Artificial Intelligence
Guest Lecture:
Game Theory I

Ariel Procaccia
Multiagent systems

An Introduction to MultiAgent Systems

Yoav Shoham, Kevin Leyton-Brown

Second Edition

Michael Wooldridge
MULTIAGENT SYSTEMS

Chapters of the Shoham and Leyton-Brown book:

1. Distributed constraint satisfaction
2. Distributed optimization
3. Games in normal form
4. Computing solution concepts of normal-form games
5. Games with sequential actions
6. Beyond the normal and extensive forms
7. Learning and teaching
8. Communication
9. Social choice
10. Mechanism design
11. Auctions
12. Coalitional game theory
13. Logics of knowledge and belief
14. Probability, dynamics, and intention

Legend:
- “Game theory”
- Not “game theory”

Game Theory I
MULTIAGENT SYSTEMS

Mike Wooldridge’s 2016 publications:

2016


- [c206] Oskar Skibski, Szymon Matejczyk, Tomasz P. Michalak, Michael Wooldridge, Makoto Yokoo: k-Coalitional Cooperative Games. AAMAS 2016: 177-185


- [c201] Haris Ariz, Paul Harrenstein, Jérôme Lang, Michael Wooldridge: Boolean Hedonic Games. KR 2016: 166-175


Game Theory I

Carnegie Mellon University
NORMAL-FORM GAME

• A game in normal form consists of:
  o Set of players $N = \{1, \ldots, n\}$
  o Strategy set $S$
  o For each $i \in N$, utility function $u_i : S^n \rightarrow \mathbb{R}$: if each $j \in N$ plays the strategy $s_j \in S$, the utility of player $i$ is $u_i(s_1, \ldots, s_n)$

• Next example created by taking screenshots of
  http://youtu.be/jILgxeNBK_8
Selling ice cream at the beach.

One day your cousin Teddy shows up.

His ice cream is identical!

You split the beach in half; you set up at 1/4.

50% of the customers buy from you.

50% buy from Teddy.

One day Teddy sets up at the 1/2 point!

Now you serve only 37.5%!
The Ice Cream Wars

- \( N = \{1,2\} \)
- \( S = [0,1] \)
- \( u_i(s_i, s_j) = \begin{cases} 
\frac{s_i + s_j}{2}, & s_i < s_j \\
1 - \frac{s_i + s_j}{2}, & s_i > s_j \\
\frac{1}{2}, & s_i = s_j 
\end{cases} \)

- To be continued...
THE PRISONER’S DILEMMA

• Two men are charged with a crime
• They are told that:
  o If one rats out and the other does not, the rat will be freed, other jailed for nine years
  o If both rat out, both will be jailed for six years
• They also know that if neither rats out, both will be jailed for one year
# The Prisoner’s Dilemma

<table>
<thead>
<tr>
<th></th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>-1, -1</td>
<td>-9, 0</td>
</tr>
<tr>
<td>Defect</td>
<td>0, -9</td>
<td>-6, -6</td>
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What would you do?
Understanding the Dilemma

• Defection is a dominant strategy
• But the players can do much better by cooperating
• Related to the tragedy of the commons
In Real Life

- Presidential elections
  - Cooperate = positive ads
  - Defect = negative ads
- Nuclear arms race
  - Cooperate = destroy arsenal
  - Defect = build arsenal
- Climate change
  - Cooperate = curb CO$_2$ emissions
  - Defect = do not curb
ON TV

http://youtu.be/S0qjK3TWZE8
THE PROFESSOR’S DILEMMA

<table>
<thead>
<tr>
<th>Professor</th>
<th>Make effort</th>
<th>Slack off</th>
</tr>
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<tbody>
<tr>
<td>Class</td>
<td>Listen</td>
<td>Sleep</td>
</tr>
<tr>
<td>Make effort</td>
<td>$10^6, 10^6$</td>
<td>$-10, 0$</td>
</tr>
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<td>$0, -10$</td>
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Dominant strategies?
Nash equilibrium

• Each player’s strategy is a best response to strategies of others

• Formally, a Nash equilibrium is a vector of strategies $s = (s_1, \ldots, s_n) \in S^n$ such that

$$\forall i \in N, \forall s'_i \in S, u_i(s) \geq u_i(s'_i, s_{-i})$$
## Nash Equilibrium

### Poll 1: How many Nash equilibria does the Professor’s Dilemma have?

1. 0  
2. 1  
3. 2  
4. 3

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Game Theory I
Nash equilibrium

http://youtu.be/CemLiSI5ox8
Russel Crowe was wrong

Hey, Dr. Nash, I think those guys over there are eyeing us. This is like your Nash equilibrium, right? One of them is hot, but we should each flirt with one of her less-desirable friends. Otherwise we risk coming on too strong to the hot one and just driving the group off.

Well, that's not really the sort of situation I wrote about. Once we're with the ugly ones, there's no incentive for one of us not to try to switch to the hot one. It's not a stable equilibrium.

Crap, forget it. Looks like all three are leaving with one guy.

DAMMIT, FEYNMAN!
END OF THE ICE CREAM WARS

The PLAN

Day 3 of the ice cream wars...

75%

Teddy sets up south of you!

You go south of Teddy.

Eventually...

Eventually...

Nash Equilibrium

Game Theory I
This is why competitors open their stores next to one another!
# Rock-Paper-Scissors

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Nash equilibrium?
**Mixed Strategies**

- A mixed strategy is a probability distribution over (pure) strategies.
- The mixed strategy of player $i \in N$ is $x_i$, where
  \[ x_i(s_i) = \Pr[i \text{ plays } s_i] \]
- The utility of player $i \in N$ is
  \[ u_i(x_1, \ldots, x_n) = \sum_{(s_1, \ldots, s_n) \in S^n} u_i(s_1, \ldots, s_n) \cdot \prod_{j=1}^{n} x_j(s_j) \]
**Exercise: Mixed NE**

- **Exercise:** player 1 plays \(\left(\frac{1}{2}, \frac{1}{2}, 0\right)\), player 2 plays \(\left(0, \frac{1}{2}, \frac{1}{2}\right)\). What is \(u_1\)?

- **Exercise:** Both players play \(\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)\). What is \(u_1\)?

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Exercise: Mixed NE

- Poll 2: Which is a NE?

1. \( \left( \left( \frac{1}{2}, \frac{1}{2}, 0 \right), \left( \frac{1}{2}, \frac{1}{2}, 0 \right) \right) \)

2. \( \left( \left( \frac{1}{2}, \frac{1}{2}, 0 \right), \left( \frac{1}{2}, 0, \frac{1}{2} \right) \right) \)

3. \( \left( \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right), \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \right) \)

4. \( \left( \left( \frac{1}{3}, \frac{2}{3}, 0 \right), \left( \frac{2}{3}, 0, \frac{1}{3} \right) \right) \)

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Any other NE?
NASH’S THEOREM

• Theorem [Nash, 1950]: In any (finite) game there exists at least one (possibly mixed) Nash equilibrium

What about computing a Nash equilibrium?
**Does NE make sense?**

- Two players, strategies are \{2, ..., 100\}
- If both choose the same number, that is what they get
- If one chooses \(s\), the other \(t\), and \(s < t\), the former player gets \(s + 2\), and the latter gets \(s - 2\)
- **Poll 3:** What would you choose?
SUMMARY

• Terminology:
  o Normal-form game
  o Nash equilibrium
  o Mixed strategies

• Nobel-prize-winning ideas:
  o Nash equilibrium 😊