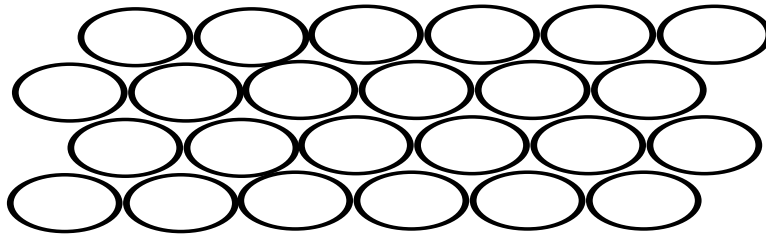


Puzzle 38: Crush the Rebellion

Ten rebel encampments have sprung up on the plane of Ushan. The Martian Federation plans to send flying saucers to deal with them. They are pretty ruthless. They will simply land on top of the encampments. The encampments are small and the saucers are huge. It must be done simultaneously, or the rebels will flee. Also, the saucers must not overlap when they land. Can the Martians prevail?

Mathematically the encampments are points in the plane and the saucers are non-overlapping disks of equal radius.

Solution: Consider the hexagonal packing of unit disks in the plane:



It is well known that in this packing, the circles cover $\frac{\pi\sqrt{3}}{6} = 0.906899\dots$ of the plane. Suppose that we place this packing randomly on the plane e.g. by choosing one circle C , choosing a point $x \in C$ and then shifting the center of C to x . Let $z_i = 1$ if the i th encampment is covered by this random placement of circles. (We don't need to assume an infinite number of saucers for this). Let $Z_1 + Z_2 + \dots + Z_{10}$ be the number of encampments covered. Then $\mathbf{P}(Z_i = 1) = \frac{\pi\sqrt{3}}{6}$ for $i = 1, 2, \dots, 10$ and so $\mathbf{E}(Z) > 9$ and so there must be a way of placing the saucers so that more than 9 i.e. 10 are covered.

Acknowledgements: We heard of this problem in a talk by Peter Winkler in the 2012 Erdős conference in Memphis.

We thank the following people for their solutions and attempted solutions: Jeremy Alm, Hui Han Chin, Jeshon Shrestha, Raghu Srinivasan, Aishwarya Venkatesh, Ke Xi.