

Nunchuk Supreme

To graduate from the Mangan KungFu Academy, one has to master the use of the Nunchuk Supreme. This is a powerful weapon that that can be used to disable an unlimited number of assailants. Think of it as a very long stick. The Nunchuk is thrown at the assailants and rotates in the air as it is thrown. It will strike an assailant, disable him/her and then using this person as a pivot, rotate about them and strike a second person and then rotate about the second person until it strikes a third person and rotates about them and so on until all of the assailants have been dealt with. Being disabled from fighting does not prevent being struck again by the Nunchuk and causing a change in the pivot of rotation.

Thinking of the assailants as fixed points in the plane, is it always possible to choose a first assailant to strike, so that if no three assailants are collinear and the Nunchuk is long enough, then all assailants will be struck down?

Solution: The nunchuk thrower must choose a point (=person) and an orientation of the line (=nunchuk) so that the numbers of points on either side of the line differ by at most one. Observe that as the line rotates, the numbers of points to the left/right of the line stay the same. Then we wait until the line has turned through an angle π . If the number of points was odd, then we end up in the same state we started in. If it was even, then in the final state the line may differ from the starting one, but only by translation across an empty region. In either case each point will have crossed from the left side of the line to the right side of the line or vice versa. This means that every point has crossed the line i.e. has been used as a pivot.

This problem was inspired by
http://michaelnielsen.org/polymath1/index.php?title=Imo_2011