Toward Sense Making with Grounded Feedback

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Abstract. Grounded feedback aims to facilitate sense making by reflecting students' symbolic input in a linked concrete representation that is easier to reason with. Initial studies led to explorations of what prior knowledge is necessary to support that reasoning. Specifically, we tested if it is obvious to students that a sum is larger than its two positive addends. It is not! Thus, concrete representations for sense making may fail because students lack prerequisite knowledge we may assume they have. More generally, these results suggest that skilled qualitative reasoning may often come after, not before, quantitative reasoning.

Keywords: grounded feedback; fraction addition; graphical representations.

Effective instruction elicits students' prior knowledge and facilitates useful connections between what students already know and what they are learning. To that end, my work presents and investigates grounded feedback, in which student inputs (e.g., 5/12) are reflected in a more familiar representation that is easier to reason with (e.g., a fraction bar). Prior work shows experimental support for such feedback over right/wrong immediate feedback (e.g., [1]), but does not provide a full theoretical characterization of grounded feedback or design recommendations for its implementation. My proposed work on grounded feedback will: continue to examine its effects with empirical, controlled classroom studies; explicitly define it and situate it in a theoretical framework; build a theoretical model of how students use grounded feedback to make sense of new information; and delineate design recommendations for its implementation. This paper focuses on the design recommendations.

Our work on grounded feedback examines middle school students learning fraction addition (Fig. 1 shows a tutor example). The grounded feedback consists of rectangular fraction bars that reflect the symbolic values that students enter. This common representation (e.g., [2]) is intended to elicit students' prior knowledge of magnitude and make salient important fractions concepts (e.g., one cannot add fractions by simply adding the numerators and denominators). An experiment with 5th graders showed student learning with grounded feedback, and some benefits over immediate right/wrong feedback [3]. However, that study also revealed students' difficulty interpreting the feedback: they often indicated that a problem was solved even when their proposed sum did not line up with the combined length of the given addends. This finding suggests that 1) the students did not realize that two addends equal their sum and/or 2) some aspect of the representation blocked students' use of that knowledge.

adfa, p. 1, 2011.

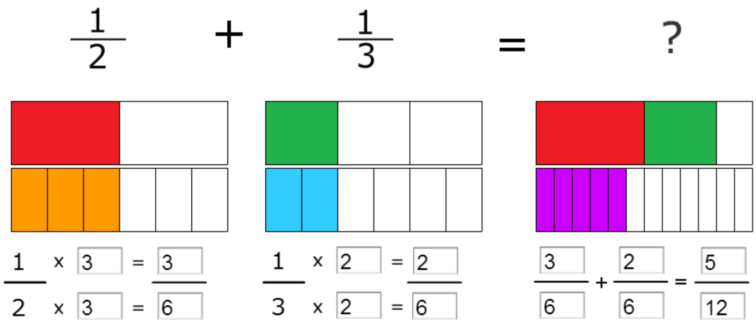

Fig. 1. Fraction Addition Tutor showing correct conversion, incorrect sum (correct sum is 5/6). Top row of fractions and red and green fraction bars are given, second row of bars dynamically shows students' inputs as they are typed in the boxes at the bottom.

A follow-up study at the same school supported hypothesis 2 [4]. 5th grade students in that study saw fraction addition equations on paper, represented with rectangles, symbols, or both. Half of the equations were correct, and half proposed a sum that was smaller than one of the addends. Students indicated if the equations were true or false. The students could tell if fraction addition equations were correct or not when the addends and sum were presented as rectangles, but performance decreased when symbolic fractions were shown with the rectangles. The fraction symbols appear to impede students' recollection or use of the addend-sum relationship.

The current study investigates students' understanding of the addend-sum relationship. 160 fifth-grade students were given 20 minutes for a 34-question test, administered by their teacher. Question order was determined randomly, and half of the test forms were printed in reverse order. This analysis is of the five questions which tested sense making with addition. Problems presented a correct addition equation with positive numbers, with a note that the equation was correct, and asked if the sum was bigger than the two addends (options: True, False, and Can't tell from the information given). Students were randomly assigned to test forms that presented this question either with whole numbers, decimals, fractions, or variables (see Fig. 2). Means for each number type reveal that students do not have a general understanding of the addend-sum relationship (percent correct for: whole numbers, 79; decimals, 75; fractions, 61; and variables, 51). An ANOVA with the total addend-sum score (dependent measure) and test form (fixed factor) showed a significant effect of number type ($p<.01$), indicating that students' recognition of the addend-sum relationship is highly dependent on the type of number in the equation.

These results indicate that many students do not have a general conception of the addend-sum relationship for positive numbers - "addition makes bigger". Because whole numbers and decimals can be compared directly, students can solve those problems by comparing the numbers (e.g., .7 > .083) without knowing the addend-sum relationship. It is harder to directly compare unlike-denominator fractions, and impossible for variables. Students' performance with fractions and variables reveals a lack of understanding that helps explain why students had trouble with the grounded feedback. Students may notice that the purple bar in Fig. 1 is smaller than the red-and-green bar, but if they do not recognize that two addends must equal their sum, they may not interpret the rectangles as indicating that 5/12 is incorrect. A corollary is, for positive numbers, that the sum is always larger than each addend. Knowledge of this relationship is not fully in place even for whole numbers. The missing understanding of the addend-sum relationship may partially explain the results from [4]. Given students' difficulty in recognizing the addend-sum relationship with fractions, perhaps those symbols distracted students from using that relationship. Additionally, this work suggests that in math, qualitative understanding might come after, not before, quantitative expertise (e.g., students may not understand the addend-sum relationship until they have extensive practice adding numbers of many types).

This body of work highlights the difficulty in designing feedback that elicits students' prior knowledge toward greater sense making (and thus greater understanding). Grounded feedback is intended to help students evaluate their own work by presenting a feedback representation that students can easily reason with. For example, it is easier for students to compare two fractions when they are represented as rectangles instead of symbols. Still, students must take the outcome of that comparison and reason with it, and if the concepts or skills needed for such reasoning are not fully in place, students may not be able to make full use of the grounded feedback. On the other hand, students' work with grounded feedback may reveal gaps in their prior knowledge that would not be evident from their interaction with immediate right/wrong feedback. If grounded feedback tutors can detect what type of prior knowledge students are missing, embedded activities could provide targeted instruction on those concepts, which students could then practice as part of the normal tutor.

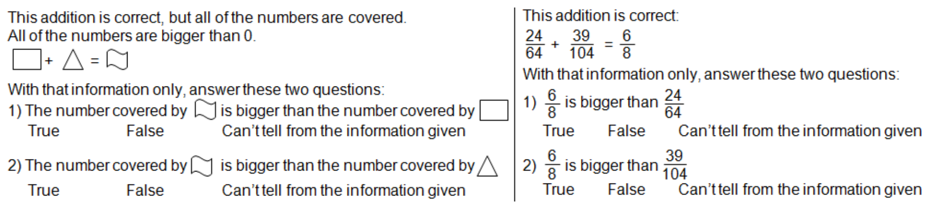

Fig. 2. Example addend-sum questions, with variables (left) and fractions (right)

References and Acknowledgements

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