Conceptual Scaffolding to Check One's Procedures

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Abstract. Our tutoring system for fraction addition uses dynamic pictorial representations that reflect student-inputted quantities. However, students had difficulty interpreting the pictorial feedback. Surprisingly, we found that including symbolic numbers with the pictures decreased performance. We hypothesize that students' difficulty may stem from insufficient domain knowledge, or insufficient metacognitive skills to use conceptual knowledge to check their work.

Keywords: graphical representation; fraction addition; symbolic fractions.

One goal of education is to foster learning with deep understanding, and one demonstration of this understanding is to check the outcome of a procedure against conceptual knowledge. For example, while tempting to say 3/4 + 1/7 = 4/11, conceptual reasoning reveals the fallacy: 3/4 is greater than half while 4/11 is smaller. Pictorial representations of each fraction may speed these comparisons. Prior work found benefits for conceptually-based pictorial feedback above right/wrong immediate feedback for college students learning algebra [1]. However, [3] found that while 6^{th} grader's math performance improved with pictorial scaffolds, 4^{th} grader's performance decreased, likely because the younger students were confused by the representations. It appears that conceptual scaffolds have great potential but are not uniformly useful.

Our tutor uses *grounded feedback*: student inputs are in the to-be-learned representation, while a linked representation reflects students' inputs in a more concrete form. In our tutor, students input numeric symbols and the tutor displays corresponding fraction bars (see fig. 1). Grounded feedback allows students to see the consequences of their errors and thus may promote students' evaluation of their own work (e.g., a student may guess that 3/6 + 2/6 = 5/12, but the fraction bars show 5/12 is too small). The link direction ensures that students engage with the more difficult to-be-learned representation instead of directly

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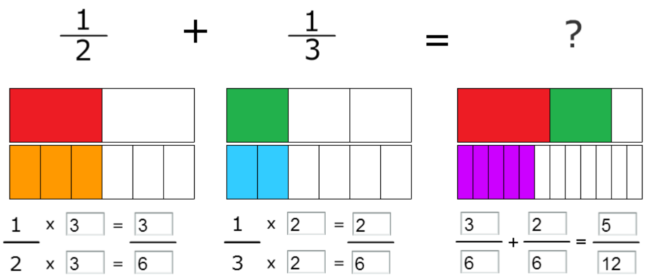

Fig. 1. Fraction Tutor. Top row of fractions and red and green fraction bars are given, second row of bars dynamically shows students' inputs as they are typed in boxes at the bottom.

manipulating the already-understood feedback representation.

Our prior work found learning gains for the grounded feedback tutor, but results also indicate that students found the feedback unclear. Participants in a think-aloud study used the fraction bar feedback to fix their own mistakes [5]. Further, a classroom study found learning gains [6]. However, the 90 5th grade students using the tutor in that study did not seem to use the fraction bars to check their work - they often clicked the "done" button when the fraction bars did not line up [6]. The next year, a paper test with 5th graders assessed the difficulty of fraction addition and fraction equivalence questions in four formats: three included fraction bars, and one was a numbers-only control (equivalence items asked if one fraction was greater than, equivalent to, or smaller than another; addition examples in fig. 2) [7]. For both question types, students performed better with the bars than with numbers alone. However, while all three fraction-bar formats were equally helpful for fraction equivalence, they were significantly different from each other for fraction addition. As the salience of the numbers increased, scores decreased (fig. 3). Students' success with the fraction equivalence items and with the pictures-only addition items indicates proficiency at using

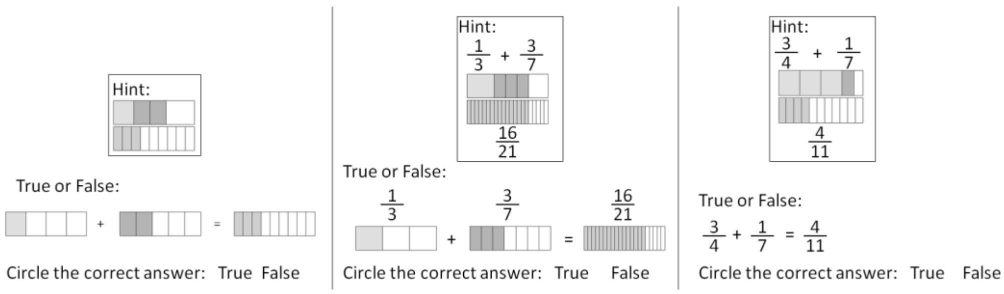

Fig. 2. Addition (from left): Pictures Only, Pictures & Numbers, Half Pictures & Numbers.

