Extensionality in Type Theory
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Abstract:

Type Theory, ala Martin-Löf, is at the same time a logic, a programming language and a set theory which has an immense potential for certifiable software engineering witnessed by the success of systems like Coq and Agda.

We will focus on the question of the role of extensionality in Type Theory which arises in many forms, e.g. should two functions which are pointwise equal be considered equal? In present day implementations this principle is unprovable and the addition of axioms is not only against the spirit of Type Theory but also destroys the fundamental property of canonicity (e.g. every closed term of type Nat is reducible to a numeral).

We argue that the lack of extensionality is a serious defect of Type Theory both from a foundational point of view, because it destroys the fundamental symmetry of data and codata, but also from a pragmatic point of view, because it prohibits abstraction and hence modularity. We discuss ways to overcome this defect such as our "Observational Type Theory" which is based on the Leibniz principle that objects should be either equal or observably different. We also discuss the relation to higher dimensional Type Theory which arises from applying the Leibniz principle to the equality of sets: since we cannot distinguish isomorphic sets we should identify them. This leads us to rejecting uniqueness of equality proofs and looking for a computationally acceptable form of Voevodsky's univalence axiom.

Host: Robert Harper
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Wednesday, February 16, 2011
Gates Hillman Center 9115
3:00 – 4:30 p.m.