

Zeroise me

A long worm crawls into a cosmetic veterinary surgery, complaining of a problem with 1's. A worm can be thought of as a string of 0's and 1's and the most beautiful worm is $00000 \cdots 0$. The procedure for removing 1's is complicated. If there is a 1 at the right hand end (where the tail is), then the surgeon can remove this 1 and place a 0 or a 1 at the left hand end (where the head is). If however there is a 0 at the right hand end then the surgeon can remove it, but he has no control of what appears in its place at the left hand end. The surgeon claims a 100% success rate. Do you believe him?

Solution: There were many solutions sent in. Here is a simple one from Peter Winkler:

“Think of the operation as a sequence of phases, each involving n steps (segment replacements). After each phase the worm has a value as a binary number. The surgeon begins each phase by converting 1's to 0's until the adversary converts a 0 to a 1; after that point the surgeon preserves all 1's. If the adversary never flips a zero, the string becomes all zeroes and the operation is over. Otherwise, the value of the binary number will be higher at the end of the phase than it was at the beginning.

Eventually the worm will become all 1's, and the surgeon, still following the algorithm, wins in n steps later. This takes time at most $n2^n$. The surgeon can do better by shifting phases in such a way as to jump ahead to higher binary values, but long worms should expect to spend a lot of time on the operating table!”

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