Seating:
A theatre has 137 seats and every ticket is sold for this evening's performance. The theatre goers arrive in random order. Unfortunately the 17th theatre goer is drunk and goes to a random seat, instead of his assigned seat. He sits down and immediately falls asleep. Subsequently, when a patron arrives and finds their seat already taken, they take a random seat. What is the probability that the last patron finds their assigned seat empty?
Solution We can ignore the first 16 patrons, they are sitting in their correct seats. Now instead of having another 121 patrons, let us assume that there are another n patrons and the first one is drunk.
Let us consider the following general situation: There are m patrons and m seats left. When $m = n$ let π_m be the probability that the last patron will be correctly seated. When $m < n$ let π_m be the probability that the last patron will be correctly seated given that a uniform single choice of the first $m-1$ patron's has his/her seat already occupied and the m'th patron's seat isn't. Then

$$\pi_m = \left(1 - \frac{2}{m}\right)\pi_{m-1} + \frac{1}{m}. \tag{1}$$

Here $1/m$ is the chance that the first patron chooses the seat not allocated to the last $m-1$ patrons. The factor $1 - 2/m$ is the chance that the first patron does not choose this seat and does not choose the last person's seat. Given this, we find we are in the same situation with $m-1$ patrons.
Finally note that $\pi_2 = \frac{1}{2}$ since the second last patron will have to choose equally from the last two seats.
That $\pi_n = \frac{1}{2}$ now follows inductively from (**??**).
Alternative Solution The process is equivalent to the process where each patron, on finding their seat occupied, forces the occupant out and sits on their seat (since patrons are indistinguishable). In this case, the drunk is the only occupant shuttling around. Now, it doesn't really matter that the drunk is shuttling around, the $n-2$ patrons between the drunk and the last patron are going to occupy their seats anyway. So we reorder the scheme of things, making the drunk choose his seat after the $n-2$ patrons are in their seat. The drunk has 2 choices, either sit on his own seat or on the other seat remaining, and thus the probability that the last patron finds his seat empty is $\frac{1}{2}$.
This is a lot neater than the first solution. Akshay Pundle found this solution on Gil Kalai's blog some time ago.

This was a popular puzzle. Thanks to the following for their attempts: Nathan Abraham, Dan Dima, Pryam Dutta, Carlos Espana, Ronald Gallagher, Shan Huang, Cijo Jose, Jyo, Jahooma, Karthik Lakshmanan, Richard Peng, Zachary Rivkin, Michael Schuresko, Ratandeep Singh and Yujia Zhai