Searching for the truth: A wise but short-sighted toad is looking for the Truth. The toad sees no more than 1 meter away while the Truth is located in the plane at distance at most D meters from the toad. The toad can make jumps, each of length at most 1 meter and after each jump she knows if she came closer to the Truth or not. Show that the toad can find the Truth (i.e. get within distance 1 meter) in at most $D + o(D)$ jumps, for large D.

Solution The toad should start by jumping one meter in some direction, then jumping right back.

She should repeat this K times (K to be determined later), equally spaced angles along the circle. This takes a total of $2K$ jumps.

At the end of this process, by noticing which of the K directions caused her to get closer to the Truth and which didn't (they form two contiguous and complementary sets), she can determine the direction of the Truth (relative to the starting point) to within $2\pi/K$ degrees. Namely, she can identify a slice of the D-circle, with angle $2\pi/K$, in which the Truth is guaranteed to lie.

She should then start and keep jumping along the bisector of that slice, until she notices she is no longer getting closer to the Truth. This will take no more than D jumps. At that point, based on simple geometry, the Truth is no further than $\pi D/K$ meters away. Now all she has to do is repeat the whole process recursively.

Let $T(D)$ be the maximum number of necessary jumps. Then the above process satisfies: $T(D) = 2K + D + T(\pi D/K)$. If we now choose, say, $K = \sqrt{D}$, we get $T(D) = 2\sqrt{D} + D + T(\pi\sqrt{D})$. $\sqrt{D}$ is of course $o(D)$. By induction, for large enough D, $T(D)$ is clearly $< 2D$, and therefore $T(\pi\sqrt{D}) < 2\pi\sqrt{D}$, which is also $o(D)$. Therefore, $T(D) = D + o(D)$.

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