$$\frac{\partial}{\partial w} = \frac{2w}{2} - \sum_i \alpha_i x_i y_i \Rightarrow$$

$$w = \sum_i x_i y_i \alpha_i$$

$$\frac{\partial}{\partial b} = \sum_i \alpha_i y_i \Rightarrow \sum_i \alpha_i y_i = 0$$

$$\frac{\partial}{\partial \alpha_i} = w^T x_i y_i + b y_i - 1 \Rightarrow$$

$$w^T x_i y_i + b y_i = 1 \Rightarrow \text{(multiply by } y_i\text{)}$$

$$b = y_i - w^T x_i$$

$$\frac{w^T w}{2} = \left(\sum \alpha_i x_i y_i\right)\left(\sum \alpha_i x_i y_i\right)$$

$$= \frac{1}{2}\sum_{i,j} \alpha_i \alpha_j x_i^T x_j y_i y_j$$

$$- \sum_i \alpha_i\left[\left(w^T x_i + b\right) y_i - 1\right] = -\left[\sum_i \alpha_i w^T x_i y_i + b\overbrace{\sum_i \alpha_i y_i}^{0} - \sum_i \alpha_i\right]$$

$$\Rightarrow = \sum_i \alpha_i - \sum_i w^T x_i y_i \alpha_i =$$

$$\sum_i \alpha_i - \left(\sum_i w^T x_i y_i\right)\left(\sum_i w^T x_i y_i\right)$$

$$= \sum \alpha_i - \sum_{i,j} \alpha_i \alpha_j x_i^T x_j y_i y_j$$

Putting the two together $\Rightarrow$

$$\sum \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i \alpha_j x_i^T x_j y_i y_j$$