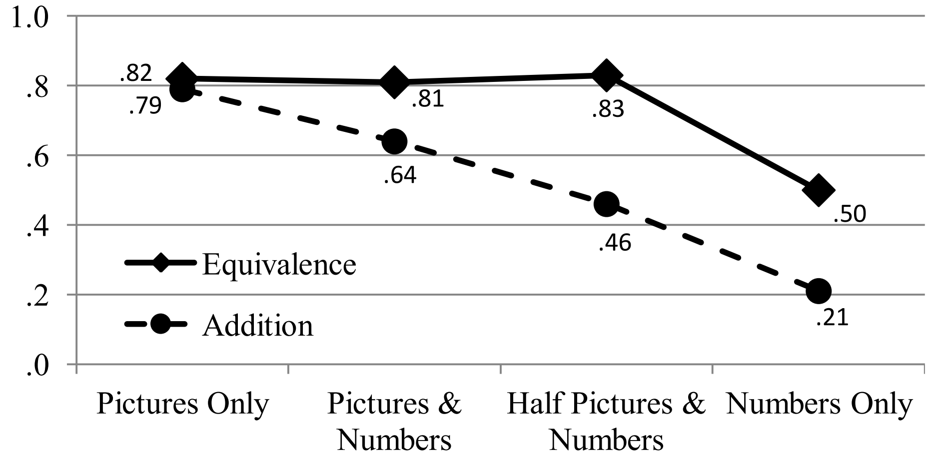

Fig. 3. Mean Scores (max. 1) on Fraction Equivalence and Addition Items by Scaffold Type.

the fraction bars to determine if two quantities are equal. Why didn't students use that skill for the other addition items? The incorrect addition items all used the common mistake of adding both numerators and both denominators. Perhaps the tempting misconception overrode the fraction bars' conceptual hint. Or, maybe students misunderstood the meaning of the equals sign and then considered the conceptual hint to be irrelevant. [2] found that 6^{th}-8^{th} grade students looking at a problem such as 3 + 4 = 7 were more likely to interpret the equals sign to mean 'write answer here' than 'both sides are equivalent.' Perhaps on the addition questions with numbers students interpreted the equals sign to mean 'put the answer here' instead of 'the two sides are equal'. In that case the fraction bars showing a sum that was not equal to the two addends would not alert the student that the answer was wrong.

This work leads to questions about the role of procedural and conceptual knowledge in problem solving. The premise of grounded feedback is that students can use their conceptual knowledge to identify errors that result from faulty procedures. However, doing that requires domain-specific skill to interpret the conceptual hint, and metacognitive skill to check one's work and fix errors before moving on. Our next study will attempt to find out if tutoring on those skills improves performance and learning with the grounded feedback tutor.

We propose a 2x2 study on grounded feedback with 1) metacognitive instruction (checking one's work with conceptual aids) and 2) domain-specific instruction (meaning of the equals sign). Figure 4 shows possible examples. The metacognitive instruction demonstrates using a conceptual aid, but does not explain why a number smaller than 15 cannot be the sum of 15+27. The domain-specific instruction explains the meaning of the equals sign, but does not demonstrate conceptual ways to check for inequalities. This experiment will determine if students need more domain-specific knowledge or more metacognitive knowledge (or both) to benefit more from the grounded feedback.

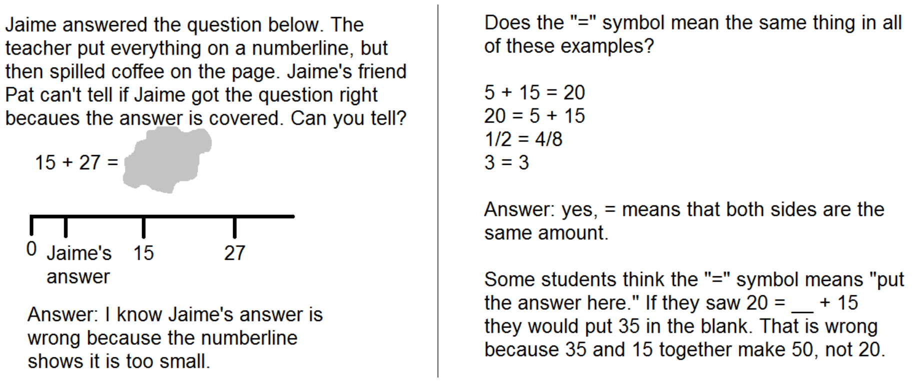

Fig. 4. Proposed Metacognitive Instruction (left) and Domain-specific Instruction (right).

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