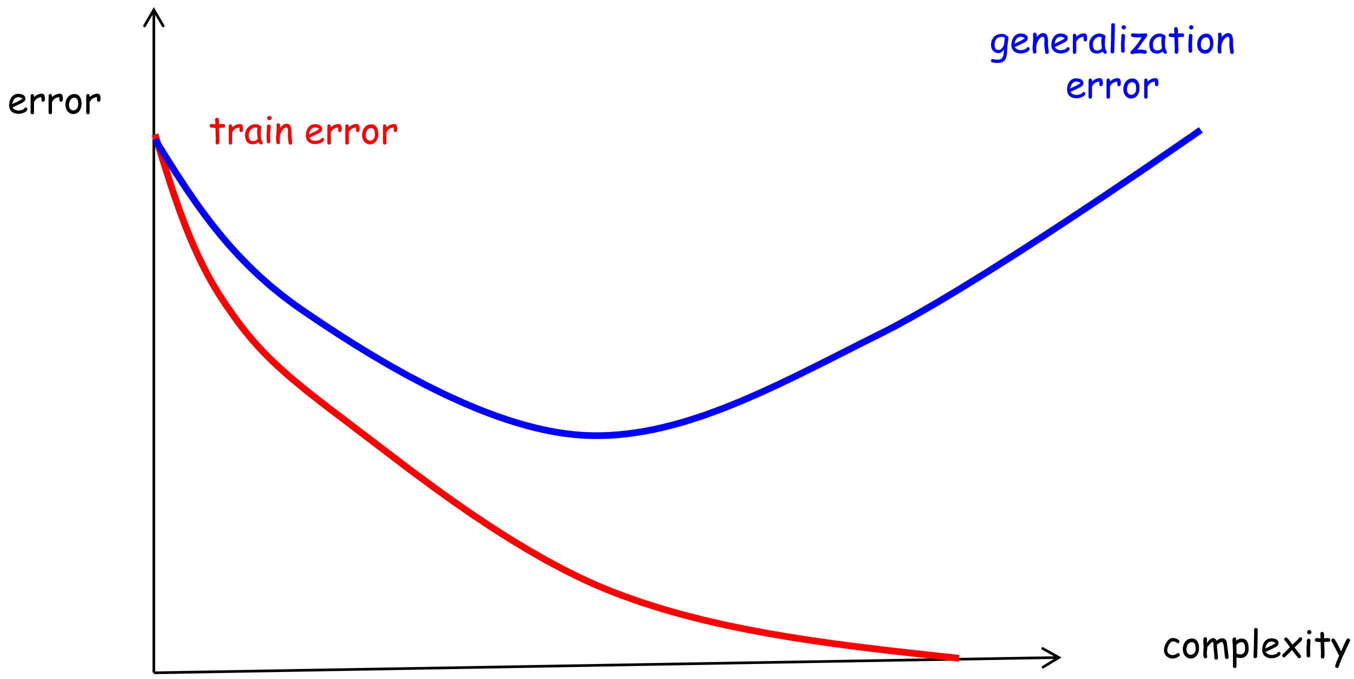
$\forall\, g \in G, err(g) \leq err_S(g) + \tilde{O}\left(\sqrt{\frac{Td}{m}}\right)$ T= # of rounds

Key reason: $\text{VCd}im(G) = \tilde{O}(dT)$ plus typical VC bounds.

Generalization Guarantees

Theorem [Freund&Schapire'97]

$$\forall\, g \in co(H), err(g) \leq err_S(g) + \tilde{O}\left(\sqrt{\frac{Td}{m}}\right)$$ where d=VCdim(H)

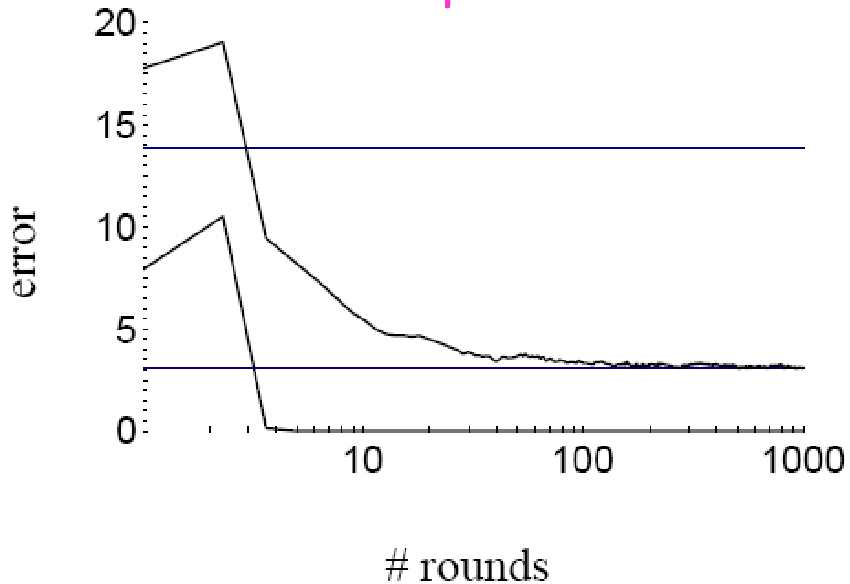


T= # of rounds

Generalization Guarantees

- Experiments with boosting showed that the test error of the generated classifier usually does not increase as its size becomes very large.

- Experiments showed that continuing to add new weak learners after correct classification of the training set had been achieved could further improve test set performance!!!



Generalization Guarantees

- Experiments with boosting showed that the test error of the generated classifier usually does not increase as its size becomes very large.

- Experiments showed that continuing to add more weak learners after correct classification of the training set had been achieved could further improve test set performance!!!

- These results seem to contradict FS'87 bound and Occam's razor (in order to achieve good test error the classifier should be as simple as possible)

How can we explain the experiments?

R. Schapire, Y. Freund, P. Bartlett, W. S. Lee. present in "*Boosting the margin: A new explanation for the effectiveness of voting methods*" a nice theoretical explanation.

Key Idea:

Training error does not tell the whole story.

We need also to consider the classification confidence!!